

- Re-write the decimal  $0.\overline{25}$  as a quotient of two integers.
- Let  $N$  denote  $2^{2015}$ . Write each of the following in terms of  $N$ .
  - $2^{2016}$
  - $2^{2018} - 2^{2016}$
  - $4^{2015}$
  - $2^{2014}$
- Simplify (or rationalize) each of the following.
  - $\frac{x^3 - 9x}{x^2 - 7x + 12}$
  - $\frac{x^2 - 2x - 8}{x^3 + x^2 - 2x}$
  - $\frac{\frac{1}{x} - \frac{1}{5}}{\frac{1}{x^2} - \frac{1}{25}}$
  - $\frac{9 - x^{-2}}{3 + x^{-1}}$
  - $\frac{5 + \sqrt{5}}{5 - \sqrt{5}}$
  - $\log_{10}(120a^3) - (\log_{10} 3a + 2 \log_{10} 2a)$
- Assume that all variables represent positive numbers. Write each of the following expressions in the form  $c a^p b^q$  where  $c, p, q$  are numbers:
  - $\frac{(2a^2)^3}{b}$
  - $\sqrt{9ab^3}$
  - $\frac{a \left(\frac{2}{b}\right)}{\frac{3}{a}}$
  - $\frac{ab - a}{b^2 - b}$
  - $\frac{a^{-1}}{(b^{-1})\sqrt{a}}$
  - $\left(\frac{a^{2/3}}{b^{1/2}}\right)^2 \left(\frac{b^{3/2}}{a^{1/2}}\right)$
- Solve the quadratic equation  $3x^2 + 4x - 1 = 0$ , **by completing the square**. Check your solutions using exact values.
- Solve each of the following equations over the real numbers. Use exact values, and show all steps. Make sure to check your solution(s).
  - $x^2 + 59 = 16x$
  - $125x + 5x^3 = 40x^2$
  - $\log_2(x - 3) - \log_2(x + 1) = 1$
  - $\sqrt{2x + 10} + \sqrt{x + 7} = 4$
  - $5^{x+2} = 2^{2x-3}$
  - $4^{x+1} - 9 \cdot 2^{x+1} = -8$
  - $3 \cdot 2^{2x-1} = 5^{2-x}$
  - $-\cos 2x = \sin x$
  - $\frac{2}{5} \ln(3x - 1) = -2$
  - $\sin 3x \cos 3x = \frac{\sqrt{3}}{4}$
  - $\log_x(20 - x) = 2$
  - $\cos x - \sqrt{3} \sin x = -\sqrt{2}$
- Solve each of the following inequalities.
  - $x^2 + 36 \leq 12x$
  - $x^2 - 4x - 5x < 0$
  - $\frac{2x - 3}{x + 5} \geq -11$
- Perform each of the following divisions.
  - $2x^5 + x^4 - 10x^3 - 2x^2 + 14x - 7$  by  $x^2 + x - 2$
  - $x^5 - 1$  by  $x + 3$
- Re-write  $\log_2 3 - \log_4 6$  as a single logarithm.
- Suppose that  $a$  and  $b$  are numbers such that  $a + b = 20$ . Find the
  - smallest value of  $3a^2 + 2b^2$
  - greatest value of  $a^2 - 3b^2$
  - greatest value of  $a - b^2$ .
- If we set the price of a ticket to \$20, we can sell 600 tickets. If we raise the price by  $x$  dollar,  $4x$  less people will buy the ticket. What is the highest possible revenue that we can obtain?
- Find the equation of the straight line passing through the intersection of the circles  $(x + 2)^2 + (y + 2)^2 = 50$  and  $(x - 2)^2 + (y - 1)^2 = 25$ .
  - Find an equation of the tangent line drawn to the graph of  $6y + x^2 + y^2 + 33 = 14x$  at the point  $(10, -7)$ .
- Find an equation for all tangent lines drawn to the graph of  $y = \frac{3}{2}x^2 - x + 3$  from the point  $P(1, -10)$ .
- Suppose that at time  $t$ , (where  $t$  is measured in hours) a sample contains  $Q(t) = 4.5(0.95^{3t})$  grams of a certain substance. How long does it take for this substance to decrease to half of its original quantity?

15. Graph each of the following.

a)  $f(x) = (3x + 24)(x + 5)(x + 8)(x + 1)(5 - x)^2(7 - x)$

b)  $10x + x^2 + y^2 = 6(y - 5)$

c)  $f(x) = \frac{49 - x^2}{2x + x^2 - 35}$

d)  $f(x) = \frac{3(x + 1)^2(x - 5)}{(x - 1)(x + 1)^8}$

e)  $f(x) = \frac{-2(x + 2)x(x - 2)^3(x - 3)^2}{(x + 1)^2x^2(x - 2)^2(x - 3)^2}$

f)  $f(x) = x^5 - 5x^3$

g)  $f(x) = \frac{3x - 1}{x + 5}$

h)  $f(x) = -2\sqrt{x + 4} - 5$

16. Simplify each of the following expressions.

a)  $\log_9\left(\frac{1}{27}\right)$

d)  $\log_3(9^k)$

g)  $25^{\log_5 7}$

j)  $e^{-2\ln 7}$

b)  $\log_{16} 4$

e)  $\log_{64}\left(\frac{1}{16}\right)$

h)  $\log_{\sqrt{27}}\left(\frac{1}{9}\right)$

k)  $3^{-2\log_3 2}$

c)  $\log_3(3^{21})$

f)  $1 + 2\log_2 3 - \log_2 36$

i)  $e^{2\ln 5}$

l)  $\log_2 5 - \log_2 40$

17. Which of the following is NOT equivalent to  $\log_8\left(\frac{50}{3}\right)$ ?

A)  $\frac{\ln\left(\frac{50}{3}\right)}{\ln 8}$

B)  $\frac{\ln 50 - \ln 3}{\ln 8}$

C)  $\frac{\ln 50 - \ln 3}{3\ln 2}$

D)  $\frac{2\ln 5 + \ln 2 - \ln 3}{3\ln 2}$

E)  $\frac{2\ln 5 - \ln 3}{3}$

18. Find the exact value for each of the following expressions.

a)  $\cos 22.5^\circ$

b)  $\cos 15^\circ \cos 75^\circ$

c)  $\frac{\tan 65^\circ - \tan 5^\circ}{1 + (\tan 65^\circ) \tan 5^\circ}$

d)  $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

e)  $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

19. Suppose that  $f(x) = 3 - x^2$  and  $g(x) = 2x - 1$ . Compute each of the following.

a)  $f(4) + g(4)$

b)  $\frac{g(2)}{f(2)}$

c)  $f(g(-1))$

d)  $g(f(-1))$

e)  $f(g(x))$

f)  $g(f(x))$

20. Find the domain for each of the following functions.

a)  $f(x) = 2^{x-1}$

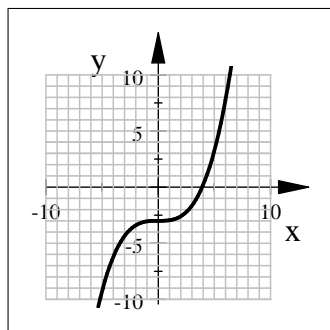
b)  $f(x) = \sqrt{10 - x^2}$

c)  $f(x) = \ln(10 - x^2)$

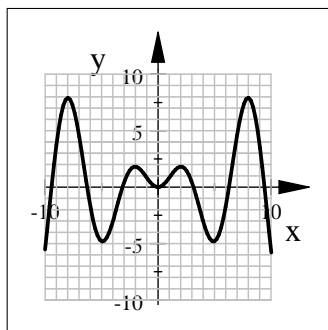
d)  $f(x) = \frac{1}{\ln(10 - x^2)}$

21. Given the graph of the function  $f(x)$ , sketch the graph of the inverse relation,  $f^{-1}(x)$  in the same coordinate system.

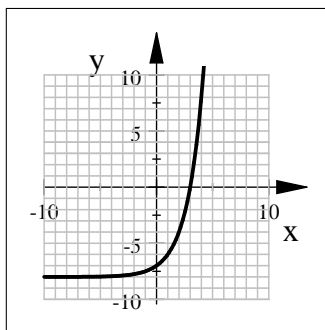
a)



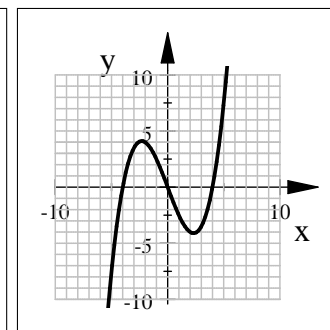
b)



c)



d)



22. Find an equation for the inverse of each of the following functions.

a)  $f(x) = 3^{5x-1}$

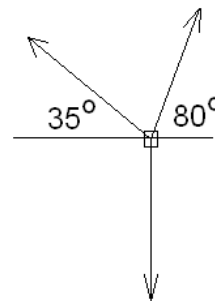
b)  $f(x) = \frac{x + 4}{3x - 5}$

c)  $f(x) = \ln(2x - 1)$

23. Consider the vectors  $\underline{u} = 3\underline{i} + 4\underline{j}$  and  $\underline{v} = 8\underline{i} - 15\underline{j}$ . Find each of the following.

- a)  $-2\underline{u}$     b)  $\|\underline{u}\|$     c)  $\|\underline{v}\|$     d)  $\underline{u} + \underline{v}$     e)  $3\underline{u} - 2\underline{v}$     f)  $\underline{u} \cdot \underline{v}$   
 g)  $(\underline{u} + \underline{v}) \cdot (\underline{u} + \underline{v})$     h) Find the angle formed by the vectors  $\underline{u}$  and  $\underline{v}$ .

24. An object is held by ropes as shown on the picture. Find the forces in the ropes if the object weighs 100 N.



25. Find the exact value of  $\sin \alpha$  where  $\alpha$  is the angle formed by the common tangent lines drawn to the graphs of  $(x - 4)^2 + y^2 = 16$  and  $x^2 + y^2 = 25$ .

26. Let  $A_1$  and  $A_2$  denote the area of two circles,  $C_1$  and  $C_2$ , respectively. Find the ratio  $\frac{A_1}{A_2}$  if an arc subtended by a central angle of  $45^\circ$  in  $C_1$  is as long as an arc subtended by a central angle of  $30^\circ$  in  $C_2$ .

27. Consider the functions  $f(x) = \log_3 x$  and  $g(x) = \log_{1/3} x$ .

- a) Graph these functions in the same coordinate system.    b) What kind of a symmetry do you notice?  
 c) What is the connection between these two functions? Justify your answer using algebra.

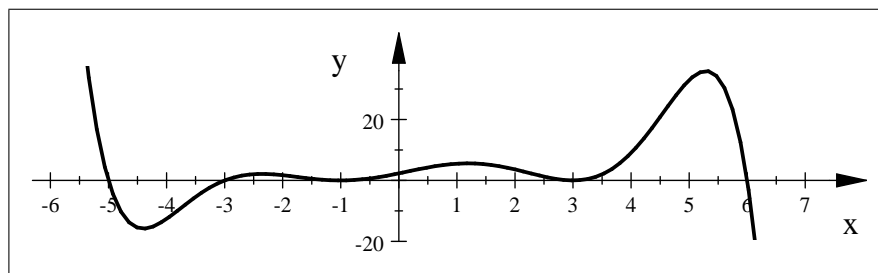
28. Redo problem 27. with the functions  $f(x) = 2^x$  and  $g(x) = \log_2 x$ .

29. Redo problem 27. with the functions  $f(x) = 2^x$  and  $g(x) = \left(\frac{1}{2}\right)^x$ .

30. Let  $C_1$  and  $C_2$  be circles defined by  $x^2 + y^2 = 64$  and  $(x - 10)^2 + y^2 = 9$ , respectively. Let  $t_1$  and  $t_2$  be the common tangent lines drawn to the circles.

- a) Find the coordinates of the point where  $t_1$  and  $t_2$  intersect each other.  
 b) Find an approximation for the acute angle formed by  $t_1$  and  $t_2$ .  
 c) Compute the exact value of the length of the line segment  $\overline{PQ}$  where  $P$  and  $Q$  are the points of tangency on  $t_1$ .

31. The picture below shows the graph of a polynomial function,  $f(x)$ .

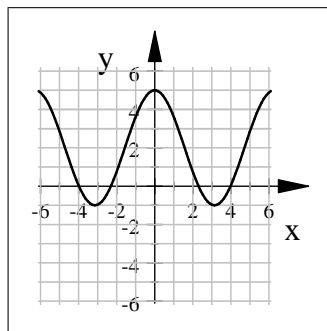
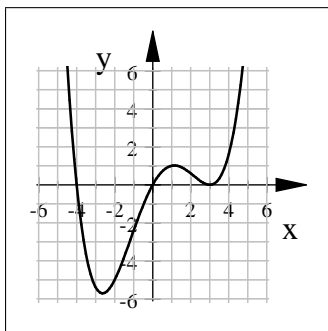
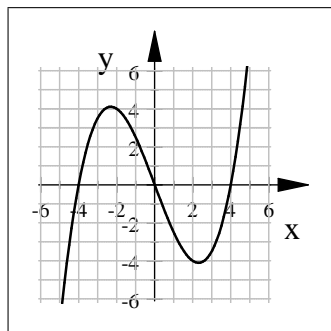


- a) What can be the degree of  $f$ ?    c) Write a possible equation for  $f$ .  
 b) Is the leading coefficient positive or negative?

32. Consider the function  $f(x) = \frac{x - 6}{2x + 5}$ .

- a) Find all horizontal asymptotes of the graph of  $f$ .    f) Find  $x$  for which  $f(x) = -\frac{4}{5}$ .  
 b) Find all vertical asymptotes of the graph of  $f$ .    g) Find the domain and range of  $f$ .  
 c) Compute the intercepts of  $f$ .  
 d) Graph  $f(x)$ .    h) Solve:  $\frac{x - 6}{2x + 5} \leq 1$   
 e) Find the inverse of  $f$ .

33. In each case, graph  $y = \frac{1}{f(x)}$ , given the graph of  $y = f(x)$ .



34. Graph each of the following functions.

a)  $f(x) = -\frac{1}{2} \sin(2x - \pi) + 1$  on  $[-2\pi, 2\pi]$

c)  $f(x) = \tan^{-1} x$

b)  $f(x) = -3 \cos\left(\frac{\pi x}{3}\right) - 2$  on  $[-9, 9]$

d)  $f(x) = \sec x$

35. Prove each of the following identities.

a)  $1 - \left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)^2 = \sin x$

b)  $\cos 4x = 8 \cos^4 x - 8 \cos^2 x + 1$

c)  $\sin 2x = \frac{1 - \tan^2\left(\frac{\pi}{4} - x\right)}{1 + \tan^2\left(\frac{\pi}{4} - x\right)}$

36. Simplify each of the following expressions.

a)  $\sin\left(\cos^{-1}\left(-\frac{3}{5}\right)\right)$

d)  $\tan\left(2 \tan^{-1}\left(\frac{3}{4}\right)\right)$

f)  $\tan\left(\frac{1}{2} \cos^{-1}\left(-\frac{1}{2}\right)\right)$

b)  $\sin(\tan^{-1}(-2))$

g)  $\sin\left(\frac{1}{2} \cos^{-1}\left(\frac{2}{3}\right)\right)$

c)  $\sin\left(2 \cos^{-1}\left(\frac{1}{3}\right)\right)$

e)  $\cos\left(2 \tan^{-1}\left(\frac{1}{3}\right)\right)$

h)  $\tan(\tan^{-1}(2) + \tan^{-1}(3))$

37. Simplify each of the following expressions.

a)  $\sin(\cos^{-1} x)$

c)  $\sin(2 \cos^{-1} x)$

e)  $\cos(2 \tan^{-1} x)$

g)  $\sin\left(\frac{1}{2} \cos^{-1} x\right)$

b)  $\sin(\tan^{-1} x)$

d)  $\tan(2 \tan^{-1} x)$

f)  $\tan\left(\frac{1}{2} \cos^{-1} x\right)$

38. Find the exact value of all solutions for each of the following equations. Present your answer in radians.

a)  $\sin x = \sin 2x$

b)  $7 \sin x + 1 = 6 \cos^2 x$

c)  $\sin x + 1 = 2 \cos^2 x$

39. Suppose that  $\sin \alpha = -\frac{5}{13}$  and  $\alpha$  is not in the fourth quadrant;  $\cos \beta = \frac{7}{25}$  and  $\beta$  is not in the first quadrant. Find the exact value for each of the following.

a)  $\tan(\alpha - \beta)$

b)  $\cos(\alpha + \beta)$

c)  $\cos 2\alpha$

d)  $\tan \frac{\alpha}{2}$

40. Let  $x$  and  $y$  be angles such that  $\sin x = -\frac{3}{5}$ ,  $\cos y = -\frac{20}{29}$ . In addition, we know that  $180^\circ \leq x \leq 270^\circ$  and  $90^\circ \leq y \leq 180^\circ$ . Find the exact value of each of the following.

a)  $\cos(x + y)$

b)  $\sin(3x)$

c)  $\tan(x - y)$

41. Express each of the following as a sum or difference.

a)  $\sin 35^\circ \cos 25^\circ$

b)  $\cos 25^\circ \cos 75^\circ$

c)  $\cos 4x \cos 2x$

42. Express each of the following as a product.

a)  $\sin 50^\circ + \sin 20^\circ$       b)  $\sin 75^\circ - \sin 35^\circ$       c)  $\cos 7x + \cos 3x$

43. Suppose that  $\tan 2x = \frac{3}{4}$ . Compute the exact value of      a)  $\cos 2x$       b)  $\sin x$

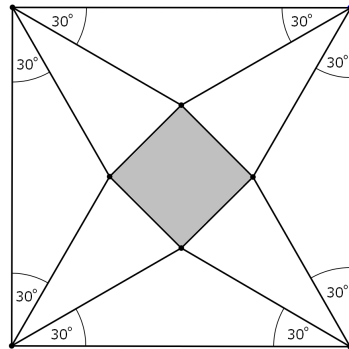
44. Find  $\tan \beta$  if  $\beta$  is the acute angle formed by  $y = \frac{2}{3}x - 5$  and  $y = -x + 1$ .

45. Solve each of the following triangles.

a)  $b = 248.6$ ,  $c = 186.2$ , and  $\gamma = 43.1^\circ$     b)  $\gamma = 42^\circ$ ,  $a = 122$  m, and  $c = 70$  m    c)  $a = 5$ ,  $b = 12$ , and  $c = 8$

46. Triangle  $ABC$  has sides of length 6, 7, and 8. Find the exact value of  $\cos \alpha + \cos \beta + \cos \gamma$ .

47. Consider the square with sides 1 meter shown on the picture. Find the exact value of the area of the shaded region.



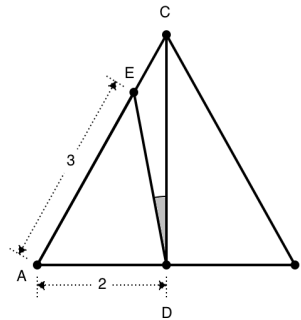
48. Prove that  $\sin 70^\circ - \sin 50^\circ = \sin 10^\circ$

49. Find the smallest and largest value of the function  $f(x) = 3 \sin x - 8 \cos x$ .

50. Solve the equation  $20x^3 - 12x^2 - 7x^4 - 2x^5 + x^6 = 0$  given that  $x = 2$  is a solution.

51. Consider the regular triangle with sides 4 meter shown on the picture.

- a) Find the exact value of the length of line segment  $CD$ .  
 b) Find the exact value of the length of line segment  $ED$ .  
 c) Find the exact value of  $\cos \delta$  if  $\delta$  is the shaded angle  $\angle EDC$ .



52. Perform the given operation over the complex numbers.

a)  $(2 - 5i)(1 + i)$     b)  $(3 - i)^2$     c)  $i^{143}$     d)  $|3 - 8i|$     e)  $(1 - 2i)^2$     f)  $\frac{-11 - 7i}{1 + 2i}$

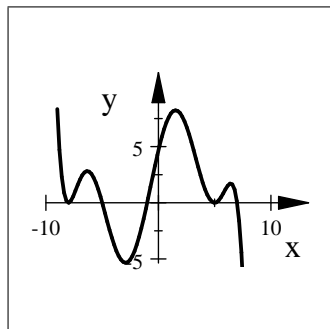
53. \* Find an equation for both tangent lines drawn to the graph of  $(x - 7)^2 + (y - 4)^2 = 50$  from the external point  $(-3, -16)$ .

## Answers

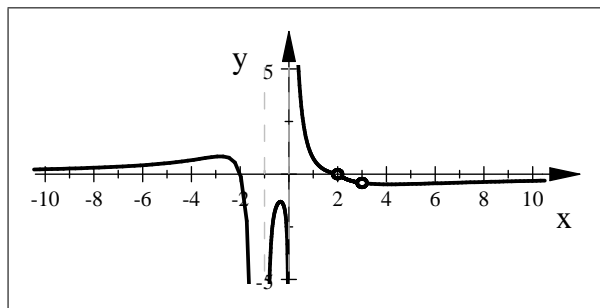
1. a)  $2N$  b)  $6N$  c)  $N^2$  d)  $\frac{N}{2}$  2.  $\frac{25}{99}$  3. a)  $\frac{x(x+3)}{x-4}$  b)  $\frac{x-4}{x(x-1)}$  c)  $\frac{5x}{x+5}$  d)  $\frac{3x-1}{x}$
- e)  $\frac{3+\sqrt{5}}{2}$  f) 1 4. a)  $8a^6b^{-1}$  b)  $3a^{\frac{1}{2}}b^{\frac{3}{2}}$  c)  $\frac{2}{3}a^2b^{-1}$  d)  $ab^{-1}$  e)  $a^{-\frac{3}{2}}b$  f)  $a^{\frac{5}{6}}b^{\frac{1}{2}}$  5.  $\frac{-2 \pm \sqrt{7}}{3}$
6. a)  $8 \pm \sqrt{5}$  b) 0 c) no solution d) -3 e)  $\log_{4/5} 200 = \frac{\ln 200}{\ln 4 - \ln 5}$  f) 2, -1
- g)  $\log_{20} \left( \frac{50}{3} \right) = \frac{\ln 50 - \ln 3}{\ln 20}$  h)  $\frac{\pi}{2} + 2k\pi, -\frac{\pi}{6} + 2k\pi, -\frac{5\pi}{6} + 2k\pi, k \in \mathbb{Z}$  i)  $\frac{1}{3} + \frac{1}{3e^5}$
- j)  $\frac{\pi}{9} + \frac{k\pi}{3}$  or  $\frac{\pi}{18} + \frac{k\pi}{3}$  where  $k \in \mathbb{Z}$  k) 4 (-5 doesn't work) l)  $\frac{5\pi}{12} + 2k\pi$   $\frac{11\pi}{12} + 2k\pi$  where  $k \in \mathbb{Z}$
7. a)  $x = 6$  b)  $0 < x < 9$  c)  $x < -5$  or  $x \geq -4$  8. a)  $2x^3 - x^2 - 5x + 1$  R  $3x - 5$
- b)  $x^4 - 3x^3 + 9x^2 - 27x + 81$  R -244 9.  $\frac{1}{2} \log_2 \left( \frac{3}{2} \right) = \log_2 \left( \sqrt{\frac{3}{2}} \right)$  10. a) 480 b) 600 c)  $\frac{81}{4}$
11. \$28 900 with price \$85 12. a)  $y = -\frac{4}{3}x + \frac{11}{3}$  b)  $\frac{3}{4}(x - 10) = y + 7$  13.  $y = -7x - 3$  and  $y = 11x - 21$
14.  $t = \frac{-\ln 2}{3 \ln 0.95}$  hours  $\approx 4.50447$  hours

15.

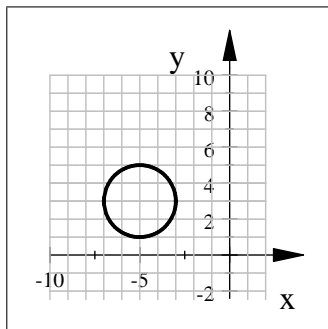
a)



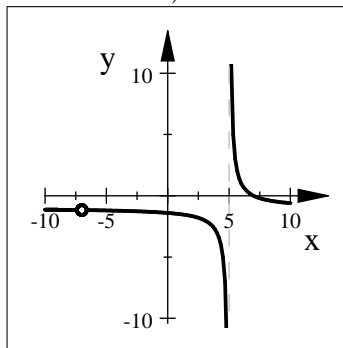
e)



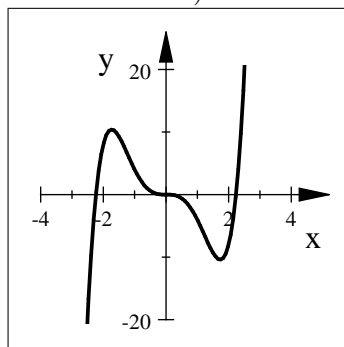
b)



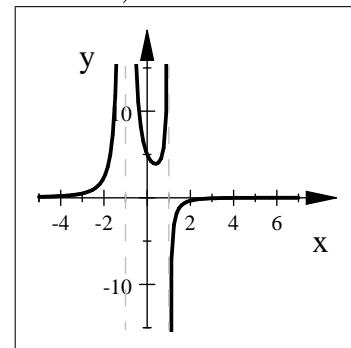
c)



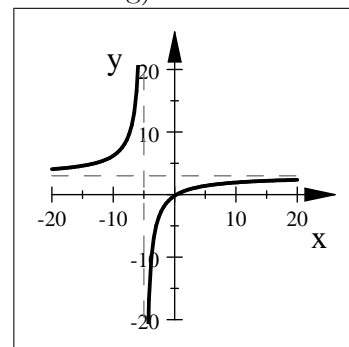
f)

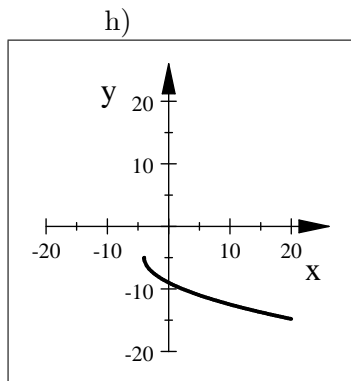


d)



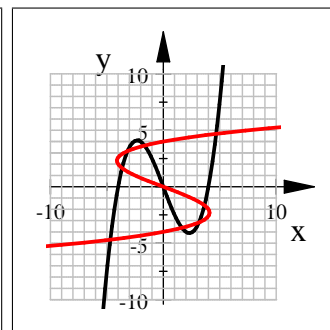
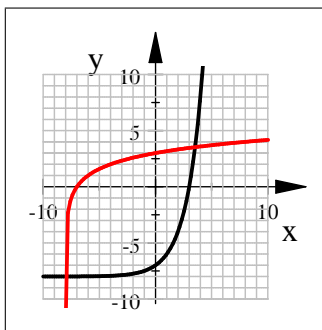
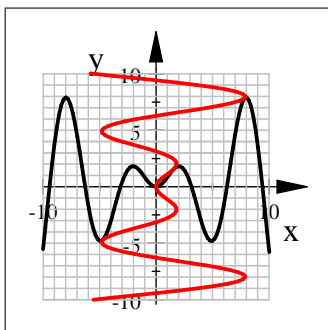
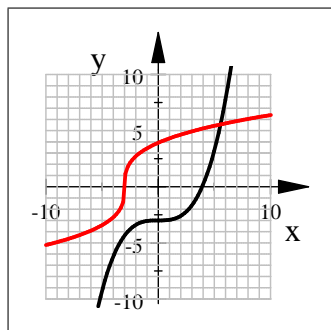
g)





16. a)  $-\frac{3}{2}$  b)  $\frac{1}{2}$  c) 21 d)  $2k$  e)  $-\frac{2}{3}$  f)  $-1$  g) 49 h)  $-\frac{4}{3}$   
 i) 25 j)  $\frac{1}{49}$  k)  $\frac{1}{4}$  l)  $-3$  17. E  
 18. a)  $-6$  b)  $-3$  c)  $-6$  d) 3 e)  $-4x^2 + 4x + 2$  f)  $-2x^2 + 5$   
 19. a)  $\mathbb{R}$  b)  $[-\sqrt{10}, \sqrt{10}]$  c)  $f(x) = (-\sqrt{10}, \sqrt{10})$   
 d)  $f(x) = (-\sqrt{10}, \sqrt{10}) \setminus \{3, -3\}$   
 20. a)  $\frac{1}{2}\sqrt{\sqrt{2}+2}$  b)  $\frac{1}{4}$  c)  $\sqrt{3}$  d)  $-\frac{\pi}{4}$  e)  $\frac{3\pi}{4}$

21. a) b) c) d)



22. a)  $f^{-1}(x) = \frac{1}{5}(\log_3 x + 1)$  b)  $f^{-1}(x) = \frac{5x + 4}{3x - 1}$  c)  $f^{-1}(x) = \frac{1}{2}(e^x + 1)$

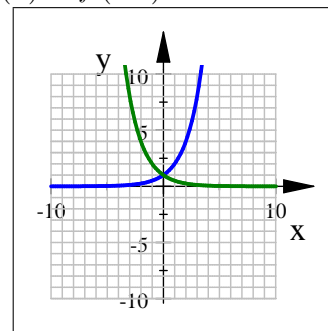
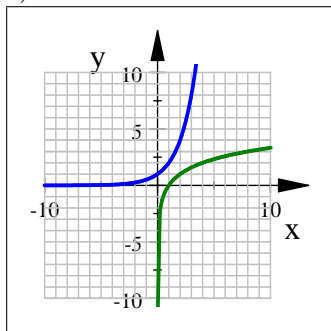
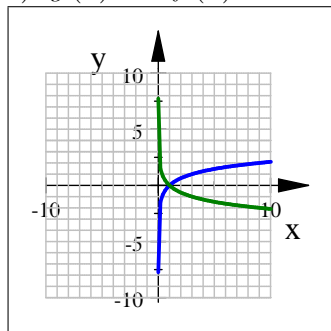
23. a)  $-6\hat{i} - 8\hat{j}$  b) 5 c) 17 d)  $11\hat{i} - 11\hat{j}$  e)  $-7\hat{i} + 42\hat{j}$  f)  $-36$  g)  $-264$  h)  $115.0576^\circ$

24. 19.160 N, 90.38343 N 25.  $\frac{\sqrt{15}}{8}$  26.  $\frac{4}{9}$

27. a)  $f$  - blue graph  
 $g$  - green graph  
 b) symmetry through the  $x$  axis  
 c)  $g(x) = -f(x)$

28. a)  $f$  - blue graph  
 $g$  - green graph  
 b) symmetry through the line  $y = x$   
 c) inverse functions

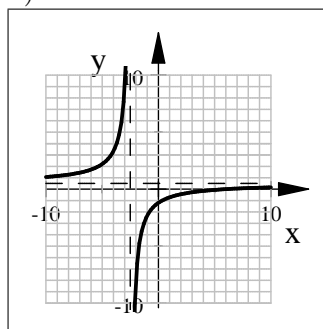
29. a)  $f$  is the blue graph,  $g$  is the green graph  
 b) symmetry through the  $y$  axis  
 c)  $g(x) = f(-x)$



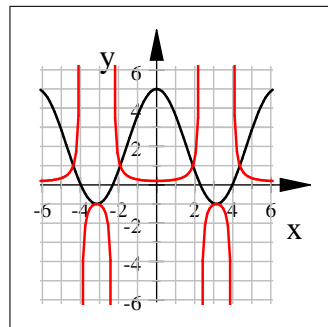
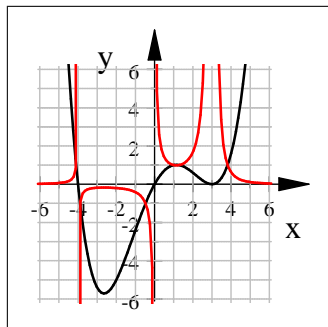
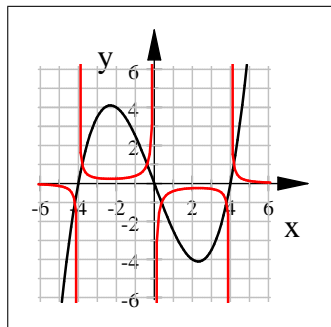
30. a) (16, 0) b)  $60^\circ$  c)  $5\sqrt{3}$  31. a) 7, 9, 11, ... b) negative c)  $y = -(x + 5)(x + 3)(x + 1)^2(x - 3)^2(x - 6)$

32. a)  $y = \frac{1}{2}$  b)  $x = -\frac{5}{2}$  c)  $(6, 0)$  and  $(0, -2.5)$   
 e)  $f^{-1}(x) = \frac{5x+6}{-2x+1}$  f)  $\frac{10}{13}$   
 g) domain:  $\left\{x : x \neq -\frac{5}{2}\right\}$  range:  $\left\{y : y \neq \frac{1}{2}\right\}$   
 h)  $x \leq -11$  or  $x > -\frac{5}{2}$

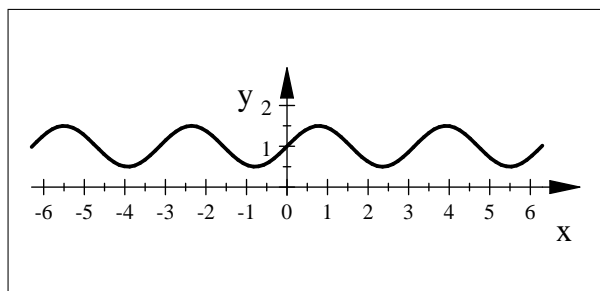
d) see below



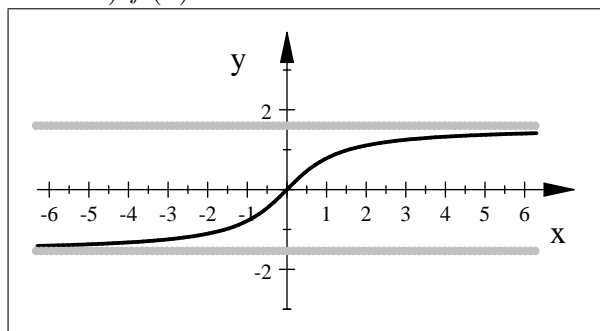
33.



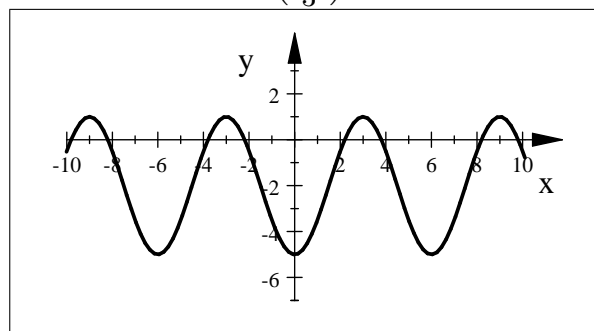
34. a)  $f(x) = -\frac{1}{2} \sin(2x - \pi) + 1$  on  $[-2\pi, 2\pi]$



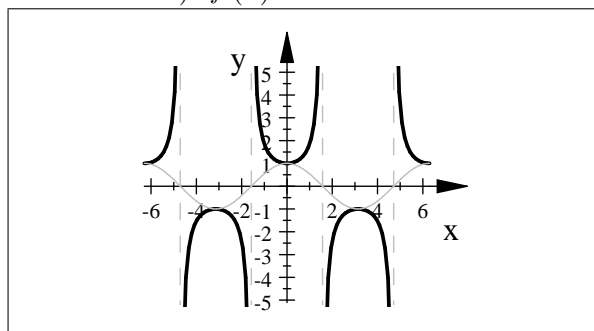
c)  $f(x) = \tan^{-1} x$



b)  $f(x) = -3 \cos\left(\frac{\pi x}{3}\right) - 2$  on  $[-9, 9]$



d)  $f(x) = \sec x$



35. see handout Trig Identities 4 36. a)  $\frac{4}{5}$  b)  $-\frac{2\sqrt{5}}{5}$  c)  $\frac{4\sqrt{2}}{9}$  d)  $\frac{24}{7}$  e)  $\frac{4}{5}$  f)  $\sqrt{3}$  g)  $\frac{\sqrt{6}}{6}$  h)  $-1$

37. a)  $\sqrt{1-x^2}$  b)  $\frac{x}{\sqrt{x^2+1}}$  c)  $2x\sqrt{1-x^2}$  d)  $\frac{2x}{1-x^2}$  e)  $\frac{1-x^2}{x^2+1}$  f)  $\frac{\sqrt{1-x^2}}{1+x}$  g)  $\sqrt{\frac{1-x}{2}}$

38. a)  $k\pi, \pm\frac{1}{3}\pi + 2k\pi$  where  $k \in \mathbb{Z}$  b)  $\frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi$  where  $k \in \mathbb{Z}$   
 c)  $-\frac{\pi}{2} + 2k\pi, \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi$   $k \in \mathbb{Z}$



39. a)  $-\frac{323}{36}$    b)  $-\frac{204}{325}$    c)  $\frac{119}{169}$    d)  $-5$    40. a)  $\frac{143}{145}$    b)  $-\frac{117}{125}$    c)  $\frac{144}{17}$

41. a)  $\frac{1}{2}(\sin 60^\circ + \sin 10^\circ)$    b)  $\frac{1}{2}(\cos 50^\circ + \cos 100^\circ)$    c)  $\frac{1}{2}(\cos 6x + \cos 2x)$    42. a)  $2 \sin 35^\circ \cos 15^\circ$

b)  $2 \cos 55^\circ \sin 20^\circ$    c)  $2 \cos 5x \cos 2x$    43. a)  $\pm \frac{4}{5}$    b)  $\pm \frac{\sqrt{10}}{10}, \pm \frac{3\sqrt{10}}{10}$    44. 5

45. a)  $\beta_1 = 65.819^\circ$ ,  $\alpha_1 = 71.081^\circ$ ,  $a_1 = 257.790$  and  $\beta_2 = 114.181^\circ$ ,  $\alpha_2 = 22.719^\circ$   $a_2 = 105.247$   
 b) no solution   c)  $\alpha = 17.612^\circ$     $\beta = 133.433^\circ$     $\gamma = 28.955^\circ$

46.  $\frac{47}{32}$    47.  $\frac{2 - \sqrt{3}}{3}$

48.  $\sin 70^\circ - \sin 50^\circ = \sin(60^\circ + 10^\circ) - \sin(60^\circ - 10^\circ) = 2 \cos 60^\circ \sin 10^\circ = 2 \cdot \frac{1}{2} \cdot \sin 10^\circ = \sin 10^\circ$

49. smallest:  $-\sqrt{73}$    largest:  $\sqrt{73}$    50.  $x = 2, 1, 0, -3$    51. a)  $\sqrt{12}$    b)  $\sqrt{7}$    c)  $\frac{3\sqrt{21}}{14}$

52. a)  $7 - 3i$    b)  $8 - 6i$    c)  $-i$    d)  $\sqrt{73}$    e)  $-3 - 4i$    f)  $-5 + 3i$    53.  $y = 7x + 5$  and  $y = x - 13$

Last revised: December 1, 2018