1. Simplify each of the following.

a) $\dfrac{2\cos^2\frac{\pi}{8}-1}{1+8\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}}$

b) $5^{(\log_{10}625/\log_{10}25)}$

c) $\log_2(\log_3 81)$

d) $\dfrac{\log_2 24-\frac{1}{2}\log_2 72}{\log_3 18-\frac{1}{3}\log_3 72}$

e) $\cos\frac{23\pi}{4}-\sin\frac{15\pi}{4}$

f) $\dfrac{\cos 35^\circ}{\sin 20^\circ\cos 35^\circ+\cos 20^\circ\sin 35^\circ}$

g) $\log_{\sin 45^\circ} 2$

h) $\sin\left(\frac{2015\pi}{6}\right)$

i) $\dfrac{1-\tan 75^\circ}{1+\tan 75^\circ}$

j) $\dfrac{\log_3 54-\frac{1}{2}\log_3 108}{\log_2 12-\frac{1}{2}\log_2 72}$

k) $\cos 75^\circ\sin 75^\circ$

l) $\left(\sin\left(\frac{11\pi}{4}\right)-\cos\left(\frac{11\pi}{4}\right)\right)^{10}$

m) $\log_8(12x^2)-\log_8(15x^3)+\log_8(20x)$

n) $2\log_3 6-\frac{1}{2}\log_3 400+3\log_3\sqrt[3]{45}$

o) $\log_{\sqrt{3}}(18x^2)-\log_{\sqrt{3}}(20x^3)+\frac{1}{2}\log_{\sqrt{3}}(900x^2)$

2. For each of the following functions given, sketch its graph and state its domain and range.

a) $f(x)=\sin^{-1}x$ b) $g(x)=\cos^{-1}x$ c) $h(x)=\tan^{-1}x$

3. Compute each of the following.

a) $\sin(37.5^\circ)$ (Hint: 37.5° is half of what angle?)

b) $\tan\alpha$ if $\sin 2\alpha=\frac{2}{3}$ and 2α is in the first quadrant.

c) $\cos\left(\frac{225^\circ}{2}\right)$

4. Simplify each of the following.

a) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

b) $\sin^{-1}(-1)$

c) $\sin\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$

d) $\cos^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right)$

e) $\cos\left(\cos^{-1}\left(\frac{1}{2}\right)\right)$

f) $\cos^{-1}\left(\cos\left(-\frac{\pi}{3}\right)\right)$

g) $\sin\left(\cos^{-1}\left(-\frac{2}{3}\right)\right)$

h) $\cos\left(\tan^{-1}\left(-\frac{1}{2}\right)\right)$

i) $\sin\left(\tan^{-1}\left(-\frac{1}{2}\right)\right)$

j) $\tan\left(\sin^{-1}\left(-\frac{3}{5}\right)\right)$

k) $\sin\left(2\cos^{-1}\left(\frac{2}{5}\right)\right)$

l) $\cos\left(2\tan^{-1}(-3)\right)$

m) $\tan\left(2\tan^{-1}(-3)\right)$

n) $\sin\left(\sin^{-1}\left(-\frac{3}{5}\right)+\cos^{-1}\left(-\frac{12}{13}\right)\right)$

o) $\sin\left(\cos^{-1}x\right)$

p) $\cos\left(\tan^{-1}x\right)$

q) $\sin\left(\tan^{-1}x\right)$

5. Prove each of the following.

a) $\sin 35^\circ+\sin 25^\circ=\cos 5^\circ$ b) $\cos 12^\circ-\cos 48^\circ=\sin 18^\circ$ c) $1+\tan\alpha\tan\beta=\dfrac{\cos(\alpha-\beta)}{\cos\alpha\cos\beta}$

6. Compute the inverse for each of the following functions.

a) $f(x)=\frac{2}{3}x-6$

b) $f(x)=\sqrt[3]{2x+1}$

c) $f(x)=(2x-3)^3+8$

d) $f(x)=\dfrac{3x-1}{7x-5}$

e) $f(x)=\dfrac{7x+10}{3x-7}$

f) $f(x)=e^{4x-1}-3$

g) $f(x)=\log_3(5x-4)$

7. Graph the function given and graph the inverse relation in the same coordinate system. You do not have to find the equation for the inverse.

a) $f(x)=\log_2 x$ b) $f(x)=|x|$ c) $y=x(x^2-9)$

8. Graph each of the following. State the steps in graphing the functions.

   a) $f(x) = 3 - \dfrac{2}{x+4}$ b) $g(x) = -\sqrt{-x+3} + 1$ c) $h(x) = 2\log_2(x-1) - 3$

9. Graph each of the following functions by first applying a division and then transformations on the graph of $y = \dfrac{1}{x}$. List the steps in graphing each function.

   a) $f(x) = \dfrac{3x-10}{x-1}$ b) $f(x) = \dfrac{5x-1}{x+2}$ c) $f(x) = \dfrac{8x}{2x-4}$

10. Graph each of the pair of functions in the same coordinate system.

    a) $f(x) = \sin x$ and $g(x) = \csc x$
    b) $f(x) = \sin x$ and $g(x) = 3\sin x$
    c) $f(x) = \sin x$ and $g(x) = \sin(3x)$
    d) $f(x) = \cos x$ and $g(x) = \cos x + 1$
    e) $f(x) = \cos x$ and $g(x) = \cos\left(x - \dfrac{\pi}{4}\right)$

11. If we know that $3\sin^2\alpha = 5\sin\alpha + 2$ and that $\cos\alpha < 0$, then find the exact value of $\sin 2\alpha$.

12. Find the sum $\cos 1° + \cos 2° + ... + \cos 359°$

13. Perform each of the following divisions.

    a) $(5x^5 - x^4 - 7x^3 + x^2 - 3x) \div (x^2 - 2)$
    b) $(x^4 - 5) \div (x + 2)$
    c) $(2x^3 - 5x^2 + 7x - 1) \div (x - 3)$
    d) $(x^7 - 1) \div (x^2 + 1)$
    e) $(x + 5) \div (x - 3)$
    f) $(2x - 1) \div (x + 1)$
    g) $(6x^4 - 15x^3 + x^2) \div (3x^2 - 1)$
    h) $x^6 \div (x - 2)$

14. a) Solve the equation $10x^5 - 426x^3 - 44x^4 - 693x^2 + x^6 = 0$ if we know that -3 is a solution of the equation.
    b) Solve the equation $48x - 44x^2 - 40x^3 + 21x^4 + 14x^5 + x^6 = 0$.

15. Suppose that $\log_7 2 = m$. Write $\log_{49} 28$ in terms of m.

16. Solve each of the following ineqalities.

    a) $\dfrac{4x}{x+5} \leq \dfrac{2}{3}$ b) $\dfrac{1}{x} \leq \dfrac{6}{x-2}$ c) $x \leq \dfrac{1}{x}$ d) $x^2 > 9$

17. Solve each of the following equations.

    a) $\log_2(3x+2) + \log_2(x-8) = 6$
    b) $\log_x(2x-1) = -1$
    c) $4^x - 2^{x+1} = 8$
    d) $\log_{x+1} 3 + \log_3(x+1) = \dfrac{5}{2}$
    e) $\sin 2x - \sqrt{3}\cos 2x = 1$
    f) $\log_2(x+1) + \log_2(x+3) = 3$
    g) $\log_3 x + \log_x 3 = \dfrac{5}{2}$
    h) $\log_3(x^3 - x) - \log_3 x = 1$
    i) $16^x - 4^{x+1} = 5$
    j) $\dfrac{\sqrt{3-x} + \sqrt{3+x}}{\sqrt{3-x} - \sqrt{3+x}} = 2$
    k) $\sin 2x - \cos x = 0$
    l) $\sin 2x = \dfrac{\sqrt{3}}{2}$
    m) $\sin x - \sqrt{3}\cos x = -1$
    n) $\sin 5x - \sqrt{3}\cos 5x = -1$

18. Find the smallest and greatest value of the function $f(x) = 3\sin x + 4\cos x$

19. Find an equation for the tangent line drawn to the circle $(x-4)^2 + (y+5)^2 = 25$ at the point $(7, -1)$.

20. Compute the exact value of each of the following.

a) $\sin\left(2\cos^{-1}\left(\frac{3}{4}\right)\right)$

b) $\sin\left(\frac{1}{2}\cos^{-1}\left(\frac{3}{4}\right)\right)$

c) $\cos\left(\tan^{-1}(-2)\right)$

d) $\tan\left(\tan^{-1}3+\tan^{-1}4\right)$

e) $\sin\left(\sin^{-1}\left(\frac{3}{5}\right)+\cos^{-1}\left(\frac{3}{5}\right)\right)$

f) $\cos\left(2\sin^{-1}x\right)$

21. Graph each of the following functions.

a) $f(x)=x^4-4x^2$

b) $g(x)=-x^3+6x$

c) $h(x)=-(x+2)^2x^3(x-2)^2$

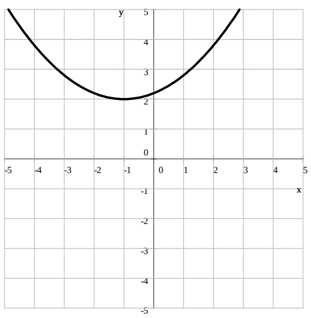
d) $f(x)=-2(x+2)^3x^2(x-2)(x-3)^2$

e) $f(x)=(x+2)x(x-3)$

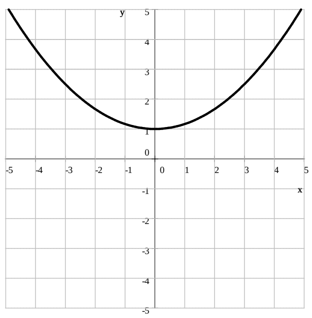
f) $f(x)=(x+2)^2x(x-2)^3$

22. Graph $y=\dfrac{1}{f(x)}$ given the graph of $y=f(x)$.

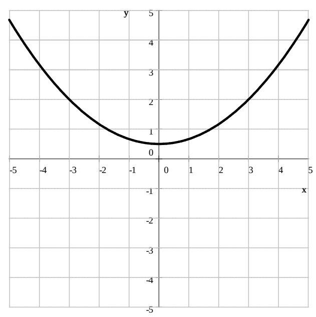
a)



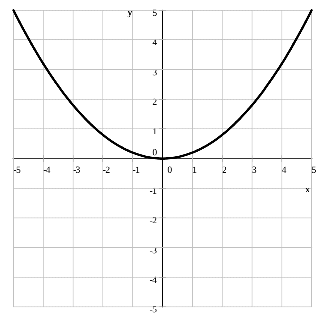
b)



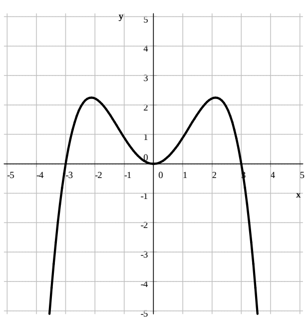
c)

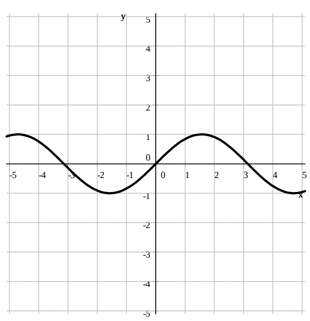


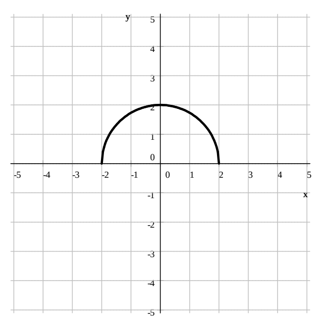
d)



23. Given the graph of a function $y=f(x)$, graph $y=\dfrac{1}{f(x)}$ in the same coordinate system.







24. Given the same graphs as in the previous problem, graph the inverse relation in the same coordinate system.

25. The number of cells in a sample at time t (measured in hours) is $N(t)=20\,000\left(1.2^{0.5t}\right)$.

a) How many cells are in the sample at $t=0$?

b) How long will it take for the sample to double from the amount that it had at $t=0$?

c) How many cells are in the sample at $t=6$?

d) How long will it take for the sample to double from the amount that it had at $t=6$?

e) What do you observe? Can we make (and perhaps prove) a general statement?

26. Is there a right triangle whose sides are consecutive integers?

27. Write $\log_3 5-\log_9 30$ as a single logrithm.

28. Classify the discontinuities of each of the following functions as a hole or a vertical asymptote.

a) $f(x) = \dfrac{x^3 - x}{x^2(x-1)}$ b) $g(x) = \dfrac{(x+3)^2 (x+1)^3 x^4 (x-2)^5 (x-5)^8}{(x+3)^3 (x+1)^2 x^4 (x-2)^8 (x-5)^9}$

29. Prove the identity $\dfrac{\tan\left(\dfrac{\pi}{4} - x\right)}{\tan\left(\dfrac{\pi}{4} + x\right)} = \dfrac{1 - \sin 2x}{1 + \sin 2x}$

30. Find the domain for each of the following functions.

a) $f(x) = \dfrac{\ln(x+4)}{\ln(x-5)}$

b) $g(x) = \ln\left(\dfrac{x+4}{x-5}\right)$

c) $h(x) = \dfrac{1}{\sin x - \cos x}$

d) $h(x) = \dfrac{\cos^4 x - \sin^4 x}{\sin 2x}$

e) $f(x) = \ln\left(\sin^2 x\right)$

f) $f(x) = \dfrac{1}{\sin x - \cos x}$

g) $f(x) = \log_5 (\sin x + \cos x)^2$

31. Compute the exact value of each of the following.

a) $\sin x$ if $\cos 2x = -\dfrac{2}{3}$ b) $\cos x$ if $\cos 2x = \dfrac{4}{5}$ c) $\cos x$ if $\sin 2x = \dfrac{5}{13}$ d) $\sin 2x$ if $\tan x = 2$

e) Compute the exact value of $\tan\alpha$ if we know that $90° \leq \alpha \leq 180°$ and $\cos 2\alpha = \dfrac{3}{5}$.

32. Solve each of the following triangles.

a) $a = 6\,\text{m}$, $c = 5\,\text{m}$, $\gamma = 38°$

b) $a = 7\,\text{m}$, $c = 3\,\text{m}$, $\gamma = 28°$

c) $a = 6\,\text{m}$, $c = 5\,\text{m}$, $\beta = 38°$

d) $a = 17$, $b = 13$, and $\beta = 20°$

e) $a = 17$, $b = 13$, and $\alpha = 20°$

f) $a = 17$, $b = 13$, and $\gamma = 20°$

33. a) Find the exact value of the cosine of the smallest angle in a triangle with sides 3, 5, and 6.

b) Find the exact value of the cosine of the largest angle in a triangle with sides 10 cm, 8 cm, and 7 cm long.

34. Find the **exact value** of the area of a triangle with sides 2, 3, and 4.

35. Two sides of a triangle are 8 ft and 15 ft long. Find the exact value of the third side if we know that the area of the triangle is 48 ft^2.

36. Triangle SML has sides of length 6, 7, and 8. Find the exact value of $\cos S + \cos M + \cos L$.

37. a) Find the exact value of $\cos\dfrac{\alpha}{2}$ if we know that $\cos\alpha = \dfrac{1}{3}$.

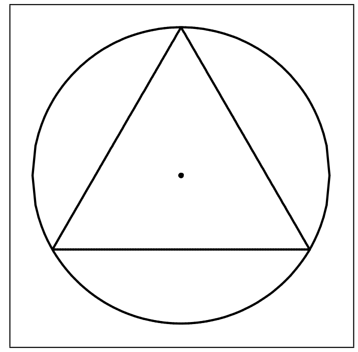
b) Find the exact value of $\sin\dfrac{\beta}{2}$ if we know that $\sin\beta = \dfrac{2}{3}$.

38. Find $\tan\beta$ if we know that $\tan\alpha = \dfrac{1}{2}$ and $\tan(\alpha+\beta) = \dfrac{13}{11}$.

39. A triangle has sides of length a, b, and c, which are consecutive integers in increasing order, and $\cos\gamma = \dfrac{5}{16}$. Find $\cos\alpha$.

40. Consider an equilateral triangle with sides 1 unit long, inscribed in a circle. Let O be the center of the circle.

a) Find the radius of the circle.
b) Find the distance between O and the side of the triangle.



41. The population of a town is growing exponentially. From 2007, it took A years for the population to double. From 2007, it took B years for the population to triple. Express B in terms of A.

Answers

1. a) $\frac{\sqrt{2}}{4}$ b) 25 C) 2 d) $\frac{9}{8}$ e) $\sqrt{2}$ f) 1 g) -2 h) $-\frac{1}{2}$ i) $-\frac{\sqrt{3}}{3}$ j) 3
k) $\frac{1}{4}$ l) 32 m) $\frac{4}{3}$ n) 4 o) 6

2. see handout

3. a) $\sqrt{\frac{\sqrt{2}+4-\sqrt{6}}{8}}$ b) $\frac{3-\sqrt{5}}{2}$ c) $-\frac{1}{2}\sqrt{2-\sqrt{2}}$

4. a) $-\frac{\pi}{6}$ b) $-\frac{\pi}{2}$ c) $\frac{\sqrt{2}}{2}$ d) $\frac{\pi}{4}$ e) $\frac{1}{2}$ f) $\frac{\pi}{3}$ g) $\frac{\sqrt{5}}{3}$ h) $\frac{2\sqrt{5}}{5}$ i) $-\frac{\sqrt{5}}{5}$
j) $-\frac{3}{4}$ k) $\frac{4\sqrt{21}}{25}$ l) $-\frac{4}{5}$ m) $\frac{3}{4}$ n) $\frac{56}{65}$ o) $\sqrt{1-x^2}$ p) $\frac{1}{\sqrt{x^2+1}}$ q) $\frac{x}{\sqrt{x^2+1}}$

5. a) $\sin 35° + \sin 25° = \cos 5°$

$$\begin{aligned}\sin 35° + \sin 25° &= \sin(30° + 5°) + \sin(30° - 5°) = \sin 30° \cos 5° + \cos 30° \sin 5° + \sin 30° \cos 5° - \cos 30° \sin 5° \\ &= 2 \sin 30° \cos 5° = 2\left(\frac{1}{2}\right)\cos 5° = \cos 5°\end{aligned}$$

b) $\cos 12° - \cos 48° = \sin 18°$

$$\begin{aligned}\cos 12° - \cos 48° &= \cos(30° - 18°) - \cos(30° + 18°) \\ &= \cos 30° \cos 18° + \sin 30° \sin 18° - (\cos 30° \cos 18° - \sin 30° \sin 18°) \\ &= \cos 30° \cos 18° + \sin 30° \sin 18° - \cos 30° \cos 18° + \sin 30° \sin 18° \\ &= 2 \sin 30° \sin 18° = 2\left(\frac{1}{2}\right)\sin 18° = \sin 18°\end{aligned}$$

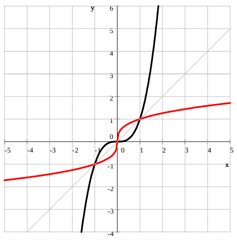
c) $1 + \tan\alpha \tan\beta = \frac{\cos(\alpha - \beta)}{\cos\alpha\cos\beta}$

$$\begin{aligned}\text{LHS} &= 1 + \tan\alpha\tan\beta = 1 + \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta} = \frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta} = \frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\cos\alpha\cos\beta} \\ &= \frac{\cos(\alpha-\beta)}{\cos\alpha\cos\beta} = \text{RHS}\end{aligned}$$

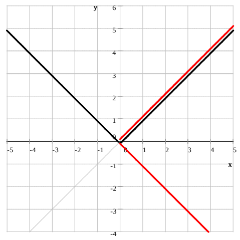
6. a) $f^{-1}(x) = \frac{3}{2}x + 9$ b) $f^{-1}(x) = \frac{1}{2}x^3 - \frac{1}{2}$ c) $f^{-1}(x) = \frac{1}{2}\sqrt[3]{x-8} + \frac{3}{2}$ d) $f^{-1}(x) = \frac{5x-1}{7x-3}$

e) $f^{-1}(x) = \frac{7x+10}{3x-7}$ f) $f^{-1}(x) = \frac{1}{4}(\ln(x+3)+1)$ g) $f^{-1}(x) = \frac{1}{5}(3^x+4)$

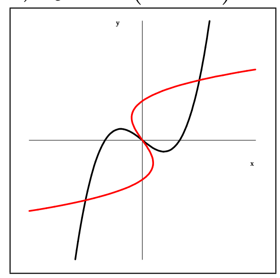
7. a) $f(x) = x^3$



b) $f(x) = |x|$

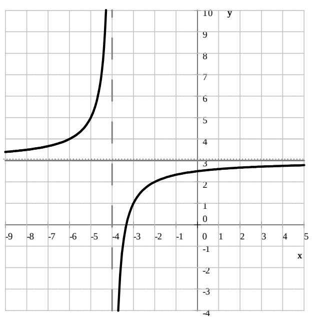


c) $y = x(x^2-9)$



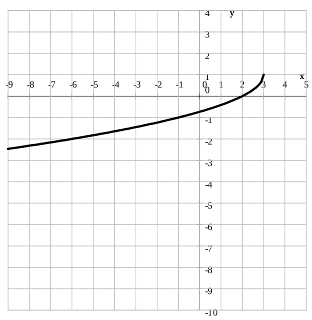
8. a) $f(x) = 3 - \frac{2}{x+4}$

Start with $y = \frac{1}{x}$
shift to the left by 4 units
stretch along by the $y-$axis by 2
reflect to the $x-$axis
shift up by 3 units



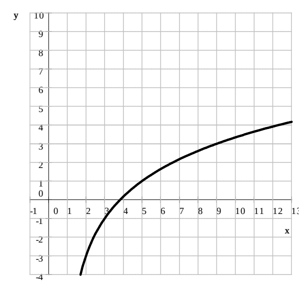
b) $g(x) = -\sqrt{-x+3} + 1$

Start with $y = \sqrt{x}$
reflect to the $y-$axis
shift to the right by 3 units
reflect to the $x-$axis
shift up by 1 unit



c) $h(x) = 2\log_2(x-1) - 3$

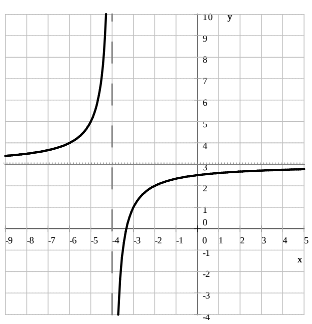
Start with $y = \log_2 x$
shift to the right by 1 unit
stretch along by the $y-$axis by 2
reflect to the $x-$axis
shift down by 3 units



9. a) $f(x) = \frac{3x-10}{x-1}$

$= -7 \cdot \frac{1}{x-1} + 3$

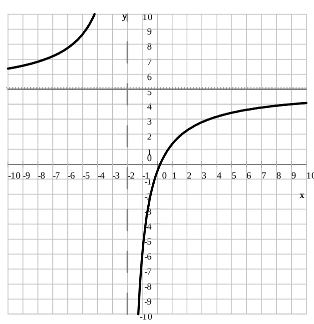
Start with $y = \frac{1}{x}$
shift to the right by 1 unit
stretch along by the $y-$axis by 7
reflect to the $x-$axis
shift up by 3 units



b) $f(x) = \frac{5x-1}{x+2}$

$= -11 \cdot \frac{1}{x+2} + 5$

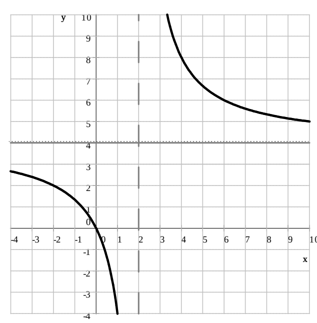
Start with $y = \frac{1}{x}$
shift to the left by 2 units
stretch along by the $y-$axis by 11
reflect to the $x-$axis
shift up by 5 units



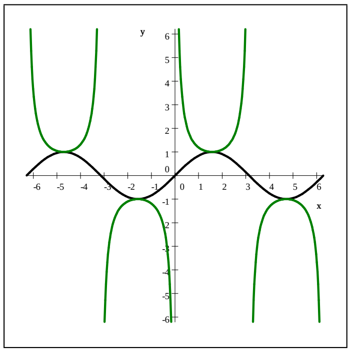
c) $f(x) = \frac{8x}{2x-4}$

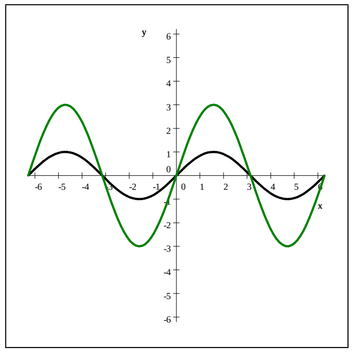
$= 8 \cdot \frac{1}{x-2} + 4$

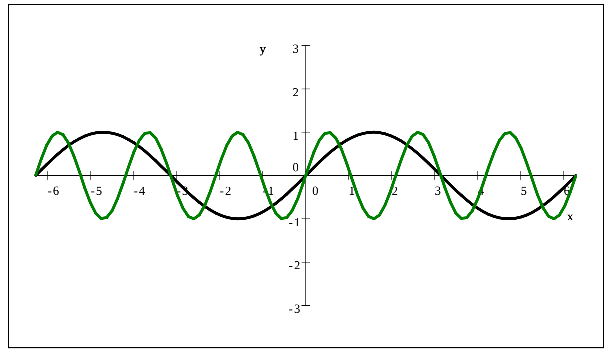
Start with $y = \frac{1}{x}$
shift to the right by 2 units
stretch along by the $y-$axis by 8
shift up by 4 units



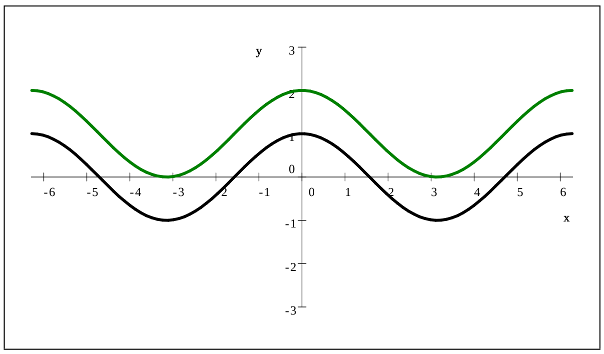
10. a) $f(x) = \sin x$ and $g(x) = \csc x$ b) $f(x) = \sin x$ and $g(x) = 3\sin x$ c) $f(x) = \sin x$ and $g(x) = \sin(3x)$

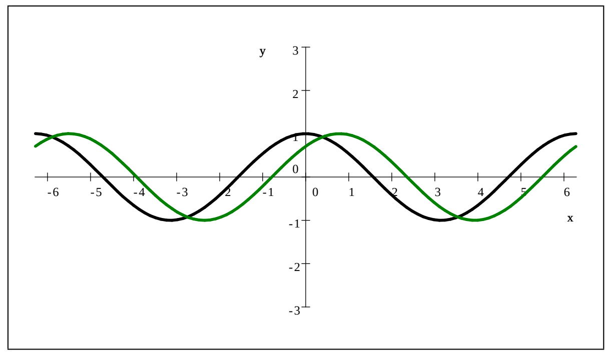






d) $f(x) = \cos x$ and $g(x) = \cos x + 1$ e) $f(x) = \cos x$ and $g(x) = \cos\left(x - \dfrac{\pi}{4}\right)$





11. $\dfrac{4}{9}\sqrt{2}$

12. -1

13. a) $5x^3 - x^2 + 3x - 1$ R $3x - 2$ b) $x^3 - 2x^2 + 4x - 8$ R 11 c) $2x^2 + x + 10$ R 29
d) $x^5 - x^3 + x$ R $-x - 1$ e) 1 R 8 f) 2 R -3 g) $2x^2 - 5x + 1$ R $-5x + 1$

h) $x^5 + 2x^4 + 4x^3 + 8x^2 + 16x + 32$ R 64

14. a) $-11, -3, 0, 7$ b) $-12, -2, 0, 1$

15. $m + \dfrac{1}{2}$ $\log_{49} 28 = \dfrac{\log_7 28}{\log_7 49} = \dfrac{\log_7 7 + \log_7 4}{2} = \dfrac{1 + 2\log_7 2}{2} = \dfrac{1 + 2m}{2} = m + \dfrac{1}{2}$

16. a) $(-5, 1]$ b) $\left[-\dfrac{2}{5}, 0\right) \cup (2, \infty)$ c) $(0, 1] \cup (-\infty, -1]$ d) $(-\infty, -3) \cup (3, \infty)$

17. a) 10 b) no solution c) 2 d) $10, \sqrt{3} - 1$ e) $\dfrac{\pi}{4} + k\pi, \dfrac{7\pi}{12} + k\pi$ where $k \in \mathbb{Z}$ f) 1 g) $9, \sqrt{3}$
h) 2 i) $\log_4 5$ j) $-\dfrac{12}{5}$ k) $\dfrac{\pi}{2} + k\pi$ $\dfrac{\pi}{6} + 2k\pi$ $\dfrac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$ l) $x = \dfrac{\pi}{6} + k\pi$ or $x = \dfrac{\pi}{3} + k\pi$ where $k \in \mathbb{Z}$

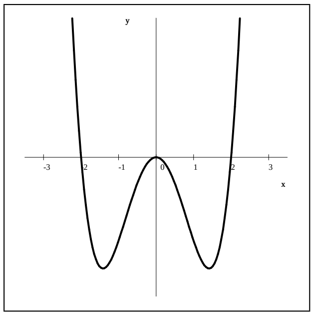
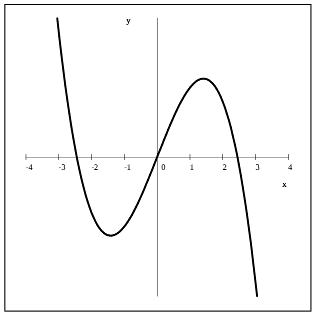
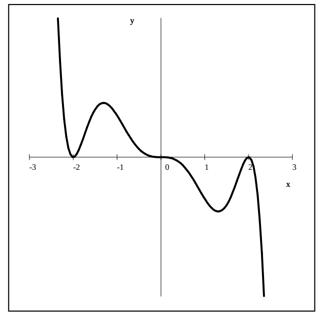
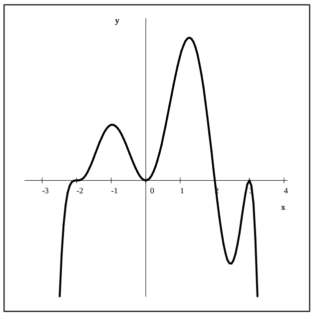
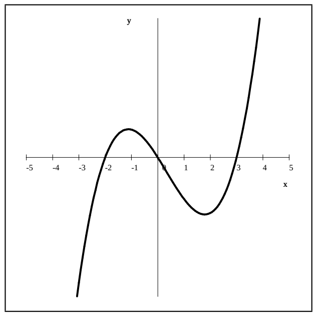
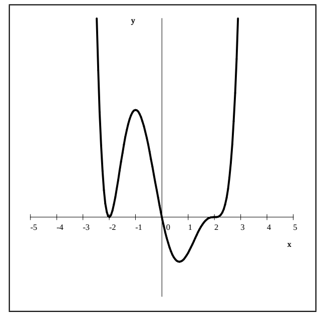
m) $-\dfrac{\pi}{2} + 2k\pi$ or $\dfrac{\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$ n) $-\dfrac{\pi}{10} + \dfrac{2k\pi}{5}$ or $\dfrac{\pi}{30} + \dfrac{2k\pi}{5}$ where $k \in \mathbb{Z}$

18. smallest value: -5 greatest value: 5

19. a) $y = -\dfrac{3}{4}x + \dfrac{17}{4}$ b) $y = -5x - 9$ and $y = x - 15$ c) $y = 4x - 15$ and $y = 8x - 35$

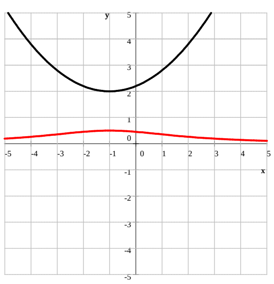
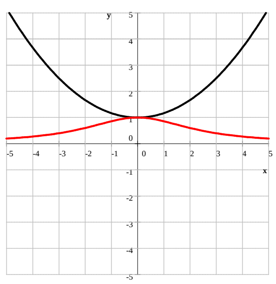
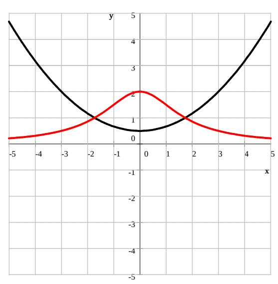
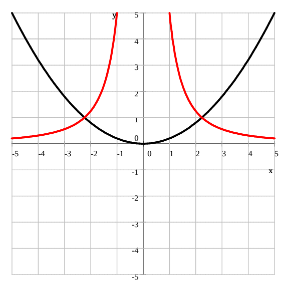
20. a) $\dfrac{3\sqrt{7}}{8}$ b) $\dfrac{\sqrt{2}}{4}$ c) $\dfrac{\sqrt{5}}{5}$ d) $-\dfrac{7}{11}$ e) 1 f) $1 - 2x^2$

21. a) $f(x) = x^4 - 4x^2$ b) $g(x) = -x^3 + 6x$ c) $h(x) = -(x+2)^2 x^3 (x-2)^2$

d) $f(x) = -2(x+2)^3 x^2 (x-2)(x-3)^2$ e) $f(x) = (x+2)x(x-3)$ f) $f(x) = (x+2)^2 x (x-2)^3$

22. a) b) c) d)

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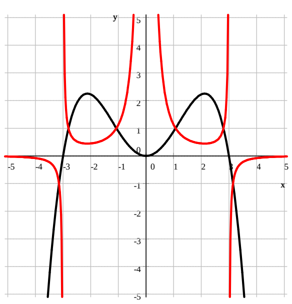
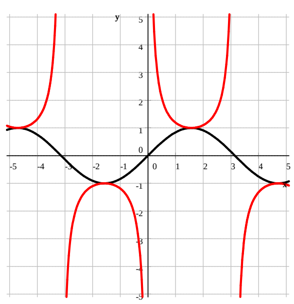
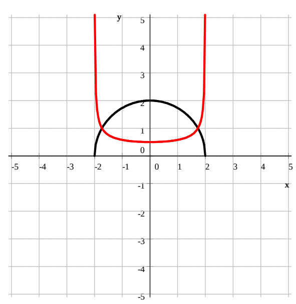

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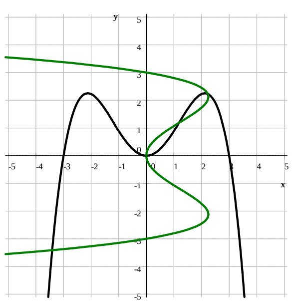
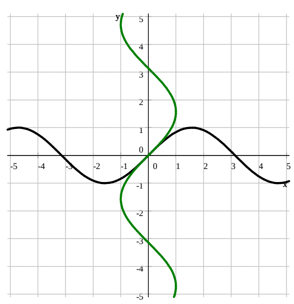
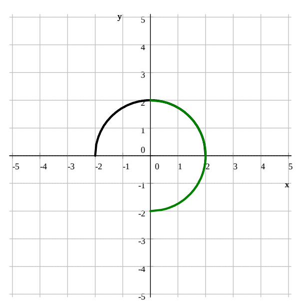

25. a) 20 000 b) 7.603 568 hours c) 34 560 d) $\dfrac{\ln 2\left(1.2^3\right)}{0.5\ln 1.2} - 6 \approx 7.603\,568$ hours

e) For any time t, we will need to wait until $t + \dfrac{2\ln 2}{\ln 1.2}$ for the sample to double. So, the doubling time is independent of t and is constant. Let t_1 be any time and t_2 is the time when the amount is doubled from $A(t_1)$. In other words, $A(t_2) = 2A(t_1)$.

$$\begin{aligned} A(t_2) &= 2A(t_1) \\ 20\,000\left(1.2^{0.5t_2}\right) &= 2\cdot 20\,000\left(1.2^{0.5t_1}\right) \\ 1.2^{0.5t_2} &= 2\left(1.2^{0.5t_1}\right) \\ \frac{1.2^{0.5t_2}}{1.2^{0.5t_1}} &= 2 \end{aligned}$$

$$\begin{aligned} 1.2^{0.5(t_2-t_1)} &= 2 \\ 0.5(t_2-t_1)\ln 1.2 &= \ln 2 \\ t_2 - t_1 &= \frac{\ln 2}{0.5\ln 1.2} = \frac{2\ln 2}{\ln 1.2} \approx 7.603\,568 \text{ hours} \end{aligned}$$

26. The sides are $3, 4,$ and 5 - there is no other one.

27. $\log_3 \dfrac{\sqrt{30}}{6}$

28. a) vertical asymptote at $x = 0$ hole at $x = 1$ b) vertical asymptote at $x = -3, 2, 5$ hole at $x = -1, 0$

29. see handout Trigonometric Identities 4.

30. a) $x > 5$ and $x \neq 6$ b) $x < -4$ or $x > 5$ c) $x \neq \dfrac{\pi}{4} + k\pi$ where $k \in \mathbb{Z}$ d) $x \neq \dfrac{k\pi}{2}$ where $k \in \mathbb{Z}$

e) $x \neq k\pi$ $k \in \mathbb{Z}$ f) $x \neq \dfrac{\pi}{4} + k\pi$ $k \in \mathbb{Z}$ g) $x \neq -\dfrac{\pi}{4} + k\pi$ $k \in \mathbb{Z}$

31. a) $\pm\dfrac{\sqrt{30}}{6}$ b) $\pm\dfrac{3\sqrt{10}}{10}$ c) $\pm\dfrac{\sqrt{26}}{26}, \pm\dfrac{5\sqrt{26}}{26}$ d) $\dfrac{4}{5}$ e) $-\dfrac{1}{2}$

32. a) $\alpha_1 \approx 47.628\,764°$, $\beta_1 \approx 94.371\,236°$, $b_1 \approx 8.097\,722$ m

$\alpha_2 \approx 132.371\,236°$, $\beta_2 \approx 9.628\,764°$, $b_2 \approx 1.358\,41$ m

b) no solution c) $\alpha \approx 85.789\,71°$, $b \approx 3.703\,965$ m, $\gamma \approx 56.210\,29°$

d) $\alpha_1 \approx 26.567\,838°$ $\gamma_1 \approx 133.432\,162°$ $c_1 \approx 27.602\,045$

$\alpha_2 \approx 153.432\,162°$ $\gamma_2 \approx 6.567\,838°$ $c_2 \approx 4.347\,503\,459$

e) $\beta \approx 15.161\,745\,79°$ $\gamma \approx 144.838\,254\,21°$ $c \approx 28.624\,256\,733\,067\,1$

f) $c = 6.531\,146$ $\alpha = 117.095\,52°$ $\beta = 42.904\,48°$

33. a) $\dfrac{13}{15}$ b) $\dfrac{13}{112}$ 34. $\dfrac{3\sqrt{15}}{4}$ 35. $\sqrt{145}$ ft or $\sqrt{433}$ ft 36. $\dfrac{47}{32}$

37. a) $\pm\dfrac{\sqrt{6}}{3}$ b) $\pm\sqrt{\dfrac{3-\sqrt{5}}{6}}$ or $\pm\sqrt{\dfrac{3+\sqrt{5}}{6}}$ 38. $\dfrac{3}{7}$ 39. $\dfrac{13}{20}$ 40. a) $\dfrac{\sqrt{3}}{3}$ b) $\dfrac{\sqrt{3}}{6}$

41. $B = \dfrac{\ln 3}{\ln 2} A$ or $B = A \log_2 3$

Solution: Let $f(x) = c \cdot d^x \quad (d > 1)$ be the exponential function expressing the population. Let x express the years passed, starting at 2007. For A, we write the equation

$$\begin{aligned} 2c &= c \cdot d^A \\ 2 &= d^A \\ \ln 2 &= A \ln d \\ \frac{\ln 2}{\ln d} &= A \end{aligned}$$

For B, we write

$$\begin{aligned} 3c &= c \cdot d^B \\ 3 &= d^B \\ \ln 3 &= B \ln d \\ \frac{\ln 3}{\ln d} &= B \end{aligned}$$

So $A = \dfrac{\ln 2}{\ln d}$ and $B = \dfrac{\ln 3}{\ln d}$. Then

$$\begin{aligned} \frac{B}{A} &= \frac{\dfrac{\ln 3}{\ln d}}{\dfrac{\ln 2}{\ln d}} = \frac{\ln 3}{\ln d} \cdot \frac{\ln d}{\ln 2} = \frac{\ln 3}{\ln 2} \\ \frac{B}{A} &= \frac{\ln 3}{\ln 2} \\ B &= A \frac{\ln 3}{\ln 2} = A \log_2 3 \end{aligned}$$