1. Simplify each of the following.

   a) $9^{-1/2}$ d) $8^{-2/3}$ g) $-8^{-2/3}$ j) $1^{3/5}$ m) $32^{-3/5}$ p) $(-16)^{3/4}$

   b) $7^{1/3}$ e) $-8^{2/3}$ h) $25^{1/4}$ k) $(-1)^{3/5}$ n) 100^{0} q) $100^{3/4}$

   c) $8^{2/3}$ f) $(-8)^{2/3}$ i) $4^{-1/2}$ l) $(-8)^{-1/3}$ o) $16^{-3/4}$ r) $36^{-1/4}$

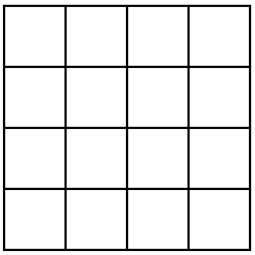
2. Compute each of the following sums.

   a) $2+4+6+...+200$ b) $11+21+31+...+651$ c) $78+83+88+...+1273$

3. How many $5-$digit numbers are there if

   a) repetition of digits is allowed

   b) repetition of digits is not allowed?



4. Use systematic listing to answer the following questions.

   a) How many rectangles are there on the picture?

   b) How many squares are there on the picture?

5. Simplify each of the following.

   a) $\sin 30^\circ - 2\cos 30^\circ + \dfrac{\sin 25^\circ}{\cos 65^\circ}$

   b) $\tan 60^\circ \cot 30^\circ - 2\sin 45^\circ \sin 60^\circ$

   c) $\sin 30^\circ \sin 45^\circ \sin 60^\circ + \cos 30^\circ \cos 45^\circ \cos 60^\circ$

   d) $\dfrac{\tan 30^\circ - \tan 45^\circ + \tan 60^\circ}{\tan 30^\circ - \tan 60^\circ}$

   e) $\cos^2 17^\circ + \sin^2 17^\circ$

   f) $\cos^2 30^\circ - \sin^2 30^\circ$

6. Suppose that α is an acute angle. (An acute angle is one between 0 and 90°)

   a) Find the exact value of $\tan\alpha$ if $\cos\alpha = \dfrac{1}{3}$.

   b) Find the exact value of $\cos\alpha$ if $\tan\alpha = 2$.

   c) Find the value of $\sin\alpha$ and $\cos\alpha$ in terms of M if we know that $\tan\alpha = M$.

7. Find an equation for each of the following lines.

   a) the line that passes through $P(-12,7)$ and is parallel to $5x-3y=15$

   b) the line that passes through $P(-12,7)$ and is perpendicular to $5x-3y=15$

   c) the line passing through $A(-5,2)$ and $B(3,-4)$

8. Prove that there is no triangle whose sides are in the ratio of $1:2:3$.

9. Solve each of the following inequalities.

   a) $x^2+4>6x$ b) $x^2 \le 6x$ c) $x^2-6x \le -11$ d) $4x-1 \ge 4x^2$

10. a) Find the exact and approximate value of the smaller angle that is formed between the line $y=2x-3$ and the positive part of the $x-$axis.

    b) Find the exact and approximate value of the smaller angle that is formed between the line $y=4x-3$ and the positive part of the $x-$axis.

    c) Based on the previous computations, determine whether doubling the slope of a line means doubling the angle or not.

11. Graph each of the following.

    a) $y=-2x^2+12x-10$ b) $(x-2)^2+(y+1)^2=9$ c) $x=y^2$

12. Consider the circle $(x-1)^2+(y+2)^2=25$. Find all points on the circle with $x-$coordinate:
    a) -9 b) -4 c) -2

13. Suppose that a and b are real numbers such that the sum of a and three times b is 12. Find each of the following.
    a) the greatest value of ab b) the smallest value of a^2+b^2 c) the greatest value of $5b^2-a^2$

14. Consider the points $A(-4,5)$ and $B(2,-9)$. Find each of the following.
    a) the length of line segment AB.
    b) the slope of the line connecting A and B
    c) the midpoint of line segment AB
    d) the equation of the perpendicular bisector of AB.

15. Consider the triangle ABC where $A(-4,4)$, $B(1,7)$, and $C(6,-1)$. Find an equation for
    a) the line that connects A and C. b) the altitude drawn to side AC.

16. Consider the circle $2y-3x+x^2+y^2=x+45$. Find an equation for the tangent line drawn to this circle at the point $P(-5,-2)$.

17. For each of the following pairs of graphs, find the coordinates of all points where they intersect.
    a) $y=x^2+7x-6$ and $y=3x-1$
    b) $y=-\frac{1}{2}x^2+5x-3$ and $y=3x-1$
    c) $y=3x^2-x+7$ and $y=x-4$
    d) $x^2+(y+4)^2=20$ and $x-3y=2$
    e) $(x-2)^2+(y+5)^2=10$ and $3y-x=-7$
    f) $x^2+(y+1)^2=16$ and $y+2x=15$

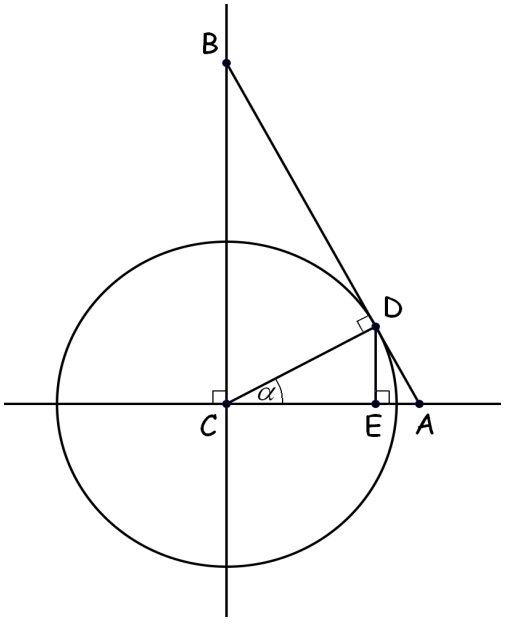
18. Consider the circles $(x+1)^2+(y-2)^2=10$ and $(x-1)^2+(y-6)^2=50$.
    a) Find the coordinates of all points where the circles intersect each other.
    b) Find an equation for the line connecting the points of intersections.

19. We are driving toward a tower. The angle of elevation is $35°$. Then we drive 50 ft toward the tower. Now the angle of elevation is $40°$. How tall is the tower? Present your answer as an approximation, accurate up to three decimal places.

20. The first row in a theater has 35 seats in it. The second row has three more seats than the first row. The third row has three more seats than the second row. And so on, each row has three more seats than the row before. If the last row has 164 seats in it, how many seats are there in the entire theater?

21. The hypotenuse of a right triangle is 50 feet long. The perimeter of the triangle is 112 feet.
    a) Find the sides of the triangle.
    b) Find the approximate value of the measure of the smallest angle in the triangle. Present your answer accurate up to three or more decimal places.

22. Consider the picture shown. The circle is centered at $C(0,0)$ and has radius 1. This circle is called the unit circle. Line AB is tangent to the unit circle, and D is the point of tangency. Let α denote angle DCE. Match each of the six trigonometric functions with the length of each of the line segments given. $\sin\alpha$, $\cos\alpha$, $\tan\alpha$, $\csc\alpha$, $\sec\alpha$, $\cot\alpha$ and AC, AD, BC, BD, CE, DE

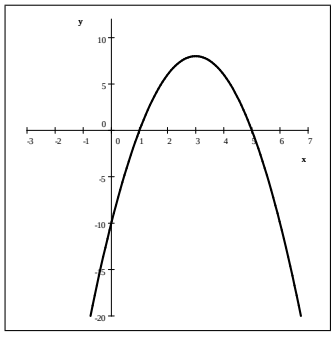
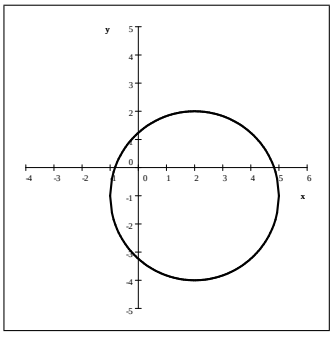
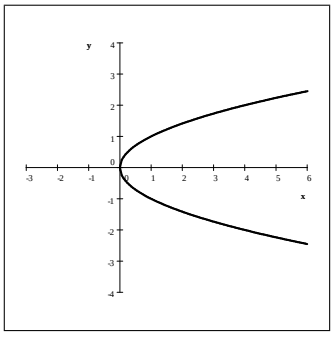


23. Find the domain for each of the given functions.
    a) $f(x)=\sqrt{x^2-4x}-\frac{1}{x-5}$
    b) $g(x)=\frac{x^2-2}{x^2+1}-\sqrt{x^2-16}$
    c) $h(x)=\frac{x^2-3x+2}{x^2-6x+5}$
    d) $m(x)=\frac{x^3-x}{3-\sqrt{2x-1}}$

24. A lattice point is a point whose both coordinates are integers. Can you find an equation of a line with
    a) no lattice points?
    b) exactly one lattice point
    c) exactly two lattice points

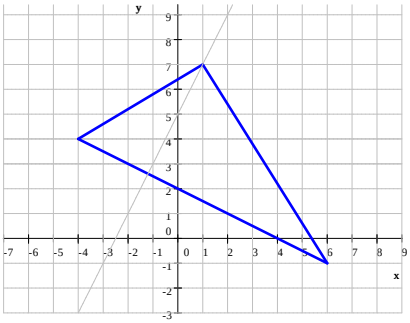
Answers

1. a) $\frac{1}{3}$ b) $\sqrt[3]{7}$ c) 4 d) $\frac{1}{4}$ e) -4 f) undefined g) $-\frac{1}{4}$ h) $\sqrt{5}$ i) $\frac{1}{2}$ j) 1
   k) undefined l) $-\frac{1}{2}$ m) $\frac{1}{8}$ n) 1 o) $\frac{1}{8}$ p) undefined q) $10\sqrt{10}$ r) $\frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}$
2. a) $10\,100$ b) $21\,515$ c) $162\,120$
3. a) $9 \cdot 10^4 = 90\,000$ b) $9 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 27\,216$
4. a) 100 b) 30
5. a) $\frac{3}{2} - \sqrt{3}$ b) $3 - \frac{\sqrt{6}}{2}$ c) $\frac{\sqrt{6}}{4}$ d) $\frac{1}{2}\sqrt{3} - 2$ e) 1 f) $\frac{1}{2}$
6. a) $2\sqrt{2}$ b) $\frac{1}{\sqrt{5}}$ c) $\sin\alpha = \frac{M}{\sqrt{M^2+1}}$ $\cos\alpha = \frac{1}{\sqrt{M^2+1}}$
7. a) $y - 7 = \frac{5}{3}(x+12)$ or $y = \frac{5}{3}x + 27$ b) $-\frac{3}{5}(x+12) = y - 7$ or $y = -\frac{3}{5}x - \frac{1}{5}$
   c) $y - 2 = -\frac{3}{4}(x+5)$ or $y + 4 = -\frac{3}{4}(x-3)$ or $y = -\frac{3}{4}x - \frac{7}{4}$
8. Such a triangle would have sides a, $2a$, and $3a$ long for some positive number a. But then the sides would fail the triangle inequality as $a + 2a \not> 3a$.
9. a) $x < 3 - \sqrt{5}$ or $x > 3 + \sqrt{5}$ b) $0 \le x \le 6$ c) no solution d) $x = \frac{1}{2}$
10. a) $\tan^{-1} 2 \approx 63.43495°$ b) $\tan^{-1} 4 \approx 75.963756\,5°$ c) no
11. a) $y = -2x^2 + 12x - 10$ b) $(x-2)^2 + (y+1)^2 = 9$ c) $x = y^2$

12. a) there is no such point b) $(-4, -2)$ c) $(-2, 2)$ and $(-2, -6)$
13. a) 12 (when $b = 2$ and $a = 6$) b) $\frac{72}{5}$ (when $b = \frac{18}{5}$ and $a = \frac{6}{5}$) c) 180 (when $b = 9$ and $a = -15$)
14. a) $\sqrt{232} = 2\sqrt{58}$ unit b) $-\frac{7}{3}$ c) $(-1, -2)$ d) $y + 1 = \frac{3}{7}(x+2)$

15. a) $-\frac{1}{2}(x+4) = y - 4$ or $y = -\frac{1}{2}x + 2$ b) $y - 7 = 2(x-1)$ or $y = 2x + 5$



16. $y + 2 = -7(x+5)$

17. a) $(1,2)$ and $(-5,-16)$ b) $(2,5)$ c) no intersection point d) $(2,0)$ and $(-4,-2)$ e) $(1,-2)$ f) no intersection point

18. a) $(0,-1)$ and $(-4,1)$ b) $y = -\frac{1}{2}x - 1$

19. $211.510\,92$ ft

20. 4378

21. a) 14 ft, 48 ft, 50 ft b) $16.260\,2°$

22. $\sin\alpha = DE$ $\cos\alpha = CE$ $\tan\alpha = AD$ $\csc\alpha = BC$ $\sec\alpha = AC$ $\cot\alpha = BD$

23. a) $(-\infty, 0] \cup [4,5) \cup (5,\infty)$ b) $(-\infty,-4] \cup [4,\infty)$ c) $(-\infty,1] \cup [5,\infty)$ d) $\left[\frac{1}{2}, 5\right) \cup (5,\infty)$

24. a) For example, $y = -\frac{1}{2}$ b) For example, $y = \sqrt{2}x$
c) There is no such line. If there are two lattice points, then there are infinitely many.

Last revised: October 10, 2018