

1. Convert each of the following to radians. Use exact values.

- a) 120° c) 45° e) 900° g) 225° i) 210°
 b) 600° d) 20° f) -50° h) -200° j) -135°

2. Convert each of the following to degrees.

- a) $\frac{5\pi}{4}$ c) $-\frac{\pi}{6}$ e) $-\frac{2}{3}\pi$ g) $-\frac{3}{2}\pi$ i) $\frac{11}{4}\pi$
 b) $\frac{7\pi}{10}$ d) $\frac{3}{5}\pi$ f) $\frac{4}{15}\pi$ h) 4π j) 1

3. Simplify each of the following.

- a) $\sin\left(\frac{3\pi}{2}\right)$ b) $\cos\left(\frac{7\pi}{4}\right)$ c) $\tan(-3\pi)$ d) $\tan\left(\frac{\pi}{2}\right)$ e) $\sec\left(-\frac{7\pi}{3}\right)$ f) $\csc\left(\frac{3\pi}{4}\right)$

4. Simplify each of the following, i.e. re-write in terms of a trigonometric function of θ .

- a) $\sin(\theta + 360^\circ)$ c) $\tan(\pi - \theta)$ e) $\sin\left(\frac{\pi}{2} - \theta\right)$ g) $\sin(\theta - \pi)$ i) $\tan(-\theta)$
 b) $\cos(\pi - \theta)$ d) $\sin(180^\circ - \theta)$ f) $\cos(\theta + \pi)$ h) $\tan(\theta + \pi)$ j) $\cos(360^\circ - \theta)$

5. Simplify each of the following.

- a) $-16^{-3/4}$ c) $(-16)^{-1/4}$ e) $\log_2 \sqrt{8}$ g) $\log_5(5^{100})$ i) $\log_{\sqrt{3}}\left(\frac{1}{9}\right)$
 b) $(-8)^{-1/3}$ d) $(-5)^0$ f) $\log_4\left(\frac{1}{\sqrt{8}}\right)$ h) $2^{\log_2 16}$ j) $\log_{0.1} 1000$

6. Simplify each of the following.

- a) $e^{-\ln A} + e^{-\ln B}$ e) $3^{\log_9 Q}$ i) $\log_2(\sec 45^\circ)$ l) $\ln\left(\frac{1}{e^{123}}\right)$
 b) $e^{-\ln A} \cdot e^{-\ln B}$ f) $(\sqrt{8})^{\log_2 T}$ j) $\log_3(\tan 60^\circ)$ m) $e^{-2 \ln 5}$
 c) $3^{\log_3 A}$ g) $\log_2(16^a)$ k) $5^{\log_{25} 3}$
 d) $9^{\log_3 P}$ h) $\log_{16}(2^a)$

7. Let $f(x) = 2x - 3$ and $g(x) = x^2 + 1$. Compute each of the following.

- a) $f(g(-2))$ b) $g(f(-2))$ c) $f(f(2))$ d) $f(f(f(2)))$ e) $f(g(f(3)))$

8. One number a is five more than twice another number b . Find the smallest value of

- a) $a^2 + b^2$ b) ab c) $a^2 - b^2$

9. Compute the exact value of each of the following.

- a) $\frac{\sin 300^\circ - \cos 450^\circ - \tan 120^\circ}{\sin 225^\circ}$ b) $\frac{\tan 240^\circ - \tan 330^\circ}{\sin 225^\circ}$ c) $\frac{\tan\left(\frac{2\pi}{3}\right) + \tan\left(\frac{\pi}{6}\right)}{1 - \tan\left(\frac{2\pi}{3}\right)\tan\left(\frac{\pi}{6}\right)}$

10. Prove each of the following identities.

a) $\sin^2 x \cos^3 x = (\sin^2 x - \sin^4 x) \cos x$

b) $(\sin x + \cos x)^2 = 1 + 2 \sin x \cos x$

c) $\cos x (\sec x - \cos x) = \sin^2 x$

d) $\frac{\sin x}{1 - \cos x} = \frac{1 + \cos x}{\sin x}$

e) $\frac{1 + \cos x}{1 - \cos x} - \frac{1 - \cos x}{1 + \cos x} = 4 \cot x \csc x$

11. a) Suppose that $\cos \alpha = -\frac{2}{3}$. Find the exact value of $\tan \alpha$.

b) Suppose that $\tan \beta = \frac{2}{3}$ and β is not in the first quadrant. Find the exact value of $\sin \beta$.

c) Suppose that $\tan \gamma = -3$. Find the exact value of $\sin \gamma \cos \gamma$

12. Solve each of the following equations. You may present your answer in degrees.

a) $\sin x = -\frac{1}{\sqrt{2}}$

c) $\tan x = -\sqrt{3}$

f) $\tan x = 0$

h) $\cos \beta = -1$

b) $\cos x = -\frac{\sqrt{3}}{2}$

d) $\sin x = -\frac{3}{2}$

g) $\sin \alpha = -\frac{\sqrt{3}}{2}$

i) $\tan \gamma = \frac{2}{3}$

13. Solve each of the following equations.

a) $\sqrt{3x+22} - \sqrt{x+8} = 2$

b) $\sqrt{3x+16} - \sqrt{x+1} = 3$

14. Solve each of the following inequalities.

a) $\frac{3x-1}{x+5} \leq 2$

b) $\frac{3}{x-7} < 1$

c) $\frac{2x+5}{x-4} > 2$

d) $\frac{5}{x-8} \leq -1$

15. Solve each of the following equations.

a) $\log_3(x-2) = 1$

d) $\frac{\log_3(x-4)}{2} - 1 = 3$

g) $3 - 5 \log_2(x+1) = 8$

b) $\ln(3x-1) = 2$

e) $\frac{2}{3} \ln(x-1) + 6 = 4$

h) $11 - 8 \ln(3x+1) = 7$

c) $\frac{\log_3(x+1) - 1}{2} = 3$

f) $\frac{2}{3} (\ln(x-1) + 6) = 4$

i) $7 - 2 \log_2(5x-1) = 3$

16. Solve each of the following equations.

a) $3^{2x-1} = \frac{1}{27}$

c) $3^{2x-1} = 10$

f) $3^{2x-10} = 3^{x-2}$

b) $2^{\frac{1}{3}x+1} = 32$

d) $e^{3x-1} = 10$

g) $3^{3x-1} = 9^{x-1}$

e) $2^{5x+1} = -1$

h) $e^{-\ln x} = 5$

17. Find the domain of each of the given functions.

a) $f(x) = \ln(x-1) + \ln(x+1)$

b) $f(x) = \frac{\ln(x+1)}{\ln(x-1)}$

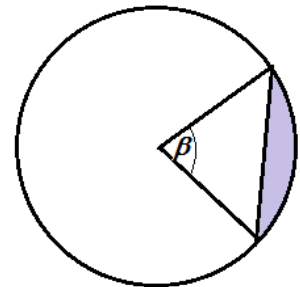
c) $f(x) = \ln\left(\frac{x+1}{x-1}\right)$

18. Suppose that C is a circle with radius 12 feet. Compute the exact and approximate values of each of the following.

a) The length of an arc subtended by a central angle of 42° .

b) The area of a sector determined by a central angle of 42° .

19. Find the radius of the circle if
- an arc subtended by a central angle of 30° has a length of 20 cm.
 - a sector of 30° has an area of 20 cm^2 .
20. Suppose that C_1 is a circle with radius 12 units and C_2 is a circle with radius 15 units. The distance between the centers is 20 units.
- Find the exact value and approximate value for the angle formed by the common tangent lines drawn to the circles.
 - Compute the distance between the two points of tangency on a common tangent line.
21. Seattle, WA and Los Angeles, CA have approximately the same longitude. The latitude of Seattle is 47.6° and that of Los Angeles is 34.1° . Find the distance to the nearest mile between the two cities. (The radius of the earth is approximately 3960 miles.)
22. a) Find the speed of the satellite that orbits the Earth above the equator at a height of 1200 miles and appears at the same point in the sky at all times. Round your answer to the nearest miles per hour. (Assume that Earth is a sphere with radius 3960 mi)
- b*) Find the speed of the satellite that orbits the Earth above Chicago at a height of 1200 miles and appears at the same point in the sky at all times. Assume that Earth is a sphere with radius 3960 mi and that the latitude of Chicago is 42°N . Round your answer to the nearest miles per hour.
23. How fast is the endpoint of the minute pointer moving on my watch if the pointer is 1.2 centimeters long? Express your answer in centimeters per minute.
24. Consider circles C_1 and C_2 with the following property. An arc subtended in C_1 by a central angle of 20° has the same length as an arc subtended in C_2 by a central angle of 15° . Find the ratio between the areas of the two circles.
25. True or false?
- If an angle α is co-terminal with 40° , then 2α is co-terminal with 80° .
 - If twice an angle 2β is co-terminal with 20° , then β is co-terminal with 10° .
26. Graph each of the following functions. State the basic properties of each of the functions.
- $f(x) = 2^x$
 - $f(x) = \log_2 x$
 - $f(x) = \cos x$
 - $f(x) = \left(\frac{1}{2}\right)^x$
 - $f(x) = \log_{1/2} x$
 - $f(x) = \tan x$
27. Compute the area of the shaded region shown on the picture. The radius of the circle is 3 meters and $\beta = 80^\circ$. Present the exact value and approximate value of the answer.
28. A water storage tank has the shape of a cylinder with diameter 10 feet. It is mounted so that the circular cross sections are vertical. If the depth of the water is 7 feet, what percentage of the total capacity is used?
29. Write an equation for the line that passes through the point $(8, -1)$ and forms a 30° angle with the positive part of the x -axis.
30. *Solve the given equation. $x^2 - 2x + 2 = \sin(xy)$



Answers

1. a) $\frac{2\pi}{3}$ b) $\frac{10\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{9}$ e) 5π f) $-\frac{5\pi}{18}$ g) $\frac{5\pi}{4}$ h) $-\frac{10\pi}{9}$ i) $\frac{7\pi}{6}$ j) $-\frac{3\pi}{4}$

2. a) 225° b) 126° c) -30° d) 108° e) -120° f) 48° g) -270° h) 720°

i) 495° j) $\left(\frac{180}{\pi}\right)^\circ \approx 57.29578^\circ$ 3. a) -1 b) $\frac{\sqrt{2}}{2}$ c) 0 d) undefined e) 2 f) $\sqrt{2}$

4. a) $\sin \theta$ b) $-\cos \theta$ c) $-\tan \theta$ d) $\sin \theta$ e) $\cos \theta$ f) $-\cos \theta$ g) $-\sin \theta$ h) $\tan \theta$
i) $-\tan \theta$ j) $\cos \theta$

5. a) $-\frac{1}{8}$ b) $-\frac{1}{2}$ c) undefined d) 1 e) $\frac{3}{2}$ f) $-\frac{3}{4}$ g) 100 h) 16 i) -4 j) -3

6. a) $\frac{1}{A} + \frac{1}{B}$ b) $\frac{1}{AB}$ c) A d) P^2 e) \sqrt{Q} f) $\sqrt{T^3}$ g) $4a$ h) $\frac{a}{4}$ i) $\frac{1}{2}$ j) $\frac{1}{2}$ k) $\sqrt{3}$ l) -123 m) $\frac{1}{25}$

7. a) 7 b) 50 c) -1 d) -5 e) 17 8. a) 5 b) $-\frac{25}{8}$ c) $-\frac{25}{3}$ 9. a) $-\frac{\sqrt{6}}{2}$ b) $-\frac{4}{3}\sqrt{6}$ c) $-\frac{\sqrt{3}}{3}$

10. a) $\sin^2 x \cos^3 x = (\sin^2 x - \sin^4 x) \cos x$

$$\text{RHS} = (\sin^2 x - \sin^4 x) \cos x = \sin^2 x (1 - \sin^2 x) \cos x = \sin^2 x \cos^2 x \cos x = \text{LHS}$$

b) $(\sin x + \cos x)^2 = 1 + 2 \sin x \cos x$

$$\text{LHS} = (\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2 \sin x \cos x = 1 + 2 \sin x \cos x = \text{RHS}$$

c) $\cos x (\sec x - \cos x) = \sin^2 x$

$$\text{LHS} = \cos x (\sec x - \cos x) = \cos x \left(\frac{1}{\cos x} - \cos x \right) = 1 - \cos^2 x = \sin^2 x = \text{RHS}$$

d) $\frac{\sin x}{1 - \cos x} = \frac{1 + \cos x}{\sin x}$

$$\text{LHS} = \frac{\sin x}{1 - \cos x} = \frac{\sin x}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} = \frac{\sin x (1 + \cos x)}{1 - \cos^2 x} = \frac{\sin x (1 + \cos x)}{\sin^2 x} = \frac{1 + \cos x}{\sin x} = \text{RHS}$$

e) $\frac{1 + \cos x}{1 - \cos x} - \frac{1 - \cos x}{1 + \cos x} = 4 \cot x \csc x$

$$\begin{aligned} \text{LHS} &= \frac{1 + \cos x}{1 - \cos x} - \frac{1 - \cos x}{1 + \cos x} = \frac{(1 + \cos x)^2 - (1 - \cos x)^2}{(1 - \cos x)(1 + \cos x)} = \frac{(1 + \cos^2 x + 2 \cos x) - (1 + \cos^2 x - 2 \cos x)}{1 - \cos^2 x} \\ &= \frac{1 + \cos^2 x + 2 \cos x - 1 - \cos^2 x + 2 \cos x}{\sin^2 x} = \frac{4 \cos x}{\sin^2 x} = 4 \frac{\cos x}{\sin x} \frac{1}{\sin x} = 4 \cot x \csc x = \text{RHS} \end{aligned}$$

11. a) $\pm \frac{\sqrt{5}}{2}$ b) $-\frac{2\sqrt{13}}{13}$ c) $-\frac{3}{10}$

12. a) $x = -45^\circ + k \cdot 360^\circ$ or $x = -135^\circ + k \cdot 360^\circ$ where $k \in \mathbb{Z}$

$$x = -\frac{\pi}{4} + 2k\pi \text{ or } x = -\frac{3\pi}{4} + 2k\pi \text{ where } k \in \mathbb{Z}$$

b) $x = \pm 150^\circ + k \cdot 360^\circ$ where $k \in \mathbb{Z}$ $x = \pm \frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$

- c) $x = -60^\circ + k \cdot 180^\circ$ where $k \in \mathbb{Z}$ $x = -\frac{\pi}{3} + k\pi$ where $k \in \mathbb{Z}$
 d) no solution e) $x = 180^\circ + k \cdot 360^\circ$ where $k \in \mathbb{Z}$ $x = \pi + 2k\pi$ where $k \in \mathbb{Z}$
 f) $x = k \cdot 180^\circ$ where $k \in \mathbb{Z}$ $x = k\pi$ where $k \in \mathbb{Z}$
 g) $\alpha = -60^\circ + k \cdot 360^\circ$ or $\alpha = -120^\circ + k \cdot 360^\circ$ where $k \in \mathbb{Z}$
 h) $\beta = 180^\circ + k \cdot 360^\circ$ where $k \in \mathbb{Z}$ i) $\gamma = 33.69007^\circ + k \cdot 180^\circ$ where $k \in \mathbb{Z}$
13. a) 1 (-7 does not work) b) 3, 0 14. a) (-5, 11] b) $(-\infty, 7) \cup (10, \infty)$ c) $(4, \infty)$ d) [3, 8)
15. a) 5 b) $\frac{1}{3}e^2 + \frac{1}{3}$ c) 2186 d) 6565 e) $\frac{1}{e^3} + 1$ f) 2 g) $-\frac{1}{2}$ h) $\frac{1}{3}\sqrt{e} - \frac{1}{3}$ i) 1
16. a) -1 b) 12 c) $\frac{1}{2}(1 + \log_3 10)$ d) $\frac{1}{3}(1 + \ln 10)$ e) no solution f) 8 g) -1 h) $\frac{1}{5}$
17. a) $(1, \infty)$ b) $(1, 2) \cup (2, \infty)$ c) $(-\infty, -1) \cup (1, \infty)$ 18. a) $\frac{14}{5}\pi \text{ ft} \approx 8.79646 \text{ ft}$ b) $\frac{84}{5}\pi \approx 52.7787566 \text{ ft}^2$
19. a) $r = \frac{120}{\pi} \text{ cm} \approx 38.1972 \text{ cm}$ b) $r = \sqrt{\frac{240}{\pi}} \text{ cm} \approx 8.740387 \text{ cm}$
20. a) $2 \sin^{-1}\left(\frac{3}{20}\right) \approx 17.253853^\circ$ b) $\sqrt{391}$ unit 21. 933 miles 22. a) $1351 \frac{\text{mi}}{\text{h}}$ b*) $1004 \frac{\text{mi}}{\text{h}}$
23. $0.04\pi \frac{\text{cm}}{\text{min}} \approx 0.1256637 \frac{\text{cm}}{\text{min}}$ 24. $\frac{A_1}{A_2} = \frac{9}{16}$

25. True or false?

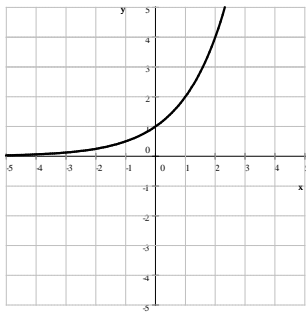
a) If an angle α is co-terminal with 40° , then 2α is co-terminal with 80° .

True: $\alpha = 40^\circ + k \cdot 360^\circ \implies 2\alpha = 80^\circ + k \cdot 720^\circ$

b) If twice an angle 2β is co-terminal with 20° , then β is co-terminal with 10° .

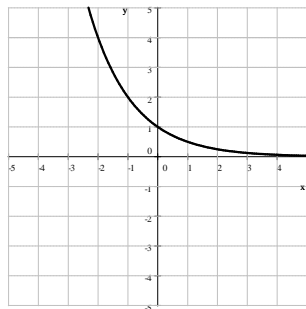
False: Let $\beta = 190^\circ$. Clearly 10° and 190° are not co-terminal, but $2\beta = 380^\circ$ which is co-terminal with 20° .

26. a) $f(x) = 2^x$



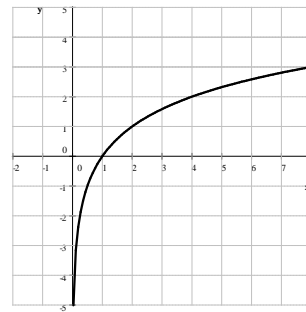
domain: \mathbb{R}
 range: $(0, \infty)$
 increasing
 one-to-one

b) $f(x) = \left(\frac{1}{2}\right)^x$



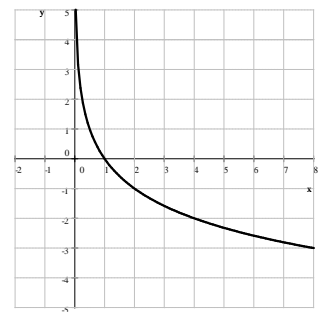
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 decreasing
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c) $f(x) = \log_2 x$



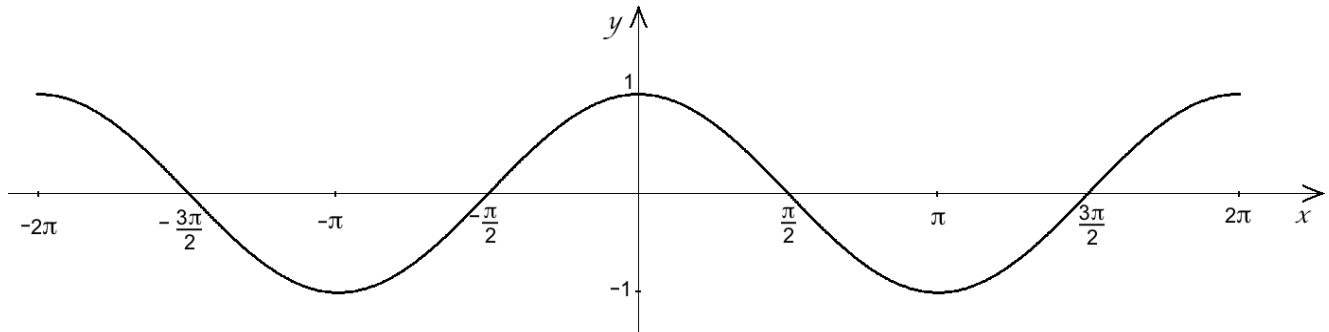
domain: $(0, \infty)$
 range: \mathbb{R}
 increasing
 one-to-one

d) $f(x) = \log_{1/2} x$

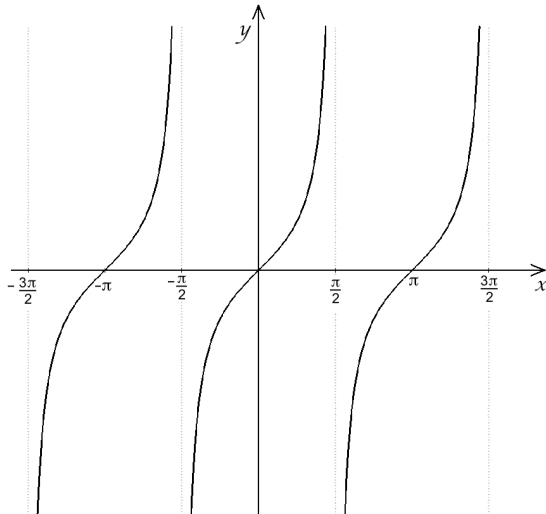


domain: $(0, \infty)$
 range: \mathbb{R}
 decreasing
 one-to-one

e) $f(x) = \cos x$



f) $f(x) = \tan x$



27. $(2\pi - 9 \sin 40^\circ \cos 40^\circ) \text{ m}^2 \approx 1.85155 \text{ m}^2$ 28. 74.77% 29. $\frac{\sqrt{3}}{3}(x - 8) = y + 1$

30. $x = 1, y = \frac{\pi}{2} + 2k\pi$ where $k \in \mathbb{Z}$

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