1. Re-write the repeating decimal $0.7\overline{16} = 0.7161616.....$ as a quotient of two integers.

2. Simplify each of the following.

   a) $\left(-2a^3b^2\right)^{-2}\left(-2a^5b^6\right)$

   b) $\dfrac{a^3b^{-2}}{a^{-1}b^{-1}}$

   c) $\dfrac{a^3-b^{-2}}{a^{-1}+b^{-1}}$

   d) $2^{-3}\left(1-3^{-2}\right)$

   e) $2^{-3}-\left(1-3^{-2}\right)$

   f) $\left(\left(2^{-2}\right)^{-2}\right)^{-2}$

   g) $\left(\dfrac{a^{-2}}{b^{-1}}\right)^{-3}$

   h) $\left(\dfrac{a^{-2}}{b^{-1}}\right)^{0}$

   i) -2^2

   j) $(-2)^{-2}$

   k) $32^{-2/5}$

   l) $-32^{-2/5}$

   m) $(-32)^{-2/5}$

   n) $\left(\dfrac{a^2b^0}{a^{-2}b^{12}}\right)^{1/2}$

   o) $\dfrac{a^{1/2}b^{2/3}}{a^{-1/2}b^{-1/3}}$

3. Let k denote 2^{1000}. Re-write each of the following in terms of k.

   a) 2^{1001} b) 2^{999} c) 2^{1002} d) 2^{3000} e) 4^{1000} f) 2^{500} g) $2^{1000}-2^{1001}+2^{1002}$

4. Let $f(x)=\dfrac{x-2}{x+5}$ and $g(x)=\dfrac{5x+2}{-x+1}$. Compute each of the following.

   a) $f(0)$

   b) $g(0)$

   c) $g(1)$

   d) $g\left(\sqrt{2}\right)$

   e) $f\left(\sqrt{20}\right)$

   f) $f(g(0))$

   g) $g(f(3))$

   h) $f(1+3)$

   i) $f(1)+f(3)$

   j) $2f(5)$

   k) $f(2\cdot 5)$

5. Simplify each of the following. Assume that x is positive.

   a) $(-16)^{-3/4}$

   b) $-16^{-3/4}$

   c) $\left(x^{2/3}\right)^{3/4}\left(\dfrac{1}{\sqrt{x}}\right)$

   d) $\dfrac{2}{\left(\sqrt{5}-1\right)^2}$

   e) $\left(1+\sqrt{3}\right)^3\left(1-\sqrt{3}\right)^3$

   f) $\left(\sqrt{3+\sqrt{5}}-\sqrt{3-\sqrt{5}}\right)^2$

   g) $\dfrac{3+\sqrt{3}}{3-\sqrt{3}}$

6. Simplify each of the following. Assume that a is positive.

   a) $\log_4\left(\dfrac{1}{2}\right)$

   b) $\log_4 8$

   c) $\log_{10} 1000$

   d) $\log_{100} 1000$

   e) $\log_2\left(\dfrac{1}{8}\right)$

   f) $\log_8 2$

   g) $\log_{\sqrt{2}} 8$

   h) $\log_4 8$

   i) $\log_4(-8)$

   j) $\log_8\left(\dfrac{1}{4}\right)$

   k) $\log_{(1/5)} 125$

   l) $\log_{0.1} 100\,000$

   m) $\log_5 1$

   n) $\log_2(\sec 45°)$

   o) $\log_a\left(a^{17}\right)$

   p) $\log_{(-5)} 25$

   q) $\log_9\left(\dfrac{1}{\sqrt{27}}\right)$

   r) $\log_a\left(\sqrt[7]{a^2}\right)$

   s) $\log_{\sqrt{3}}(81)$

   t) $\log_2\left(16^2\right)$

   u) $\log_2\left(8^x\right)$

   v) $\log_8\left(2^x\right)$

7. Simplify each of the following. Assume that x represents a positive number.

   a) $\left(\sqrt{3}\right)^6$

   b) $\left(\sqrt[7]{x}\right)^{21}$

   c) $\sqrt{\sqrt{\sqrt{x}}}$

   d) $\left(\sqrt[12]{x}\right)^4$

   e) $\sqrt[2]{\sqrt[3]{x}}$

   f) $(\sec 45°)^{10}$

   g) $\log_3(\tan 60°)$

   h) $\left(\dfrac{1}{\sqrt[3]{x}}\right)^{-12}$

   i) $\dfrac{\sqrt{x}}{\sqrt[3]{\sqrt{x}}\cdot\sqrt{\sqrt{x}}}$

   j) $\log_3\left(\dfrac{\sqrt[3]{9}}{\sqrt{3}}\right)$

   k) $\ln(\tan 45°)$

   l) $\log_2\left(\log_2\left(\sin 45° + \cos 45°\right)\right)$

   m) $\log_3(\tan 30°)$

8. Which of the following angles are co-terminal? Group co-terminal angles together.

-585° 630° 270° -225° -180° 660° 180° -540°
135° 300° 540° -90° 495° -780° -60°

9. Suppose that a and b are real numbers so that a is eight less than three times b. Find the greatest possible value of $5b^2 - a^2$.

10. Suppose that α is the smallest angle in a right triangle with sides $8\,\text{cm}$, $15\,\text{cm}$, and $17\,\text{cm}$ long. Compute the exact value of each of the following.

a) $\sin\alpha$ b) $\cos\alpha$ c) $\tan\alpha$ d) $\sin^2\alpha + \cos^2\alpha$ e) $\sec^2 x - \tan^2 x$

11. Find the exact value of each of the following expressions and simplify.

a) $\sin 30^\circ - \cos 45^\circ \tan 60^\circ$

b) $\sin^2 45^\circ \cos 45^\circ - \cos^2 30^\circ$

c) $(\tan 30^\circ + \tan 60^\circ)^2$

d) $\sin 60^\circ \sin 30^\circ + \cos 60^\circ \cos 30^\circ$

e) $(\sin 45^\circ + \cos 45^\circ)^2$

f) $\sin^2 45^\circ + \cos^2 45^\circ$

g) $(\sin 45^\circ + \cos 45^\circ)^{10}$

h) $\sin^{10} 45^\circ + \cos^{10} 45^\circ$

i) $\log_3(\tan 60^\circ)$

j) $\log_2(\sin 45^\circ)$

k) $\dfrac{\tan 45^\circ - \tan 60^\circ}{1 + \tan 60^\circ \tan 45^\circ}$

l) $\dfrac{\tan^2 60^\circ - \tan^2 30^\circ}{\tan^2 60^\circ \tan^2 30^\circ}$

12. Approximate each of the following by placing them between two consecutive integers.

a) $\log_2 100$ b) $\log_2 200$ c) $\log_{10} 2014$ d) $\log_5 2014$ e) $\log_2\left(\dfrac{1}{3}\right)$

13. Re-write each of the following as an exponential statement.

a) $x = \log_7(2y - 1)$
b) $3 = \log_B(A - 1)$
c) $p = \log_q T$
d) $x + 2 = \log_a\left(x^2 + 1\right)$
e) $2 = \log_x 20$

14. Re-write each of the following as a logarithmic statement. Assume that all variables represent positive numbers.

a) $3^{x-2} = 60$ b) $10^{2x-5} = 2012$ c) $\left(\dfrac{1}{3}\right)^{a+1} = b - 2$ d) $A^B = C$

15. Solve each of the following equations. Make sure to check your solutions.

a) $\dfrac{2x+1}{5} - \dfrac{x-1}{3} = 2x + 16$

b) $(2x-1)^2 - (x-2)^2 = x + 1$

c) $2^{x^2-5x-50} = 1$

d) $\log_6(7x + 1) = 2$

e) $5^{(x+2)/3} - 25 = 100$

f) $e^{x-7} = \dfrac{1}{e^2}$

g) $2x^5 = 32x^4$

h) $2x^5 = 32x^3$

i) $2x^5 = 32x$

j) $x^2 + x = 1$

16. Solve each of the following basic logarithmic equations.

a) $\log_3(2x - 7) = 2$
b) $\log_3\left(x^2 - 19\right) = 4$
c) $\ln(1 - 3x) = 28$
d) $\ln(2x + 1) = -3$
e) $\log_2(3x + 4) = -1$
f) $3\log_2(7x - 1) - 2 = 10$
g) $\dfrac{2}{3}\log_2(x + 2) - 1 = 7$
h) $\dfrac{2\log_2(x+2) - 1}{3} = 7$

17. Solve each of the following basic exponential equations.

a) $2^x = \dfrac{1}{16}$ c) $2^x = \dfrac{1}{\sqrt{2}}$ f) $3^{5x-1} = \sqrt{27}$ i) $3^{5x-1} = 40$ l) $e^{2x+8} = 1$

d) $2^x = 21$ g) $3^{5x-1} = -9$ j) $e^{2x+8} = 0$

b) $2^x = -8$ e) $2^x = -4$ h) $3^{5x-1} = \dfrac{1}{9}$ k) $e^{2x+8} = \dfrac{1}{e^5}$ m) $e^{2x+8} = 10$

18. Solve each of the following inequalities.

a) $3x^2 \geq 15x$ b) $28x + 88 \leq -2x^2$ c) $\dfrac{1}{3}x^2 < 4x - 12$ d) $\dfrac{1}{3}x^2 \leq 4x - 12$

19. Find the domain of each of the following functions.

a) $f(x) = \dfrac{1}{5x+3}$ d) $f(x) = \sqrt{x^2 - 10x + 24}$ g) $f(x) = \dfrac{3x-1}{x^2+4}$

b) $f(x) = \sqrt{5x+3}$ e) $f(x) = \sqrt{x-3} - 2 + \dfrac{1}{x-10}$ h) $f(x) = \sqrt{9-x^2} - \sqrt{x+1}$

c) $f(x) = \dfrac{1}{x^2 - 10x + 24}$ f) $f(x) = \dfrac{1}{\sqrt{x-3} - 2}$

20. The sum of a and three times b is 100. What is the smallest possible value of $a^2 - (2b)^2$?

21. Find the height of a buiding if the angle of elevation from the ground to its top changes from $15°$ to $25°$ as the observer advances $120\,\text{ft}$ toward it.

22. A straight pyramid has a square base with sides 18 units long. All other edges are 20 units long. Compute the angle formed by the base and a triangular face.

23. Compute the area and perimeter of a regular $12-$sided polygon written into a circle with radius $9\,\text{cm}$. Present the exact value and approximation of your answer.

24. Solve each of the given non-linear systems.

a) $\begin{cases} (x+4)^2 + (y-4)^2 = 25 \\ 2y - 1 = y + 7x + 6 \end{cases}$ b) $\begin{cases} \dfrac{2}{y} - \dfrac{1}{x} = \dfrac{6}{xy} \\ (x-6)^2 + (y-1)^2 = 10 \end{cases}$ c) $\begin{cases} x + y = -x - 1 \\ (x-2)^2 + y^2 = 5 \end{cases}$

25. Consider the circle $x^2 - 20x + y^2 = 21 + 4y$. Find the equation of the tangent line drawn to the circle at $(0, 7)$.

26. Compute the area of a triangle with sides $15\,\text{m}$, $15\,\text{m}$, and $22\,\text{m}$.

27. Suppose that α is an acute angle. (An acute angle is one between 0 and $90°$)

a) Find the exact value of $\tan\alpha$ if $\cos\alpha = \dfrac{1}{3}$.

b) Find the exact value of $\cos\alpha$ if $\tan\alpha = 2$.

c) Find the value of $\sin\alpha$ and $\cos\alpha$ in terms of M if we know that $\tan\alpha = M$.

28. Find all real numbers that are exactly four greater then their own reciprocal.

29. A circle has a radius of 10 units. A point P is 17 units away from the center of the circle. Find the measure of the angle formed by the tangent lines drawn from P to the circle.

30. A person is standing 35 feet away from a street light that is 28 ft tall. How long is his shadow if he is 5.6 feet tall?

31. Find an equation for the line that
    a) is parallel to $5x + 3y = -15$ and passes through the point $(-6, 2)$.
    b) is perpendicular to $5x + 3y = -15$ and passes through the point $(-6, 2)$.
    c) passes through the points $(3, 8)$ and $(-1, 2)$.

32. An arch is in the shape of a semicircle. At a point along the base 2 feet from an end of the arch, the height of the arch is 8 feet. Find the maximum height of the arch.

33. The shortest side of a right triangle is $12\,\text{cm}$. The difference between the other two sides is $2\,\text{cm}$. Find an approximate value for the smallest angle in the triangle.

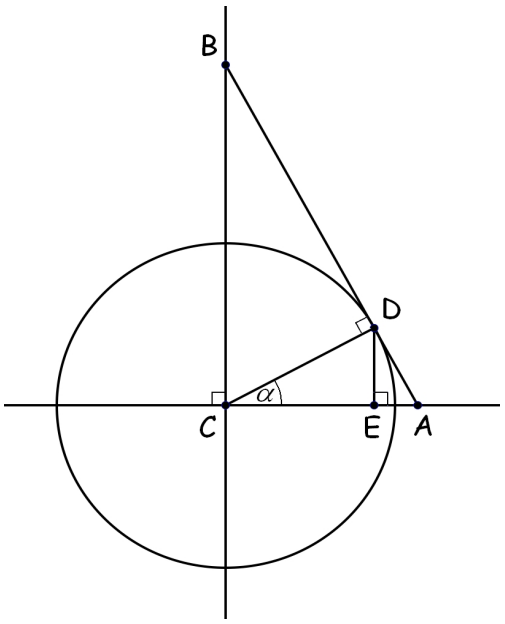
34. Graph each of the following.
    a) $f(x) = \frac{1}{2}x^2 + 2x - \frac{5}{2}$ b) $y = 2^x$ c) $(x-2)^2 + (y-1)^2 = 25$ d) $f(x) = \left(\frac{1}{2}\right)^x$

35. a) Find the exact and approximate values of the smaller angle that is formed between the positive part of the $x-$axis and the line $y = 2x - 1$.
    b) Find an equation for the straight line that passes through the point $P(4, -2)$ and forms a 60° angle with the positive part of the $x-$axis.

36. Suppose that C_1 is a circle with radius 12 units and C_2 is a circle with radius 15 units. The distance between the centers is 20 units.
    a) Find the exact value and approximate value for the angle formed by the common tangent lines drawn to the circles.
    b) Compute the distance between the two points of tangency on a common tangent line.

37. The hypotenuse of a right triangle is 50 feet long. The perimeter of the triangle is 112 feet.
    a) Find the sides of the triangle.
    b) Find the approximate value of the measure of the smallest angle in the triangle. Present your answer accurate up to three or more decimal places.

38. Consider the picture shown. The circle is centered at $C(0, 0)$ and has radius 1. This circle is called the unit circle. Line AB is tangent to the unit circle, and D is the point of tangency. Let α denote angle DCE. Match each of the six trigonometric functions with the length of each of the line segments given. $\sin\alpha, \cos\alpha, \tan\alpha, \csc\alpha, \sec\alpha, \cot\alpha$ and $AC,\ AD, BC, BD, CE, DE$



39. Prove each of the given identities.
    a) $\tan x + \dfrac{\cos x}{1 + \sin x} = \dfrac{1}{\cos x}$ c) $\dfrac{1}{1 - \sin x} - \dfrac{1}{1 + \sin x} = 2\tan x \sec x$ e) $\dfrac{1 + \tan^2 x}{1 - \tan^2 x} = \dfrac{1}{\cos^2 x - \sin^2 x}$
    b) $\tan^2 x + 1 = \sec^2 x$ d) $\tan x + \cot x = \sec x \csc x$

40. * Suppose that $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$.
    a) How many relations are possible with domain A and range any non-empty subset of B?
    b) How many fuctions are possible with domain A and range any non-empty subset of B?
    c) How many one-to-one fuctions are possible with domain A and range any non-empty subset of B?

41. * Find the equation of the circle that passes through $A(1, -3)$, $B(3, 3)$, and $C(7, -5)$.

42. * Solve the equation $\sqrt{x^2+x+3}+x^2+x-7=2$

43. * Simplify $\dfrac{\sqrt{1009+\sqrt{2017}}-\sqrt{1009-\sqrt{2017}}}{\sqrt{2}}$

Answers

1. $\dfrac{709}{990}$ 2. a) $-\dfrac{b^2}{2a}$ b) $\dfrac{a^4}{b}$ c) $\dfrac{a^4b^2-a}{ab+b^2}$ d) $\dfrac{1}{9}$ e) $-\dfrac{55}{72}$ f) $\dfrac{1}{256}$ g) $\dfrac{a^6}{b^3}$ h) 1 i) -4 j) $\dfrac{1}{4}$ k) $\dfrac{1}{4}$ l) $-\dfrac{1}{4}$ m) undefined n) $\dfrac{a^2}{b^6}$ o) ab

3. a) $2k$ b) $\dfrac{k}{2}$ c) $4k$ d) k^3 e) k^2 f) $\sqrt{k}$ g) $3k$

4. a) $-\dfrac{2}{5}$ b) 2 c) undefined d) $-7\sqrt{2}-12$ e) $\dfrac{14}{5}\sqrt{5}-6$ f) 0 g) 3 h) $\dfrac{2}{9}$ i) $-\dfrac{1}{24}$ j) $\dfrac{3}{5}$ k) $\dfrac{8}{15}$

5. a) undefined b) $-\dfrac{1}{8}$ c) 1 d) $\dfrac{\sqrt{5}+3}{4}$ e) -8 f) 2 g) $\sqrt{3}+2$

6. a) $-\dfrac{1}{2}$ b) $\dfrac{3}{2}$ c) 3 d) $\dfrac{3}{2}$ e) -3 f) $\dfrac{1}{3}$ g) 6 h) $\dfrac{3}{2}$ i) undefined j) $-\dfrac{2}{3}$ k) -3 l) -5 m) 0 n) $\dfrac{1}{2}$ o) 17 p) undefined q) $-\dfrac{3}{4}$ r) $\dfrac{2}{7}$ s) 8 t) 8 u) $3x$ v) $\dfrac{x}{3}$

7. a) 27 b) x^3 c) $\sqrt[8]{x}$ d) $\sqrt[3]{x}$ e) $\sqrt[6]{x}$ f) 32 g) $\dfrac{1}{2}$ h) x^4 i) $\sqrt[12]{x}$ j) $\dfrac{1}{6}$ k) 0 l) -1 m) $-\dfrac{1}{2}$

8. Group 1: $135°, 495°, -225°, -585°$ Group 2: $-60°, 300°, 660°, -780°$ Group 3: $180°, -180°, 540°, -540°$ Group 4: $-90°, 270°, 630°$ 9. 80 10. a) $\dfrac{8}{17}$ b) $\dfrac{15}{17}$ c) $\dfrac{8}{15}$ d) 1 e) 1

11. a) $\dfrac{1}{2}\left(1-\sqrt{6}\right)$ b) $\dfrac{1}{2}-\dfrac{1}{4}\sqrt{3}$ c) $\dfrac{16}{3}$ d) $\dfrac{\sqrt{3}}{2}$ e) 2 f) 1 g) 32 h) $\dfrac{1}{16}$ i) $\dfrac{1}{2}$ j) $-\dfrac{1}{2}$ k) $\sqrt{3}-2$ l) $\dfrac{8}{3}$

12. a) $6<\log_2 100<7$ b) $7<\log_2 200<8$ c) $3<\log_{10} 2014<4$ d) $4<\log_5 2014<5$ e) $-2<\log_2\left(\dfrac{1}{3}\right)<-1$ 13. a) $7^x=2y-1$ b) $B^3=A-1$ c) $q^p=T$ d) $a^{x+2}=x^2+1$ e) $x^2=20$

14. a) $x-2=\log_3 60$ b) $2x-5=\log_{10} 2012$ c) $a+1=\log_{1/3}(b-2)$ d) $B=\log_A C$

15. a) -8 b) $-1, \dfrac{4}{3}$ c) $-5, 10$ d) 5 e) 7 f) 5 g) $0, 16$ h) $-4, 0, 4$ i) $-2, 0, 2$ j) $\dfrac{-1\pm\sqrt{5}}{2}$

16. a) 8 b) ± 10 c) $\dfrac{1}{3}-\dfrac{e^{28}}{3}$ d) $\dfrac{1}{2e^3}-\dfrac{1}{2}$ e) $-\dfrac{7}{6}$ f) $\dfrac{17}{7}$ g) 4094 h) 2046

17. a) -4 b) no solution c) $-\dfrac{1}{2}$ d) $\log_2 21$ e) no solution f) $\dfrac{1}{2}$ g) no solution h) $-\dfrac{1}{5}$ i) $\dfrac{1}{5}(1+\log_3 40)$ j) no solution k) $-\dfrac{13}{2}$ l) -4 m) $\dfrac{1}{2}(-8+\ln 10)$

18. a) $(-\infty, 0]\cup[5, \infty)$ b) $\left[-7-\sqrt{5}, -7+\sqrt{5}\right]$ c) no solution d) 6

19. a) $x\neq -\dfrac{3}{5}$ b) $x\geq -\dfrac{3}{5}$ c) $x\neq 4, 6$ d) $x\leq 4$ or $x\geq 6$ e) $x\geq 3$ and $x\neq 10$ f) $x\geq 3$ and $x\neq 7$ g) $\mathbb{R}$ h) $-1\leq x\leq 3$

20. -8000 21. $75.588\,461$ ft 22. $\cos^{-1}\left(\dfrac{9}{\sqrt{319}}\right) \approx 59.74142°$

23. $P = 216\sin 15°$ cm $\approx 55.904\,91$ cm $\quad A = 972\sin 15°\cos 15°$ cm$^2 \approx 243$ cm^2

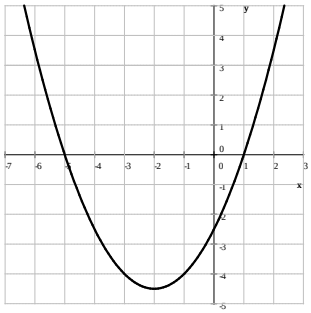
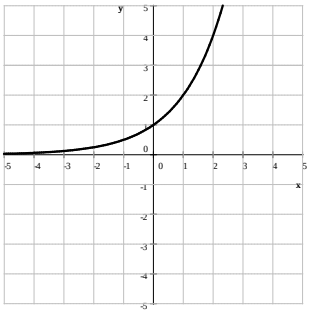
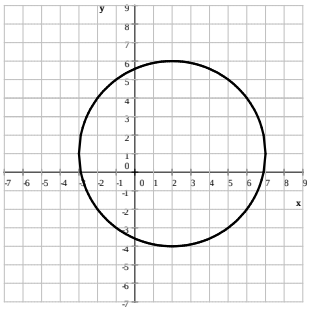
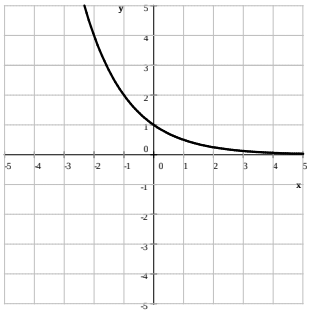
24. a) $(0,7)$ and $(-1,0)$ b) $(5,4)$ and $(3,0)$ c) $(0,-1)$ d) $(-2,1)$

25. $y = 2x+7$ 26. $A = 22\sqrt{26}$ m$^2 \approx 112.178\,43$ m^2 27. a) $2\sqrt{2}$ b) $\dfrac{1}{\sqrt{5}}$ c) $\sin\alpha = \dfrac{M}{\sqrt{M^2+1}}$ $\cos\alpha = \dfrac{1}{\sqrt{M^2+1}}$

28. $2-\sqrt{5}, 2+\sqrt{5}$ 29. $2\sin^{-1}\left(\dfrac{10}{17}\right) \approx 72.063\,758°$ 30. 8.75 ft

31. a) $y = -\dfrac{5}{3}x - 8$ b) $y = \dfrac{3}{5}x + \dfrac{28}{5}$ c) $y = \dfrac{3}{2}x + \dfrac{7}{2}$ 32. 17 feet 33. $\sin^{-1}\left(\dfrac{12}{37}\right) \approx 18.924\,64°$

34. a) $f(x) = \dfrac{1}{2}x^2 + 2x - \dfrac{5}{2}$ b) $y = 2^x$ c) $(x-2)^2 + (y-1)^2 = 25$ d) $f(x) = \left(\dfrac{1}{2}\right)^x$

35. a) $\tan^{-1} 2 \approx 63.434949$ b) $y + 2 = \sqrt{3}(x-4)$ 36. a) $2\sin^{-1}\left(\dfrac{3}{20}\right) \approx 17.253\,853°$ b) $\sqrt{391}$ unit

37. a) $\tan x + \dfrac{\cos x}{1+\sin x} = \dfrac{1}{\cos x}$

$$\text{LHS} = \tan x + \frac{\cos x}{1+\sin x} = \frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x}$$

bring to common denominator:

$$\begin{aligned}\frac{\sin x}{\cos x} + \frac{\cos x}{1+\sin x} &= \frac{\sin x\,(1+\sin x)}{\cos x\,(1+\sin x)} + \frac{\cos x\,(\cos x)}{(1+\sin x)\,(\cos x)} = \frac{\sin x\,(1+\sin x) + \cos^2 x}{\cos x\,(1+\sin x)} \\ &= \frac{\sin x + \sin^2 x + \cos^2 x}{\cos x\,(1+\sin x)} = \frac{\sin x + 1}{\cos x\,(1+\sin x)} = \frac{1}{\cos x} = \text{RHS}\end{aligned}$$

b) $\tan^2 x + 1 = \sec^2 x$

$$\text{LHS} = \tan^2 x + 1 = \left(\frac{\sin x}{\cos x}\right)^2 + 1 = \frac{\sin^2 x}{\cos^2 x} + 1$$

bring to common denominator

$$\frac{\sin^2 x}{\cos^2 x} + 1 = \frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{\sin^2 x + \cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x = \text{RHS}$$

c) $\dfrac{1}{1-\sin x} - \dfrac{1}{1+\sin x} = 2\tan x \sec x$

Solution: We will again bring the two fractions on the left-hand side to the common denominator. Because $\sin^2 x + \cos^2 x = 1$, we also have that $1 - \sin^2 x = \cos^2 x$

$$\begin{aligned} \text{LHS} &= \frac{1}{1-\sin x} - \frac{1}{1+\sin x} = \frac{1+\sin x}{(1-\sin x)(1+\sin x)} - \frac{1-\sin x}{(1+\sin x)(1-\sin x)} \\ &= \frac{1+\sin x-(1-\sin x)}{(1-\sin x)(1+\sin x)} = \frac{1+\sin x-1+\sin x}{1-\sin^2 x} = \frac{2\sin x}{\cos^2 x} = 2\cdot\frac{\sin x}{\cos x}\cdot\frac{1}{\cos x} \\ &= 2\tan x \sec x = \text{RHS} \end{aligned}$$

d) $\tan x + \cot x = \sec x \csc x$

Solution: We will bring the two fractions to the common denominator.

$$\begin{aligned} \text{LHS} &= \tan x + \cot x = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x} = \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} \\ &= \frac{1}{\sin x \cos x} = \frac{1}{\sin x}\cdot\frac{1}{\cos x} = \sec x \csc x = \text{RHS} \end{aligned}$$

e) $\dfrac{1+\tan^2 x}{1-\tan^2 x} = \dfrac{1}{\cos^2 x - \sin^2 x}$

Solution: We will bring fractions to the common denominator:

$$\begin{aligned} \text{LHS} &= \frac{1+\tan^2 x}{1-\tan^2 x} = \frac{1+\left(\dfrac{\sin x}{\cos x}\right)^2}{1-\left(\dfrac{\sin x}{\cos x}\right)^2} = \frac{1+\dfrac{\sin^2 x}{\cos^2 x}}{1-\dfrac{\sin^2 x}{\cos^2 x}} = \frac{\dfrac{\cos^2 x}{\cos^2 x}+\dfrac{\sin^2 x}{\cos^2 x}}{\dfrac{\cos^2 x}{\cos^2 x}-\dfrac{\sin^2 x}{\cos^2 x}} \\ &= \frac{\dfrac{\cos^2 x+\sin^2 x}{\cos^2 x}}{\dfrac{\cos^2 x-\sin^2 x}{\cos^2 x}} = \frac{\dfrac{1}{\cos^2 x}}{\dfrac{\cos^2 x-\sin^2 x}{\cos^2 x}} \end{aligned}$$

To divide is to multiply by the reciprocal:

$$\frac{\dfrac{1}{\cos^2 x}}{\dfrac{\cos^2 x-\sin^2 x}{\cos^2 x}} = \frac{1}{\cos^2 x}\cdot\frac{\cos^2 x}{\cos^2 x-\sin^2 x} = \frac{1}{\cos^2 x-\sin^2 x} = \text{RHS}$$

38. a) 15^4 b) 4^4 c) 24 39. $(x-5)^2+(y+1)^2 = 20$ 40. Hint: try introducing a new variable!

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