1. a) Compute the exact value of $\sin\alpha$ if we know that $\tan\alpha = \frac{5}{2}$.

   b) Suppose that $\tan\beta = S$. Express $\cos\beta$ in terms of S.

2. Simplify each of the given expressions. Rationalize the denominators.

   a) $\sin^2 27^\circ + \cos^2 27^\circ$ b) $\dfrac{\sin 45^\circ - \tan 30^\circ}{\sin 45^\circ + \tan 30^\circ}$ c) $\sin 30^\circ \cos 60^\circ - \cos 30^\circ \sin 60^\circ$

3. Compute each of the following sums.

   a) $210 + 223 + 236 + ... + 1302$ c) $120 + 130 + 140 + ... + 1010$

   b) $100 + 103 + 106 + ... + 847$ d) $\dfrac{1}{2019} + \dfrac{2}{2019} + \dfrac{3}{2019} + ... + \dfrac{2018}{2019}$

4. Find all points of intersections of the circles given.

   a) $(x+4)^2 + (y-4)^2 = 25$ and $(x-10)^2 + (y-2)^2 = 125$

   b) $x^2 + (y-4)^2 = 25$ and $(x-6)^2 + (y-1)^2 = 10$

   c) $(x+4)^2 + (2x-7)^2 = 40$ and $(x+10)^2 + (y-5)^2 = 175$

   d) $(x+5)^2 + (y-3)^2 = 13$ and $(x-4)^2 + (y+3)^2 = 52$

5. The sum of a and three times b is 20. What is the smallest possible value of $a^2 + b^2$?

6. Use rational exponents to simplify each of the following. Assume that x is positive.

   a) $\sqrt[3]{x\sqrt[4]{x^3\sqrt{x}}}$ b) $\sqrt[7]{\sqrt{x^3}\sqrt[3]{x^2}\sqrt[6]{x}}$ c) $\dfrac{\sqrt[3]{x\sqrt{x^7}\sqrt[5]{x^2}}}{\sqrt[6]{x^5}}$ d) $\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{x}}}}}$

7. Simplify each of the following.

   a) $\log_3(9^x)$ c) $\log_2\left(\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{2}}}}}\right)$ e) $2^{\log_2 4} + 2^{\log_2 8}$ g) $\log_9\left(\dfrac{1}{\sqrt{3}}\right)$

   b) $\log_8(2^y)$ d) $\log_2\left(\log_2\left(\sqrt{\sqrt{\sqrt{\sqrt{\sqrt{2}}}}}\right)\right)$ f) $2^{\log_2 4 + \log_2 8}$ h) $\log_8(-32)$

8. Solve each of the following basic logarithmic equations.

   a) $\dfrac{2\log_2(3x-1) - 4}{3} = 2$ c) $4\ln(x-2) + 1 = -2$ f) $\dfrac{2\log_3(8x+1) - 1}{5} = 3$

   d) $4(\ln(x-2) + 1) = -2$

   b) $\dfrac{2}{3}\log_2(3x-1) - 4 = 2$ e) $\dfrac{2}{5}\log_3(8x+1) - 1 = 3$ g) $\log_5(3x-1) = -2$

9. Solve each of the following basic exponential equations.

   a) $3^x = 243$ d) $2^{4x-1} = 42$ g) $e^{2x-8} = e^{10}$

   b) $3^x = 38$ e) $10^{3x-1} = 10^{41}$ h) $e^{3x-7} = 5$

   c) $2^{4x-1} = 128$ f) $10^{3x-1} = 19$ i) $3^{2x-5} = -27$

10. Solve each of the given inequalities.

    a) $x^2 > 25$ b) $x^2 < x$ c) $x^2 - 4x > -13$ d) $x^2 - 6x \geq 1$

11. A number is 3 greater than its own reciprocal. Find this number.

12. Prove the given identities.

a) $\dfrac{1+\sin x}{1-\sin x}-\dfrac{1-\sin x}{1+\sin x}=4\tan x\sec x$

b) $1+\tan^2 x=\sec^2 x$

c) $\dfrac{\cot x-1}{\cot x+1}=\dfrac{1-\tan x}{1+\tan x}$

d) $\sin^2 x\cos^3 x=\left(\sin^2 x-\sin^4 x\right)\cos x$

e) $(\sin x+\cos x)^2=1+2\sin x\cos x$

f) $\cos x\left(\sec x-\cos x\right)=\sin^2 x$

g) $\dfrac{\sin x}{1-\cos x}=\dfrac{1+\cos x}{\sin x}$

13. Label each of the following as true or false.

a) $\sin(30^\circ+30^\circ)=\sin 30^\circ+\sin 30^\circ$

b) $\log_3 9+\log_3 27=\log_3 36$

c) $\log_2 4+\log_2 8=\log_2 32$

d) If α and β are co-terminal angles, then 2α and 2β are also co-terminal.

e) If 2α and 2β are co-terminal angles, then α and β are also co-terminal.

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e) If 2α and 2β are co-terminal angles, then α and β are also co-terminal.

14. Find the height of a tree if the angle of elevation of its top changes from 18° to 31° as the observer advances $200\,\text{ft}$ toward its base. Present an approximate value.

15. Consider the parabola $y=\dfrac{1}{2}x^2-4x+7$. Find the equation of all tangent lines drawn to the parabola that have slope $m=2$.

16. Graph each of the following functions.

a) $f(x)=\dfrac{1}{x}$

b) $f(x)=\dfrac{1}{x^2}$

c) $f(x)=|x|$

d) $f(x)=x^3$

e) $f(x)=2^x$

f) $f(x)=\left(\dfrac{1}{2}\right)^x$

g) $f(x)=\log_2 x$

h) $f(x)=\log_{1/2} x$

17. Graph the following functions in the same coordinate system

$f(x)=3^x$ $\quad g(x)=\left(\dfrac{1}{3}\right)^x$ $\quad h(x)=\log_3 x$ $\quad j(x)=\log_{(1/3)} x$

18. Find the domain of each of the given functions.

a) $f(x)=\dfrac{3x-5}{x^2+4}$

b) $g(x)=\sqrt{x-5}+\sqrt{12-x}$

c) $h(x)=\dfrac{x^2-36}{5-\log_2(x-1)}$

19. The hypotenuse of a right triangle is 29 units long. Find the other two sides if we also know that the perimeter of the triangle is 70 units.

20. We draw the common tangent lines to two circles. Circle 1 has radius $r_1=8$ units. The distance between the centers of the two circle is 6 units. The angle formed by the two common tangent lines is 60°. Find the exact value of the radius of circle 2.

21. A 5.6 ft tall person is standing $40\,\text{ft}$ away from a street light that is $28\,\text{ft}$ tall. How long is her shadow?

22. Solve: $\log_x(x+20)=2$

23. The circumference of a circle with radius r is $C=2\pi r$. Suppose the
Earth was a perfect sphere with a perfectly fitting belt of $24\,000$ miles surrounding it along a great circular path. Suppose the belt was cut, and one hundred feet of additional material was added to the belt, with the "loose fit" evenly distributed around the earth so that the new belt was still circular with its center at the center of the earth. Which of the following best describes the resulting situation?

A) You colud slip a piece of paper between the belt and the earth.

B) You could get your fingers under the belt.

C) You could crawl under the belt.

D) You could walk upright under the belt.

E) You could drive a truck under the belt.

24. Given numbers a and b, the arithmetic mean of a and b is $\frac{a+b}{2}$. The arithmetic mean can be defined for three numbers: $\frac{a+b+c}{3}$, four numbers: $\frac{a+b+c+d}{4}$, five numers, and so on.

Given positive numbers a and b, the geometric mean of a and b is $\sqrt{ab}$. The geometric mean can be defined for three numbers: $\sqrt[3]{abc}$, four numbers: $\sqrt[4]{abcd}$, five numers, and so on.

a) Compute the arithmetic and geometric means of 8 and 32.

b) Prove the following. If a and b are any positive numbers, then $\sqrt{ab} \leq \frac{a+b}{2}$

and the two sides are equal if and only if $a = b$.

Answers

1. a) $\frac{5}{\sqrt{29}}$ b) $\frac{1}{\sqrt{S^2+1}}$ 2. a) 1 b) $5-2\sqrt{6}$ c) $-\frac{1}{2}$ 3. a) 64 260 b) 118 375 c) 50 850 d) 1009

4. a) $(0,7)$ and $(-1,0)$ b) $(5,4)$ and $(3,0)$ c) no intersection point d) $(-2,1)$ 5. 40

6. a) $\sqrt[8]{x^5}$ b) $\sqrt[3]{x}$ c) $\sqrt[5]{x^4}$ d) $\sqrt[32]{x}$ 7. a) $2x$ b) $\frac{y}{3}$ c) $\frac{1}{32}$ d) -5 e) 12 f) 32 g) $-\frac{1}{4}$ h) undefined

8. a) 11 b) 171 c) $e^{-3/4}+2=\frac{1}{\left(\sqrt[4]{e}\right)^3}+2$ d) $e^{-3/2}+2=\frac{1}{\left(\sqrt{e}\right)^3}+2$ e) 7381 f) 820 g) $\frac{26}{75}$

9. a) 5 b) $\log_3 38$ c) 2 d) $\frac{1}{4}\left(1+\log_2 42\right)$ e) 14 f) $\frac{1}{3}\left(1+\log_{10} 19\right)$ g) 9 h) $\frac{1}{3}\left(7+\ln 5\right)$ i) no solution

10. a) $(-\infty,-5)\cup(5,\infty)$ b) $(0,1)$ c) $\mathbb{R}$ d) $\left(-\infty,-\sqrt{10}+3\right]\cup\left[\sqrt{10}+3,\infty\right)$ 11. $\frac{3-\sqrt{13}}{2}$, $\frac{3+\sqrt{13}}{2}$

12. a) $\frac{1+\sin x}{1-\sin x}-\frac{1-\sin x}{1+\sin x}=4\tan x\sec x$

$$\begin{aligned}\text{LHS} &= \frac{1+\sin x}{1-\sin x}-\frac{1-\sin x}{1+\sin x}=\frac{(1+\sin x)^2-(1-\sin x)^2}{(1-\sin x)(1+\sin x)}=\frac{\left(1+\sin^2 x+2\sin x\right)-\left(1+\sin^2 x-2\sin x\right)}{1-\sin^2 x}\\ &= \frac{1+\sin^2 x+2\sin x-1-\sin^2 x+2\sin x}{\cos^2 x}=\frac{4\sin x}{\cos^2 x}=4\frac{\sin x}{\cos x}\frac{1}{\cos x}=4\tan x\sec x=\text{RHS}\end{aligned}$$

b) $1+\tan^2 x=\sec^2 x$

$$\text{LHS}=1+\tan^2 x=1+\frac{\sin^2 x}{\cos^2 x}=\frac{\cos^2 x}{\cos^2 x}+\frac{\sin^2 x}{\cos^2 x}=\frac{1}{\cos^2 x}=\sec^2 x=\text{RHS}$$

c) $\frac{\cot x-1}{\cot x+1}=\frac{1-\tan x}{1+\tan x}$

$$\text{LHS}=\frac{\cot x-1}{\cot x+1}=\frac{\frac{1}{\tan x}-1}{\frac{1}{\tan x}+1}\cdot\frac{\tan x}{\tan x}=\frac{1-\tan x}{1+\tan x}=\text{RHS}$$

d) $\sin^2 x\cos^3 x=\left(\sin^2 x-\sin^4 x\right)\cos x$

$$\text{RHS}=\left(\sin^2 x-\sin^4 x\right)\cos x=\sin^2 x\left(1-\sin^2 x\right)\cos x=\sin^2 x\cos^2 x\cos x=\text{LHS}$$

e) $(\sin x+\cos x)^2=1+2\sin x\cos x$

$$\text{LHS}=(\sin x+\cos x)^2=\sin^2 x+\cos^2 x+2\sin x\cos x=1+2\sin x\cos x=\text{RHS}$$

f) $\cos x\left(\sec x-\cos x\right)=\sin^2 x$

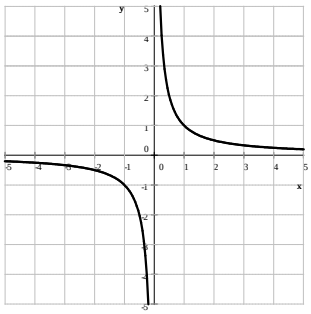
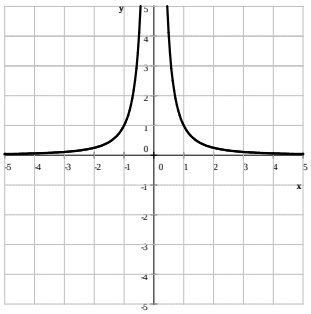
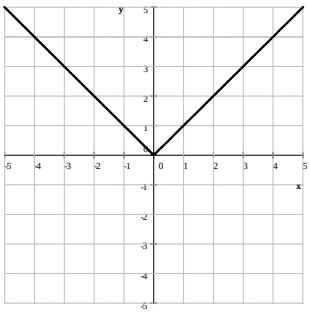
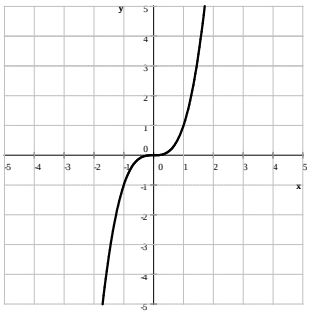
$$\text{LHS}=\cos x\left(\sec x-\cos x\right)=\cos x\left(\frac{1}{\cos x}-\cos x\right)=1-\cos^2 x=\sin^2 x=\text{RHS}$$

g) $\frac{\sin x}{1-\cos x}=\frac{1+\cos x}{\sin x}$

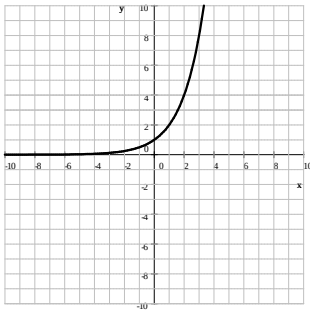
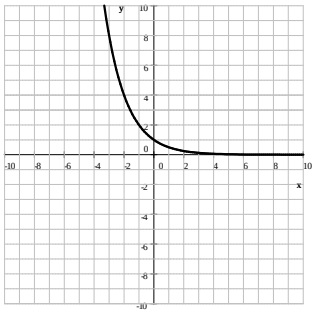
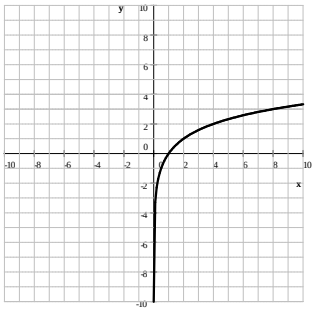
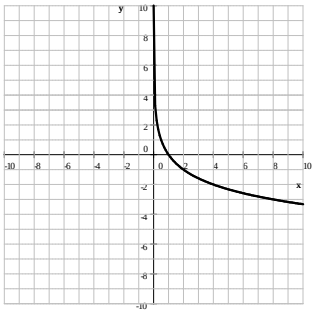
$$\text{LHS}=\frac{\sin x}{1-\cos x}=\frac{\sin x}{1-\cos x}\cdot\frac{1+\cos x}{1+\cos x}=\frac{\sin x\left(1+\cos x\right)}{1-\cos^2 x}=\frac{\sin x\left(1+\cos x\right)}{\sin^2 x}=\frac{1+\cos x}{\sin x}=\text{RHS}$$

13. a) false b) false c) true d) true e) false 14. 141.502 35 ft 15. $y=2x-11$

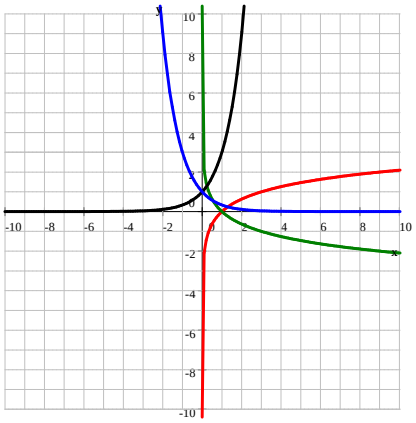
16. a) $f(x) = \dfrac{1}{x}$ b) $f(x) = \dfrac{1}{x^2}$ c) $f(x) = |x|$ d) $f(x) = x^3$

e) $f(x) = 2^x$ f) $f(x) = \left(\dfrac{1}{2}\right)^x$ g) $f(x) = \log_2 x$ h) $f(x) = \log_{1/2} x$

17. $f(x) = 3^x$ is the black graph
$g(x) = \left(\dfrac{1}{3}\right)^x$ is the blue graph
$h(x) = \log_3 x$ is the red graph
$j(x) = \log_{(1/3)} x$ is the green graph



18. a) $\mathbb{R}$ b) $[5, 12]$ c) $(1, 33) \cup (33, \infty)$

19. 20 units and 21 units 20. 11 units or 5 units

21. 10 ft 22. 5

23. E)

The radius of Earth: $R = \dfrac{24000}{2\pi}$ The new radius: $\dfrac{24 + 100}{2\pi}$. The difference: $\dfrac{24100}{2\pi} - \dfrac{24000}{2\pi} \approx 15.915\,5$

24. a) arithmetic mean: 20 b) geometric mean: 16

b) $\left(\sqrt{a} - \sqrt{b}\right)^2 \geq 0$

$a + b - 2\sqrt{ab} \geq 0$

$a + b \geq 2\sqrt{ab}$

$\dfrac{a+b}{2} \geq \sqrt{ab}$