1. Convert each of the following to radians. Use exact values.

   a) $120°$ c) $45°$ e) $900°$ g) $225°$ i) $210°$
   b) $600°$ d) $20°$ f) $-50°$ h) $-200°$ j) $-135°$

2. Convert each of the following to degrees.

   a) $\frac{5\pi}{4}$ c) $-\frac{\pi}{6}$ e) $-\frac{2}{3}\pi$ g) $-\frac{3}{2}\pi$ i) $\frac{11}{4}\pi$
   b) $\frac{7\pi}{10}$ d) $\frac{3}{5}\pi$ f) $\frac{4}{15}\pi$ h) 4π j) 1

3. Find the exact value of each of the following.

   a) $\sin 210°$ f) $\tan 315°$ k) $\cos 180°$ p) $\sin 600°$ u) $\tan(-90°)$
   b) $\cos 210°$ g) $\sin(-60°)$ l) $\tan 180°$ q) $\cos 600°$ v) $\sin(-225°)$
   c) $\tan 210°$ h) $\cos(-60°)$ m) $\sin(-150°)$ r) $\tan 600°$ w) $\cos(-225°)$
   d) $\sin 315°$ i) $\tan(-60°)$ n) $\cos(-150°)$ s) $\sin(-90°)$ x) $\tan(-225°)$
   e) $\cos 315°$ j) $\sin 180°$ o) $\tan(-150°)$ t) $\cos(-90°)$

4. Simplify each of the following.

   a) $\sin\left(\frac{3\pi}{2}\right)$ b) $\cos\left(\frac{7\pi}{4}\right)$ c) $\tan(-3\pi)$ d) $\tan\left(\frac{\pi}{2}\right)$ e) $\sec\left(-\frac{7\pi}{3}\right)$ f) $\csc\left(\frac{3\pi}{4}\right)$

5. Simplify each of the following.

   a) $-16^{-3/4}$ c) $(-16)^{-1/4}$ e) $\log_2\sqrt{8}$ g) $\log_5\left(5^{100}\right)$ i) $\log_{\sqrt{3}}\left(\frac{1}{9}\right)$
   b) $(-8)^{-1/3}$ d) $(-5)^0$ f) $\log_4\left(\frac{1}{\sqrt{8}}\right)$ h) $2^{\log_2 16}$ j) $\log_{0.1} 1000$

6. Simplify each of the following.

   a) $e^{-\ln A} + e^{-\ln B}$ e) $3^{\log_9 Q}$ i) $\log_2(\sec 45°)$ l) $\ln\left(\frac{1}{e^{123}}\right)$
   b) $e^{-\ln A} \cdot e^{-\ln B}$ f) $\left(\sqrt{8}\right)^{\log_2 T}$ j) $\log_3(\tan 60°)$ m) $e^{-2\ln 5}$
   c) $3^{\log_3 A}$ g) $\log_2(16^a)$ k) $5^{\log_{25} 3}$
   d) $9^{\log_3 P}$ h) $\log_{16}(2^a)$

7. Re-write each of the following as a single (or no) logarithm.

   a) $\log_2 96 - \log_2 6$ b) $\log_6 2 + \log_6 3$ c) $\log_2 5 + \log_4 3$ d) $\log_2 80 - \log_4 5$

8. Suppose that $x = \log_2 5$. Express each of the following in terms of x.

   a) $\log_2 10$ b) $\log_2 200$ c) $\log_2 160$ d) $\log_2\left(\frac{25}{8}\right)$ e) $\log_5 2$ f) $\log_{160} 200$

9. Let $f(x) = 2x - 3$ and $g(x) = x^2 + 1$. Compute each of the following.

   a) $f(g(-2))$ b) $g(f(-2))$ c) $f(f(2))$ d) $f(f(f(2)))$ e) $f(g(f(3)))$

10. One number a is five more than twice another number b. Find the smallest value of

a) a^2+b^2 b) ab c) a^2-b^2

11. Compute the exact value of each of the following.

a) $\dfrac{\sin 300^\circ - \cos 450^\circ - \tan 120^\circ}{\sin 225^\circ}$

b) $\dfrac{\tan 240^\circ - \tan 330^\circ}{\sin 225^\circ}$

c) $\dfrac{\tan\left(\frac{2\pi}{3}\right)+\tan\left(\frac{\pi}{6}\right)}{1-\tan\left(\frac{2\pi}{3}\right)\tan\left(\frac{\pi}{6}\right)}$

d) $\sin^2\left(-\frac{\pi}{4}\right)+\cos^2\left(-\frac{\pi}{4}\right)$

e) $\sin\left(\frac{\pi}{5}\right)-\cos\left(\frac{3\pi}{10}\right)$

f) $\tan 28^\circ \cdot \tan 62^\circ$

12. Prove each of the following identities.

a) $\sin^2 x\cos^3 x = \left(\sin^2 x - \sin^4 x\right)\cos x$

b) $(\sin x+\cos x)^2 = 1+2\sin x\cos x$

c) $\cos x\,(\sec x - \cos x) = \sin^2 x$

d) $\dfrac{\sin x}{1-\cos x} = \dfrac{1+\cos x}{\sin x}$

e) $\dfrac{1+\cos x}{1-\cos x} - \dfrac{1-\cos x}{1+\cos x} = 4\cot x\csc x$

13. Solve each of the following equations.

a) $\sqrt{3x+22}-\sqrt{x+8}=2$

b) $\sqrt{3x+16}-\sqrt{x+1}=3$

c) $\sqrt{x+6}+\sqrt{11-x}=5$

d) $\sqrt{3x+10}-\sqrt{x+4}=2$

e) $\sqrt{x-3}+1=\sqrt{x+2}$

f) $\sqrt{x-1}-1=\sqrt{2x-9}$

14. Solve each of the following equations.

a) $\log_3(x-2)=1$

b) $\ln(3x-1)=2$

c) $\dfrac{\log_3(x+1)-1}{2}=3$

d) $\dfrac{\log_3(x-4)}{2}-1=3$

e) $\dfrac{2}{3}\ln(x-1)+6=4$

f) $\dfrac{2}{3}(\ln(x-1)+6)=4$

g) $3-5\log_2(x+1)=8$

h) $11-8\ln(3x+1)=7$

i) $7-2\log_2(5x-1)=3$

j) $\log_3(x+20)+\log_3(x+2)=5$

k) $\log_6(2x+18)+\log_6(x+2)=2$

l) $\log_2(5x+1)-\log_2(x+2)=3$

m) $\log_3(4x-11)+\log_3(-4x+21)=2$

n) $\log_2(7x-17)-\log_2(x+4)=2$

15. Solve each of the following equations.

a) $3^{2x-1}=\dfrac{1}{27}$

b) $2^{\frac{1}{3}x+1}=32$

c) $3^{2x-1}=10$

d) $e^{3x-1}=10$

e) $2^{5x+1}=-1$

f) $3^{2x-10}=3^{x-2}$

g) $3^{3x-1}=9^{x-1}$

h) $e^{-\ln x}=5$

16. Find all points of intersections of the circles given.

a) $(x+4)^2+(y-4)^2=25$ and $(x-10)^2+(y-2)^2=125$

b) $(x+1)^2+(y-6)^2=25$ and $(x-5)^2+(y-3)^2=10$

c) $(x+3)^2+(y+7)^2=5$ and $(x-1)^2+(y+5)^2=5$

17. Find the domain of each of the given functions.

a) $f(x)=\ln(x-1)+\ln(x+1)$

b) $f(x)=\dfrac{\ln(x+1)}{\ln(x-1)}$

c) $f(x)=\ln\left(\dfrac{x+1}{x-1}\right)$

18. Suppose that C is a circle with radius 12 feet. Compute the exact and approximate values of each of the following.
    a) The length of an arc subtended by a central angle of $42°$.
    b) The area of a sector determined by a central angle of $42°$.

19. Find the radius of the circle if
    a) an arc subtended by a central angle of $30°$ has a length of $20\,\mathrm{cm}$.
    b) a sector of $30°$ has an area of $20\,\mathrm{cm}^2$.

20. Suppose that C_1 is a circle with radius 12 units and C_2 is a circle with radius 15 units. The distance between the centers is 20 units.
    a) Find the exact value and approximate value for the angle formed by the common tangent lines drawn to the circles.
    b) Compute the distance between the two points of tangency on a common tangent line.

21. Seattle, WA and Los Angeles, CA have approximately the same longitude. The latitude of Seattle is $47.6°$ and that of Los Angeles is $34.1°$. Find the distance to the nearest mile between the two cities. (The radius of the earth is approximately 3960 miles.)

22. a) Find the speed of the sattelite that orbits the Earth above the equator at a height of 1200 miles and appears at the same point in the sky at all times. Round your answer to the nearest miles per hour. (Assume that Earth is a sphere with radius $3960\,\mathrm{mi}$)

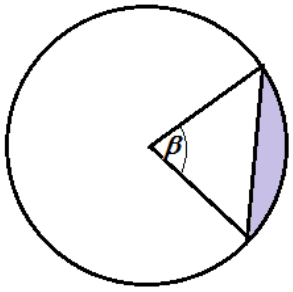
    b*) Find the speed of the sattelite that orbits the Earth above Chicago at a height of 1200 miles and appears at the same point in the sky at all times. Assume that Earth is a sphere with radius $3960\,\mathrm{mi}$ and that the latitude of Chicago is $42°\mathrm{N}$. Round your answer to the nearest miles per hour.

23. How fast is the endpoint of the minute pointer moving on my watch if the pointer is 1.2 centimeters long? Express your answer in centimeters per minute.

24. Consider circles C_1 and C_2 with the following property. An arc subtended in C_1 by a central angle of $20°$ has the same length as an arc subtended in C_2 by a central angle of $15°$. Find the ratio between the areas of the two circles.

25. We place $\$5000$ in a bank ccout with an annual compound interest rate of 4%. How long do we need to wait until the account has $\$10\,000$?

26. Graph each of the following functions. State the basic properties of each of the functions.
    a) $f(x) = 2^x$ b) $f(x) = \left(\frac{1}{2}\right)^x$ c) $f(x) = \log_2 x$ d) $f(x) = \log_{1/2} x$



27. Compute the area of the shaded region shown on the picture. The radius of the circle is 3 meters and $\beta = 80°$. Present the exact value and approximate value of the answer.

28. A water storage tank has the shape of a cylinder with diameter 10 feet. It is mounted so that the circular cross sections are vertical. If the depth of the water is 7 feet, what percentage of the total capacity is used?

29. Write an equation for the line that passes through the point $(8, -1)$ and forms a $30°$ angle with the positive part of the $x-$axis.

Answers

1. a) $\frac{2\pi}{3}$ b) $\frac{10\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{9}$ e) 5π f) $-\frac{5\pi}{18}$ g) $\frac{5\pi}{4}$ h) $-\frac{10\pi}{9}$ i) $\frac{7\pi}{6}$ j) $-\frac{3\pi}{4}$

2. a) $225°$ b) $126°$ c) $-30°$ d) $108°$ e) $-120°$ f) $48°$ g) $-270°$ h) $720°$ i) $495°$ j) $\left(\frac{180}{\pi}\right)^\circ \approx 57.295\,78°$

3. a) $-\frac{1}{2}$ b) $-\frac{\sqrt{3}}{2}$ c) $\frac{\sqrt{3}}{3}$ d) $-\frac{\sqrt{2}}{2}$ e) $\frac{\sqrt{2}}{2}$ f) -1 g) $-\frac{\sqrt{3}}{2}$ h) $\frac{1}{2}$ i) $-\sqrt{3}$ j) 0 k) -1
   l) 0 m) $-\frac{1}{2}$ n) $-\frac{\sqrt{3}}{2}$ o) $\frac{\sqrt{3}}{3}$ p) $-\frac{\sqrt{3}}{2}$ q) $-\frac{1}{2}$ r) $\sqrt{3}$ s) -1 t) 0 u) undefined
   v) $\frac{\sqrt{2}}{2}$ w) $-\frac{\sqrt{2}}{2}$ x) -1 4. a) -1 b) $\frac{\sqrt{2}}{2}$ c) 0 d) undefined e) 2 f) $\sqrt{2}$

5. a) $-\frac{1}{8}$ b) $-\frac{1}{2}$ c) undefined d) 1 e) $\frac{3}{2}$ f) $-\frac{3}{4}$ g) 100 h) 16 i) -4 j) -3

6. a) $\frac{1}{A}+\frac{1}{B}$ b) $\frac{1}{AB}$ c) A d) P^2 e) $\sqrt{Q}$ f) $\sqrt{T^3}$ g) $4a$ h) $\frac{a}{4}$ i) $\frac{1}{2}$ j) $\frac{1}{2}$ k) $\sqrt{3}$ l) -123 m) $\frac{1}{25}$

7. a) 4 b) 1 c) $\log_4(75)$ d) $\log_4 1280$ or $\log_2\left(16\sqrt{5}\right)$

8. a) $x+1$ b) $2x+3$ c) $x+5$ d) $2x-3$ e) $\frac{1}{x}$ f) $\frac{2x+3}{x+5}$ 9. a) 7 b) 50 c) -1 d) -5 e) 17

10. a) 5 b) $-\frac{25}{8}$ c) $-\frac{25}{3}$ 11. a) $-\frac{\sqrt{6}}{2}$ b) $-\frac{4}{3}\sqrt{6}$ c) $-\frac{\sqrt{3}}{3}$ d) 1 e) 0 f) 1

12. a) $\sin^2 x\cos^3 x = \left(\sin^2 x - \sin^4 x\right)\cos x$

$$\text{RHS} = \left(\sin^2 x - \sin^4 x\right)\cos x = \sin^2 x\left(1-\sin^2 x\right)\cos x = \sin^2 x\cos^2 x\cos x = \text{LHS}$$

b) $(\sin x+\cos x)^2 = 1+2\sin x\cos x$

$$\text{LHS} = (\sin x+\cos x)^2 = \sin^2 x+\cos^2 x+2\sin x\cos x = 1+2\sin x\cos x = \text{RHS}$$

c) $\cos x\,(\sec x-\cos x) = \sin^2 x$

$$\text{LHS} = \cos x\,(\sec x-\cos x) = \cos x\left(\frac{1}{\cos x}-\cos x\right) = 1-\cos^2 x = \sin^2 x = \text{RHS}$$

d) $\dfrac{\sin x}{1-\cos x} = \dfrac{1+\cos x}{\sin x}$

$$\text{LHS} = \frac{\sin x}{1-\cos x} = \frac{\sin x}{1-\cos x}\cdot\frac{1+\cos x}{1+\cos x} = \frac{\sin x\,(1+\cos x)}{1-\cos^2 x} = \frac{\sin x\,(1+\cos x)}{\sin^2 x} = \frac{1+\cos x}{\sin x} = \text{RHS}$$

e) $\dfrac{1+\cos x}{1-\cos x}-\dfrac{1-\cos x}{1+\cos x} = 4\cot x\csc x$

$$\begin{aligned}\text{LHS} &= \frac{1+\cos x}{1-\cos x}-\frac{1-\cos x}{1+\cos x} = \frac{(1+\cos x)^2-(1-\cos x)^2}{(1-\cos x)(1+\cos x)} = \frac{\left(1+\cos^2 x+2\cos x\right)-\left(1+\cos^2 x-2\cos x\right)}{1-\cos^2 x}\\ &= \frac{1+\cos^2 x+2\cos x-1-\cos^2 x+2\cos x}{\sin^2 x} = \frac{4\cos x}{\sin^2 x} = 4\frac{\cos x}{\sin x}\frac{1}{\sin x} = 4\cot x\csc x = \text{RHS}\end{aligned}$$

13. a) 1 (-7 does not work) b) $3, 0$ c) $-5, 10$ d) 5 (-3 does not work) e) 7 f) 5 (17 does not work)

14. a) 5 b) $\frac{1}{3}e^2+\frac{1}{3}$ c) 2186 d) 6565 e) $\frac{1}{e^3}+1$ f) 2 g) $-\frac{1}{2}$ h) $\frac{1}{3}\sqrt{e}-\frac{1}{3}$ i) 1
j) 7 (-29 does not work) k) 0 (-11 does not work) l) no solution (-5 does not work) m) $3, 5$ n) 11

15. a) -1 b) 12 c) $\frac{1}{2}(1+\log_3 10)$ d) $\frac{1}{3}(1+\ln 10)$ e) no solution f) 8 g) -1 h) $\frac{1}{5}$

16. a) $(-4,6)$ and $(-5,-1)$ b) $(4,6)$ and $(2,2)$ c) $(-1,-6)$ 17. a) $(1,\infty)$ b) $(1,2)\cup(2,\infty)$ c) $(-\infty,-1)\cup(1,\infty)$

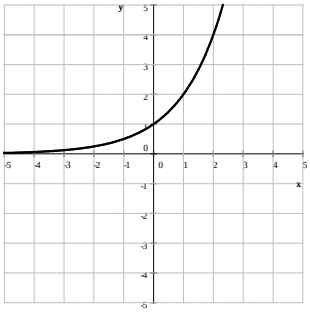
18. a) $\frac{14}{5}\pi$ ft ≈ 8.79646 ft b) $\frac{84}{5}\pi \approx 52.778\,757$ ft^2

19. a) $r=\frac{120}{\pi}$ cm $\approx 38.197\,2$ cm b) $r=\sqrt{\frac{240}{\pi}}$ cm $\approx 8.740\,39$ cm

20. a) $2\sin^{-1}\left(\frac{3}{20}\right) \approx 17.253\,853°$ b) $\sqrt{391}$ unit 21. 933 miles 22. a) $1351\frac{\text{mi}}{\text{h}}$ b*) $1004\frac{\text{mi}}{\text{h}}$

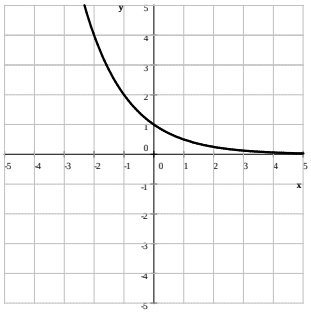
23. $0.04\pi\frac{\text{cm}}{\text{min}} \approx 0.12566\,\frac{\text{cm}}{\text{min}}$ 24. $\frac{A_1}{A_2}=\frac{9}{16}$ 25. $\log_{1.04} 2=\frac{\ln 2}{\ln 1.04} \approx 17.673$ we need to wait 18 years

26. a) $f(x)=2^x$



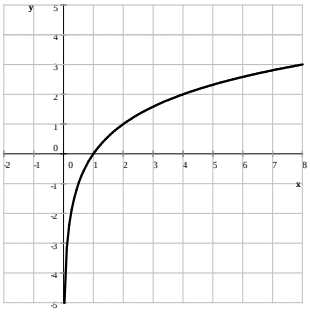
domain: $\mathbb{R}$
range: $(0,\infty)$
increasing
one-to-one

b) $f(x)=\left(\frac{1}{2}\right)^x$



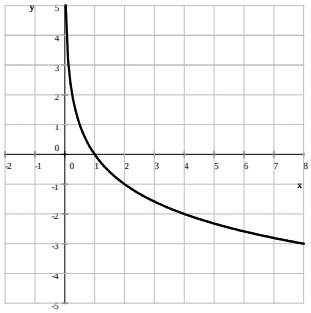
domain: $\mathbb{R}$
range: $(0,\infty)$
decreasing
one-to-one

c) $f(x)=\log_2 x$



domain: $(0,\infty)$
range: $\mathbb{R}$
increasing
one-to-one

d) $f(x)=\log_{1/2} x$



domain: $(0,\infty)$
range: $\mathbb{R}$
decreasing
one-to-one

27. $(2\pi-9\sin 40°\cos 40°)$ m$^2 \approx 1.851\,55$ m^2 28. 74.77% 29. $\frac{\sqrt{3}}{3}(x-8)=y+1$

Last revised: March 14, 2019