1. a) Suppose that α is NOT an acute angle. Find the exact value of $\cos\alpha$ and $\tan\alpha$ if $\sin\alpha = \frac{1}{3}$. Rationalize the denominator in your answers.

   b) Suppose that β is an angle with $90^\circ < \beta < 270^\circ$. Find the value of $\sin\beta$ and $\cos\beta$ if $\tan\beta = 4$. Rationalize the denominator in your answer.

   c) Suppose that γ is an angle not in the third quadrant . Find the value of $\sin\gamma$, $\cos\gamma$, and $\tan\gamma$ if $\sec\gamma = -\frac{5}{2}$. Rationalize the denominator in your answer.

   d) Suppose that $\tan\theta = -\frac{3}{2}$. Compute $\sin 2\theta$.

2. Express each of the following in terms of x if $x = 2^{2018}$.

   a) $2^{2018} + 2^{2019} + 2^{2020}$ b) $2^{2018} + 4^{2018}$

3. We deposited \$2000 in a bank account with an annual compound interest rate of 7%. If we do not withdraw or add money to this account, how long do we need to wait until there is \$4000 in the account?

4. Compute the exact value of each of the following. Rationalize denominators and simplify your answer.

   a) $\cos 15^\circ$ b) $\sin 15^\circ$ c) $\sin 75^\circ$ d) $\cos 75^\circ$

5. Prove each of the following identities.

   a) $\tan x + \dfrac{\cos x}{1+\sin x} = \sec x$ b) $\dfrac{\cos x}{1-\sin x} = \sec x + \tan x$ c) $(\sin x - \cos x)^2 = 1 - \sin 2x$

6. Prove each of the following co-function identities.

   a) $\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$ b) $\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$

7. Suppose that α and β are acute angles with $\sin\alpha = \frac{1}{3}$ and $\cos\beta = \frac{5}{13}$. Compute the exact value of each of the following.

   a) $\cos\alpha$ c) $\sin\beta$ e) $\sin 2\alpha$ g) $\sin(\alpha+\beta)$ i) $\cos(\alpha+\beta)$

   b) $\tan\alpha$ d) $\tan\beta$ f) $\cos 2\beta$ h) $\sin(\alpha-\beta)$ j) $\cos(\alpha-\beta)$

8. Suppose that $\sin A = -\frac{3}{5}$. Compute the exact value of each of the following.

   a) $\cos A$ b) $\tan A$ c) $\sin 2A$ d) $\cos 2A$

9. Suppose that $\sin B = \frac{1}{3}$ and B is not in the first quadrant. Compute the exact value of each of the following.

   a) $\sec B$ b) $\sin 2B$ c) $\cos 2B$

10. Compute the exact value of $\sin\left(\frac{4\pi}{3}\right)\cos\left(\frac{\pi}{12}\right) - \cos\left(\frac{4\pi}{3}\right)\sin\left(\frac{\pi}{12}\right)$

11. Graph each of the following functions.

    a) $f(x) = \log_2 x$
    b) $f(x) = \log_{1/2} x$
    c) $f(x) = x^2 + 6x + 5$
    d) $f(x) = \dfrac{1}{x}$
    e) $f(x) = \sqrt[3]{x}$
    f) $f(x) = \sqrt{4 - x^2}$
    g) $f(x) = -|x|$
    h) $f(x) = \left(\frac{1}{2}\right)^x$
    i) $f(x) = \cos x$
    j) $f(x) = \sin xx$

12. Prove that $\log_{3/4}\left(\frac{15}{8}\right) = \dfrac{\ln 3 + \ln 5 - 3\ln 2}{\ln 3 - 2\ln 2}$.

13. Find the exact value of each of the following expressions.

a) $\sin\left(-\frac{5\pi}{2}\right) - \cos\left(\frac{7\pi}{3}\right) + \tan\left(\frac{3\pi}{4}\right) - \cos(7\pi)$

b) $\log_{10}(0.01) + \log_8\left(\frac{1}{2}\right) - \ln\left(\frac{1}{e^5}\right)$

c) $\cos 420^\circ - \tan 210^\circ + \sec 240^\circ + \cot 135^\circ$

d) $\cos 100^\circ \cos 40^\circ + \sin 100^\circ \sin 40^\circ$

14. Simplify each of the following.

a) $\ln\left(e^{-5}\right)$

b) $e^{-\ln 3}$

c) $3^{\log_9 A}$

d) $\log_m\left(m^4\right)$

e) $\log_{27} 9$

f) $\ln\left(-e^3\right)$

g) $\ln\left(e^{-3}\right)$

h) $9^{\log_3 7}$

i) $e^{3\ln 2}$

j) $\ln 1$

k) $25^{\log_5 10}$

l) $5^{\log_{25} 10}$

m) $2^{\log_8 x}$

n) $\log_5\left(5^b\right)$

o) $\log_5\left(125^m\right)$

p) $\log_2\left[\left(\frac{1}{8}\right)^p\right]$

q) $\log_{16}\left(2^x\right)$

15. Simplify each of the following.

a) $\log_{10} 2 + \log_{10} 5$

b) $\log_6 180 - \log_6 5$

c) $\log_2 24 - \log_2 3$

d) $\log_{10} 40 - 2\log_{10} 2$

e) $\log_3 18 + \log_3 24 - 4\log_3 2$

f) $\log_5 0.4 + \log_5 2.5$

16. Write each of the following as a single logarithm. Assume that all variables represent positive numbers.

a) $\log_{10} a + 2\log_{10} b$

b) $\frac{1}{3}\log_2 a - 3\log_2 b$

c) $1 + \log_3 2$

d) $2 + \log_5 3$

e) $\frac{1}{2} - \log_7 x$

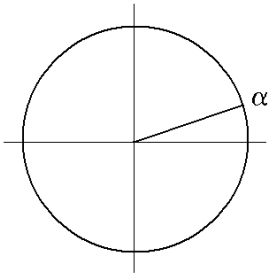
f) $2 + \ln 3 + \ln x - 2\ln y$

17. a) Compute the area of the triangle determined by the points $A(5, 2)$, $B(10, 2)$, and $C(10, 6)$.

b) Compute the area of the triangle determined by the points $A(\log_2 x, \log_2 y)$, $B(\log_2 8x, \log_2 y)$, and $C(\log_2 x, \log_2 4y)$.

18. The picture shows an angle α on the unit circle. Draw each of the following angles in the same circle.

a) $-\alpha$ b) $\alpha - 180^\circ$ c) $\alpha + 90^\circ$



19. Simplify $\log_3 6 - \log_9 12$

20. Solve each of the following inequalities.

a) $\frac{3x-1}{x+5} \le 2$ b) $\frac{3}{x-7} < 1$ c) $\frac{2x+5}{x-4} > 2$ d) $\frac{5}{x-8} \le -1$

21. Find the radius of the circle in which a sector subtended by a central angle of 50° has area $14\,\text{cm}^2$.

22. Simplify each of the following. (i.e. write it in terms of trigonometric functions of α.)

a) $\sin(90^\circ - \alpha)$

b) $\sin(180^\circ - \alpha)$

c) $\sin(-\alpha)$

d) $\sin(\alpha + 180^\circ)$

e) $\cos(90^\circ - \alpha)$

f) $\cos(180^\circ - \alpha)$

g) $\cos(-\alpha)$

h) $\cos(\alpha + 180^\circ)$

i) $\tan(90^\circ - \alpha)$

j) $\tan(180^\circ - \alpha)$

k) $\tan(-\alpha)$

l) $\tan(\alpha + 180^\circ)$

23. Find all angles β so that twice β is coterminal with 120°. Express your answer

a) in degrees b) in radians

c) Find all coterminal angles β such that $-500^\circ < \beta < 500^\circ$.

24. Solve each of the following.

a) $\sqrt{2x+5}-\sqrt{x+6}=1$ b) $\sqrt{3x+4}=x-2$

25. Solve each of the following equations. You may present your answer in degrees.

a) $\sin x=-\dfrac{1}{\sqrt{2}}$

b) $\cos x=-\dfrac{\sqrt{3}}{2}$

c) $\tan x=-\sqrt{3}$

d) $\sin x=-\dfrac{3}{2}$

e) $\cos x=-1$

f) $\tan x=0$

g) $\sin\alpha=-\dfrac{\sqrt{3}}{2}$

h) $\cos\beta=-1$

i) $\tan\gamma=\dfrac{2}{3}$

26. Solve each of the following equations for x. Present the exact value of each solution.

a) $2^{3x-1}=4$ b) $3^{2x-1}=5$ c) $e^{t-2}=5$ d) $2+5e^{x-1}=2012$

27. Solve each of the following equations.

a) $\log_2(x-1)+\log_2(x-5)=5$

b) $\log_6(3-x)+\log_6(-x-2)=2$

c) $\log_2(x+1)+\log_2(x-1)=3$

d) $\log_5(x-16)-\log_5(4x+20)=-2$

e) $\log_2(3x+6)-\log_2(x+7)=3$

f) $\log_2(1-x)+\log_2(9-x)=7$

g) $\log_6(x-3)+\log_6 3+\log_6(x+1)=2$

h) $\log_2(x-5)-\log_2(2x-14)=-2$

i*) $\log_{x-3}(x+20)\cdot\log_x(x-3)=2$

28. Compare the domains of $f(x)=\log_3\left(x^2-4\right)$ and $g(x)=\log_3(x+2)+\log_3(x-2)$.

29. Graph each of the following pairs of functions together, in the same coordinate system.

a) $f(x)=x^3$ and $g(x)=\sqrt[3]{x}$ b) $f(x)=2^x$ and $g(x)=\left(\dfrac{1}{2}\right)^x$ c) $f(x)=2^x$ and $g(x)=\log_2 x$

30. *Which is greater: $2^{\log_{10}3}$ or $3^{\log_{10}2}$?

31. *Solve each of the following.

a) $\sqrt{x^2-3x+8}+3x=x^2-22$ b) $\sqrt{x^2-9}-x=3-\sqrt{9-x^2}$ c) $5^{\log_2(x-1)}+(x-1)^{\log_2 5}=50$

Answers

1. a) $\cos\alpha=-\dfrac{\sqrt{8}}{3}=-\dfrac{2\sqrt{2}}{3}$ $\tan\alpha=-\dfrac{\sqrt{2}}{4}$ b) $\sin\beta=-\dfrac{4\sqrt{17}}{17}$ $\cos\beta=-\dfrac{\sqrt{17}}{17}$

c) $\sin\gamma=\dfrac{\sqrt{21}}{5}$ $\cos\gamma=-\dfrac{2}{5}$ $\tan\gamma=-\dfrac{\sqrt{21}}{2}$ d) $-\dfrac{12}{13}$

2. a) $7x$ b) $x+x^2$ 3. It will happen sometime during the 11th year. $\log_{1.07}2=\dfrac{\ln 2}{\ln 1.07}\approx 10.24477$

4. a) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ b) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ c) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ d) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

5. a) $\tan x+\dfrac{\cos x}{1+\sin x}=\sec x$

$$\begin{aligned}\text{RHS} &= \tan x+\frac{\cos x}{1+\sin x}=\frac{\sin x}{\cos x}+\frac{\cos x}{1+\sin x}=\frac{\sin x(1+\sin x)+\cos^2 x}{\cos x(1+\sin x)}\\ &= \frac{\sin x+\sin^2 x+\cos^2 x}{\cos x(1+\sin x)}=\frac{\sin x+1}{\cos x(1+\sin x)}=\frac{1}{\cos x}=\sec x=\text{LHS}\end{aligned}$$

b) $\dfrac{\cos x}{1-\sin x} = \sec x + \tan x$

$$\text{LHS} = \frac{\cos x}{1-\sin x} = \frac{\cos x}{1-\sin x}\cdot\frac{1+\sin x}{1+\sin x} = \frac{\cos x\,(1+\sin x)}{1-\sin^2 x} = \frac{\cos x\,(1+\sin x)}{\cos^2 x} = \frac{1+\sin x}{\cos x}$$
$$= \frac{1}{\cos x} + \frac{\sin x}{\cos x} = \sec x + \tan x = \text{RHS}$$

c) $(\sin x - \cos x)^2 = 1 - \sin 2x$

$$\text{LHS} = (\sin x - \cos x)^2 = \sin^2 x + \cos^2 x - 2\sin x\cos x = 1 - \sin 2x = \text{RHS}$$

6. a) $\sin\left(\dfrac{\pi}{2} - \alpha\right) = \cos\alpha$

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \sin\frac{\pi}{2}\cos\alpha - \cos\frac{\pi}{2}\sin\alpha = 1\cdot\cos\alpha - 0\cdot\sin\alpha = \cos\alpha$$

b) $\cos\left(\dfrac{\pi}{2} - \alpha\right) = \sin\alpha$

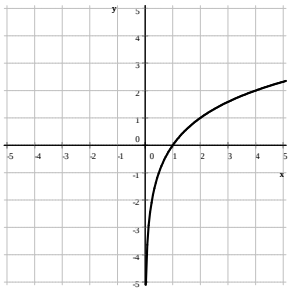
$$\cos\left(\frac{\pi}{2} - \alpha\right) = \cos\frac{\pi}{2}\cos\alpha + \sin\frac{\pi}{2}\sin\alpha = 0\cdot\cos\alpha + 1\cdot\sin\alpha = \sin\alpha$$

7. a) $\dfrac{2\sqrt{2}}{3}$ b) $\dfrac{\sqrt{2}}{4}$ c) $\dfrac{12}{13}$ d) $\dfrac{12}{5}$ e) $\dfrac{4\sqrt{2}}{9}$ f) $-\dfrac{119}{169}$ g) $\dfrac{5+24\sqrt{2}}{39}$ h) $\dfrac{5+24\sqrt{2}}{39}$

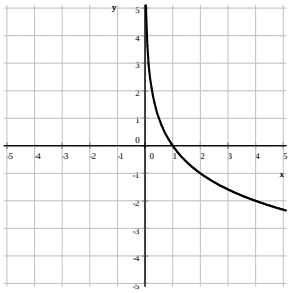
i) $\dfrac{10\sqrt{2}-12}{39}$ j) $\dfrac{10\sqrt{2}+12}{39}$

8. a) $\pm\dfrac{4}{5}$ b) $\pm\dfrac{3}{4}$ c) $\pm\dfrac{24}{25}$ d) $\dfrac{7}{25}$ 9. a) $-\dfrac{3\sqrt{2}}{4}$ b) $\dfrac{4}{9}\sqrt{2}$ c) $\dfrac{8}{9}$ 10. $-\dfrac{1}{\sqrt{2}}$

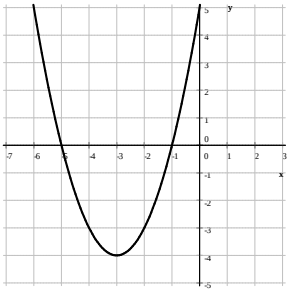
11. a) $f(x) = \log_2 x$



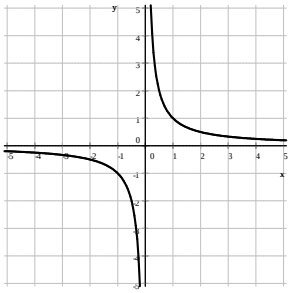
b) $f(x) = \log_{1/2} x$



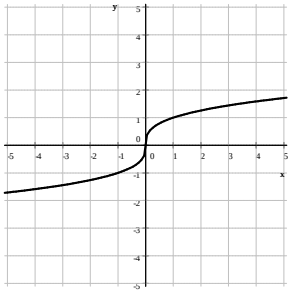
c) $f(x) = x^2 + 6x + 5$



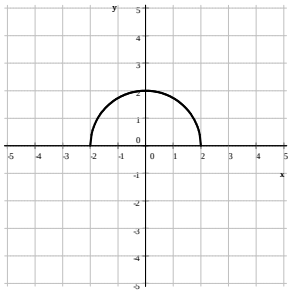
d) $f(x) = \dfrac{1}{x}$



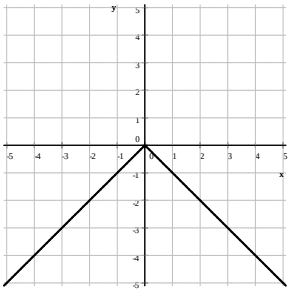
e) $f(x) = \sqrt[3]{x}$



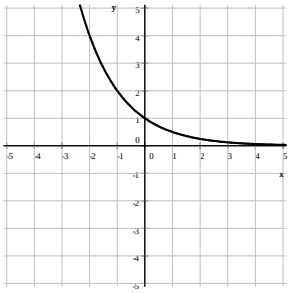
f) $f(x) = \sqrt{4-x^2}$



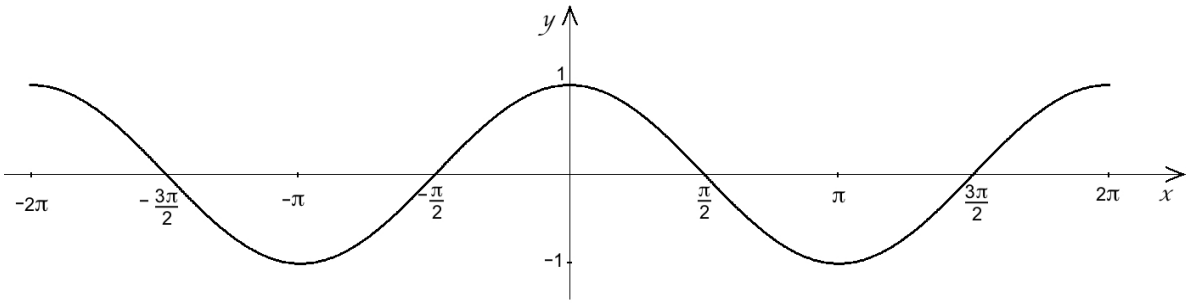
g) $f(x) = -|x|$



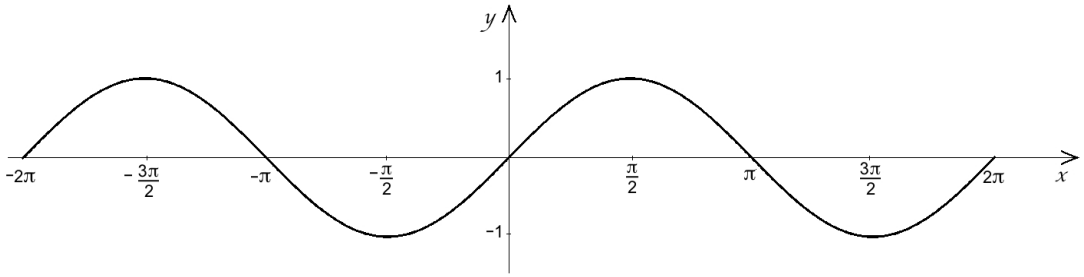
h) $f(x) = \left(\dfrac{1}{2}\right)^x$



i) $f(x) = \cos x$



j) $f(x) = \sin x$



12. $\log_{3/4}\left(\frac{15}{8}\right) = \frac{\ln\left(\frac{15}{8}\right)}{\ln\left(\frac{3}{4}\right)} = \frac{\ln 15 - \ln 8}{\ln 3 - \ln 4} = \frac{(\ln 3 + \ln 5) - \ln\left(2^3\right)}{\ln 3 - \ln\left(2^2\right)} = \frac{\ln 3 + \ln 5 - 3\ln 2}{\ln 3 - 2\ln 2}$

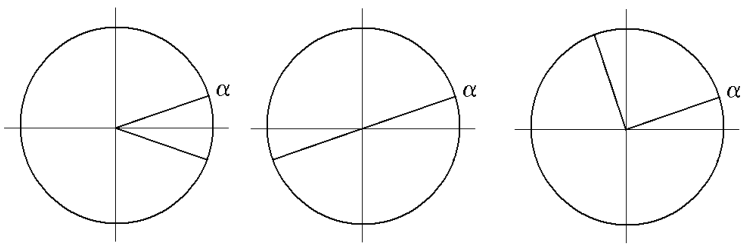
13. a) $-\frac{3}{2}$ b) $\frac{8}{3}$ c) $-\frac{\sqrt{3}}{3} - \frac{5}{2}$ d) $\frac{1}{2}$

14. a) -5 b) $\frac{1}{3}$ c) $\sqrt{A}$ d) 4 e) $\frac{2}{3}$ f) undefined g) -3 h) 49 i) 8 j) 0 k) 100 l) $\sqrt{10}$
m) $\sqrt[3]{x}$ n) b o) $3m$ p) $-3p$ q) $\frac{1}{4}x$ 15. a) 1 b) 2 c) 3 d) 1 e) 3 f) 0

16. a) $\log_{10} ab^2$ b) $\log_2\left(\frac{\sqrt[3]{a}}{b^3}\right)$ c) $\log_3 6$ d) $\log_5 75$ e) $\log_7\left(\frac{\sqrt{7}}{x}\right)$ f) $\ln\left(\frac{3e^2x}{y^2}\right)$

17. a) 10 unit^2 b) 3 unit^2

18. a) b) c)



19. $\frac{1}{2}$ 20. a) $(-5, 11]$ b) $(-\infty, 7) \cup (10, \infty)$ c) $(4, \infty)$ d) $[3, 8)$ 21. $\sqrt{\frac{504}{5\pi}}$ cm $\approx 5.664\,42$ cm

22. a) $\cos\alpha$ b) $\sin\alpha$ c) $-\sin\alpha$ d) $-\sin\alpha$ e) $\sin\alpha$ f) $-\cos\alpha$ g) $\cos\alpha$ h) $-\cos\alpha$ i) $\cot\alpha$
j) $-\tan\alpha$ k) $-\tan\alpha$ l) $\tan\alpha$

23. a) $60^\circ + k \cdot 180^\circ$ where $k \in \mathbb{Z}$ b) $\frac{\pi}{3} + k\pi$ where $k \in \mathbb{Z}$ c) $-480^\circ, -300^\circ, -120^\circ, 60^\circ, 240^\circ, 420^\circ$

24. a) 10 (-2 does not work) b) 7 (0 doesn't work)

25. a) $x = -45^\circ + k \cdot 360^\circ$ or $x = -135^\circ + k \cdot 360^\circ$ where $k \in \mathbb{Z}$

$x = -\frac{\pi}{4} + 2k\pi$ or $x = -\frac{3\pi}{4} + 2k\pi$ where $k \in \mathbb{Z}$

b) $x = \pm 150^\circ + k \cdot 360^\circ$ where $k \in \mathbb{Z}$ $x = \pm\frac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$

c) $x = -60^\circ + k \cdot 180^\circ$ where $k \in \mathbb{Z}$ $x = -\frac{\pi}{3} + k\pi$ where $k \in \mathbb{Z}$

d) no solution e) $x = 180^\circ + k \cdot 360^\circ$ where $k \in \mathbb{Z}$ $x = \pi + 2k\pi$ where $k \in \mathbb{Z}$

f) $x = k \cdot 180^\circ$ where $k \in \mathbb{Z}$ $x = k\pi$ where $k \in \mathbb{Z}$

g) $\alpha = -60^\circ + k \cdot 360^\circ$ or $\alpha = -120^\circ + k \cdot 360^\circ$ where $k \in \mathbb{Z}$

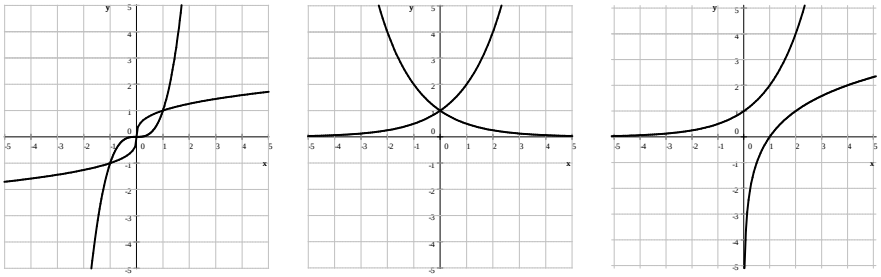
h) $\beta = 180^\circ + k \cdot 360^\circ$ where $k \in \mathbb{Z}$ i) $\gamma = 33.690\,07^\circ + k \cdot 180^\circ$ where $k \in \mathbb{Z}$

26. a) 1 b) $\frac{1}{2}(1 + \log_3 5)$ or $\log_9 15$ c) $2 + \ln 5$ d) $1 + \ln 402$

27. a) 9 (-3 does not work) b) -6 (7 does not work) c) 3 (-3 does not work) d) 20 e) no solution
f) -7 (16 does not work) g) 5 (-3 does not work) h) no solution (3 does not work) i) 5

28. The domain of f is $(-\infty, -2) \cup (2, \infty)$ and the domain of g is $(2, \infty)$

29. a) b) c)



30. Hint: take the common logarithm of both sides and compare. 31. a) $-4, 7$ b) -3 c) 5