1. Find the inverse for each of the following functions.

   a) $f(x) = \dfrac{3x-1}{5x+2}$ b) $f(x) = 5e^{3x-1} + 2$ c) $g(x) = 4 - \log_3\left(x^7 + 1\right)$

2. Solve each of the following triangles:

   a) $b = 16\,\text{ft}$, $\alpha = 38°$, and $\beta = 83°$.
   b) $a = 4\,\text{cm}$, $b = 7\,\text{cm}$, and $\alpha = 58°$.
   c) $a = 6\,\text{in}$, $b = 4\sqrt{3}\,\text{in}$, and $\alpha = 60°$.
   d) $a = 12\,\text{m}$, $c = 15\,\text{m}$, and $\alpha = 32°$
   e) $a = 4\,\text{m}$, $c = 7\,\text{m}$ and $\gamma = 70°$
   f) $\beta = 78°$ $\gamma = 49°$ and $c = 15\,\text{ft}$
   g) $\alpha = 62°$ $a = 7\,\text{m}$ and $b = 12\,\text{m}$
   h) $\alpha = 31°$ $a = 4\,\text{cm}$ and $c = 5\,\text{cm}$

3. a) Compute $\sin\alpha$ and $\cos\alpha$ in terms of M if we know that α is an acute angle and that $\tan\alpha = M$.

   b) Compute $\sin\beta$ and $\cos\beta$ in terms of T if we know that β is an angle in the fourth quadrant that $\tan\beta = T$.

4. Solve each of the following.

   a) $\dfrac{7}{2x-1} \le -3$ b) $\dfrac{x-1}{3x+1} \le \dfrac{1}{3}$ c) $\dfrac{x}{x+4} \ge \dfrac{2}{3}$ d) $\dfrac{x+3}{x-3} < 2$

5. We draw tangent lines to a circle from a point P outside of the circle. The line segment between P and a point of tangency is 3 units long. The line segment connecting P and the center of the circle intersects the circle in point Q. Line segment PQ is $\sqrt{3}$ units long. Compute the angle formed between the two tangent lines.

6. Compute the exact value of each of the following. Rationalize denominators and simplify your answer.

   a) $\cos 15°$ b) $\sin 15°$ c) $\tan 75°$

7. Find an equation for the tangent line drawn to the circle $16x - 2y + x^2 + y^2 + 15 = 0$ to the point $P(-1, 2)$.

8. Prove each of the following identities.

   a) $\cot 2x = \dfrac{\cot^2 x - 1}{2\cot x}$
   b) $4\sin^4 x = 1 - 2\cos 2x + \cos^2 2x$
   c) $\cos 3x = 4\cos^3 x - 3\cos x$
   d) $\sin\left(\dfrac{\pi}{2} - x\right) = \cos x$
   e) $\dfrac{\tan(45° - x)}{\tan(45° + x)} = \dfrac{1 - \sin 2x}{1 + \sin 2x}$
   f) $\sin(30° + \beta) + \sin(30° - \beta) = \cos\beta$

9. Simplify completely or re-write each of the following as a single logarithm.

   a) $\log_3 8 - \log_9 24$ b) $\log_2 3 - \log_4 6$ c) $(\log_2 3)(\log_3 5)(\log_5 8)$

10. Suppose that $\tan x = -\dfrac{4}{5}$. Compute the exact value of $\sin 2x$.

11. Given $A(5, 2)$ and $B(-11, 6)$, find an equation for the perpendicular bisector of A and B.

12. Simplify each of the following. Use exact values.

    a) $\dfrac{\sin\left(\dfrac{7\pi}{2}\right)\cos\left(-\dfrac{11\pi}{6}\right)}{\tan\left(\dfrac{13\pi}{3}\right)}$ b) $-\cos(12°)\sin(72°) + \sin(12°)\cos(72°)$

13. Suppose that $a = \log_2 5$ and $b = \log_3 6$. Write $\log_{12} 60$ in terms of a and b.

14. Simplify each of the following. (Write it in terms of $\sin\alpha$, $\cos\alpha$, and/or $\tan\alpha$.

    a) $\sin(\alpha + 90°)$ b) $\cos(\alpha + 90°)$ c) $\tan(\alpha - 180°)$ d) $\cos(\alpha - 180°)$

15. Which of the following functions are not one-to-one?

a) $f(x) = -\ln x$ b) $g(x) = \dfrac{1}{3}x + 8$ c) $h(x) = x^3 - x$ d) $f(x) = \dfrac{3x-8}{x+3}$

16. Simplify each of the following.

a) $\log_3(\tan 60°) + \log_2(\sin 45°) - \ln(\tan 45°)$

b) $\sec 330° - \csc(-315°) + \cot 60°$

c) $\dfrac{\tan 160° - \tan 25°}{1 + \tan 160° \tan 25°}$

d) $\dfrac{\tan 15° + 1}{\tan 15° - 1}$

17. Find the domain of $f(x) = \dfrac{x^2 - 4}{5 - \sqrt{2x-4}}$

18. Prove each of the following co-function identities.

a) $\sin\left(\dfrac{\pi}{2} - \alpha\right) = \cos\alpha$ b) $\cos\left(\dfrac{\pi}{2} - \alpha\right) = \sin\alpha$ c) $\tan\left(\dfrac{\pi}{2} - \alpha\right) = \cot\alpha$

19. Suppose that α and β are acute angles with $\sin\alpha = \dfrac{1}{3}$ and $\cos\beta = \dfrac{5}{13}$. Compute the exact value of each of the following.

a) $\cos\alpha$
b) $\tan\alpha$
c) $\sin\beta$
d) $\tan\beta$
e) $\sin 2\alpha$
f) $\cos 2\beta$
g) $\sin(\alpha + \beta)$
h) $\sin(\alpha - \beta)$
i) $\cos(\alpha + \beta)$
j) $\cos(\alpha - \beta)$
k) $\tan 2\alpha$
l) $\tan 2\beta$

20. Suppose that $\sin A = -\dfrac{3}{5}$. Compute the exact value of each of the following.

a) $\sin 2A$ b) $\cos 2A$

21. Suppose that $\sin B = \dfrac{1}{3}$ and B is not in the first quadrant. Compute the exact value of each of the following.

a) $\sec B$ b) $\tan 2B$ c) $\cos 2B$ d) $\tan\left(B - \dfrac{\pi}{4}\right)$

22. The expression $\dfrac{2\sin x}{\cos x - \sin x \tan x}$ is equivalent to which of the following?

A) $\tan 2x$ B) $\cot 2x$ C) $\tan x$ D) $\cot x$ E) $\sec x$

23. Compute the sum $83 + 86 + 89 + \ldots + 203$.

24. Compute the exact value of $\dfrac{\tan\left(\dfrac{4\pi}{3}\right) - \tan\left(\dfrac{\pi}{12}\right)}{1 + \tan\left(\dfrac{4\pi}{3}\right)\tan\left(\dfrac{\pi}{12}\right)}$. (Hint: there is an easy way and also a difficult way to do this.)

25. Solve each of the following equations.

a) $\tan^2 x + \tan x = 0$
b) $\sin^2 x = \dfrac{1}{2}$
c) $\sin x = \sin 2x$
d) $\sin x + \cos 2x = 0$
e) $\cos x - 2\sin^2 x = 1$

26. Find the domain for each of the following functions.

a) $f(x) = \log_2(x^2 - 9)$ b) $\log_2(x+3) + \log_2(x-3)$ c) $\ln\left(\dfrac{x+1}{x-1}\right)$ d) $\dfrac{\ln(x+1)}{\ln(x-1)}$

27. Circle C_1 has a radius 5 unit long. Circle C_2 has a radius 11 unit long. The centers are at a distance of 12 units from each other. We draw the lines tangent to both circles.

   a) Find an approximation of the angle formed by the two tangent lines.

   b) Compute the distance between the two points of tangency on one of the common tangent lines.

28. Compute the exact value of $\tan\alpha$ if we know that $\tan 2\alpha = \dfrac{8}{15}$.

29. A lattice point is a point on the coordinate system of whose both coordinates are integers. Can you find the equation of a line with

   a) No lattice points? b) Exactly one lattice point? c) Exactly two lattice points?

30. We leave $\$1$ in a bank that promises an annual compound interest rate of 5%. We notify the bank that a relative will withdraw all money and close the account once it reached $\$1000\,000$. How long will the bank have to maintain the account?

Answers

1. a) $f'(x) = \dfrac{2x+1}{-5x+3}$ b) $f'(x) = \dfrac{1+\ln\left(\dfrac{x-2}{5}\right)}{3}$ c) $g'(x) = \sqrt[7]{3^{4-x}-1}$

2. a) $\gamma = 59°$, $a \approx 9.924\,56\,\text{ft}$, $c \approx 13.817\,671\,64\,\text{ft}$ b) no solution

   c) $\beta = 90°$, $\gamma = 30°$, $c = 2\sqrt{3}\,\text{in}$

   d) $\gamma_1 \approx 41.483\,098°$ $\beta_1 \approx 106.516\,902°$ and $b_1 \approx 21.710\,535\,35\,\text{m}$
   $\gamma_2 \approx 138.516\,902°$ $\beta_2 \approx 9.483\,098°$ and $b_2 \approx 3.730\,907\,55\,\text{m}$

   e) $\alpha \approx 32.477\,421°$ $\beta \approx 77.522\,579°$ $b \approx 7.273\,302\,4\,\text{m}$

   f) $\alpha = 53°$ $b \approx 19.44087\,\text{ft}$ $a \approx 15.8730364\,\text{ft}$ g) no solution

   h) $\gamma_1 \approx 40.075\,583°$, $\beta_1 \approx 108.924\,417°$, $b_1 \approx 7.346\,62\,\text{cm}$ or
   $\gamma_2 \approx 139.924\,417°$, $\beta_2 \approx 9.075\,583°$, $b_2 \approx 1.225\,0532\,\text{cm}$

3. a) $\sin\alpha = \dfrac{M}{\sqrt{M^2+1}}$ and $\cos\alpha = \dfrac{1}{\sqrt{M^2+1}}$ b) T is negative. $\sin\beta = \dfrac{T}{\sqrt{T^2+1}}$ and $\cos\beta = \dfrac{1}{\sqrt{T^2+1}}$

4. a) $\left[-\dfrac{2}{3}, \dfrac{1}{2}\right)$ b) $\left(-\dfrac{1}{3}, \infty\right)$ c) $(-\infty,-4) \cup [8,\infty)$ d) $(-\infty,3) \cup (9,\infty)$ 5. $60°$

6. a) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ b) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ c) $2+\sqrt{3}$ 7. $-7(x+1) = y-2$

8. a) $\cot 2x = \dfrac{\cot^2 x - 1}{2\cot x}$

$$\text{RHS} = \frac{\cot^2 x - 1}{2\cot x} = \frac{\dfrac{\cos^2 x}{\sin^2 x} - 1}{2\dfrac{\cos x}{\sin x}} = \frac{\dfrac{\cos^2 x}{\sin^2 x} - \dfrac{\sin^2 x}{\sin^2 x}}{2\dfrac{\cos x}{\sin x}} = \frac{\dfrac{\cos^2 x - \sin^2 x}{\sin^2 x}}{\dfrac{2\cos x}{\sin x}} = \frac{\cos^2 x - \sin^2 x}{\sin^2 x} \cdot \frac{\sin x}{2\cos x}$$

$$= \frac{\cos^2 x - \sin^2 x}{2\sin x\cos x} = \frac{\cos 2x}{\sin 2x} = \cot 2x = \text{LHS}$$

b) $4\sin^4 x = 1 - 2\cos 2x + \cos^2 2x$

$$\begin{aligned} \text{LHS} &= 4\sin^4 x = 4\left(\sin^2 x\right)^2 = \left(2\sin^2 x\right)^2 = \left(2\sin^2 x - 1 + 1\right)^2 = (-\cos 2x + 1)^2 = \\ &= \cos^2 2x - 2\cos 2x + 1 = \text{RHS} \end{aligned}$$

c) $\cos 3x = 4\cos^3 x - 3\cos x$

$$\begin{aligned} \text{LHS} &= \cos 3x = \cos(x + 2x) = \cos x \cos 2x - \sin x \sin 2x = \cos x\left(2\cos^2 x - 1\right) - \sin x\,(2\sin x \cos x) = \\ &= 2\cos^3 x - \cos x - 2\sin^2 x \cos x = 2\cos^3 x - \cos x - 2\left(1 - \cos^2 x\right)\cos x = \\ &= 2\cos^3 x - \cos x - 2\cos x\left(1 - \cos^2 x\right) = 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x = 4\cos^3 x - 3\cos x = \text{RHS} \end{aligned}$$

d) $\sin\left(\frac{\pi}{2} - x\right) = \cos x$

$$\text{LHS} = \sin\left(\frac{\pi}{2}\right)\cos x - \cos\left(\frac{\pi}{2}\right)\sin x = 1\cos x - 0\sin x = \cos x = \text{RHS}$$

e) $\dfrac{\tan(45^\circ - x)}{\tan(45^\circ + x)} = \dfrac{1 - \sin 2x}{1 + \sin 2x}$

$$\begin{aligned} \text{LHS} &= \frac{\tan(45^\circ - x)}{\tan(45^\circ + x)} = \frac{\dfrac{\tan 45^\circ - \tan x}{1 + \tan 45^\circ \tan x}}{\dfrac{\tan 45^\circ + \tan x}{1 - \tan 45^\circ \tan x}} = \frac{\dfrac{1 - \tan x}{1 + 1\cdot\tan x}}{\dfrac{1 + \tan x}{1 - 1\cdot\tan x}} = \frac{1 - \tan x}{1 + \tan x}\cdot\frac{1 - \tan x}{1 + \tan x} = \left(\frac{1 - \dfrac{\sin x}{\cos x}}{1 + \dfrac{\sin x}{\cos x}}\right)^2 \\ &= \frac{\left(1 - \dfrac{\sin x}{\cos x}\right)^2}{\left(1 + \dfrac{\sin x}{\cos x}\right)^2} = \frac{\left(1 - \dfrac{\sin x}{\cos x}\right)^2}{\left(1 + \dfrac{\sin x}{\cos x}\right)^2}\cdot\frac{\cos^2 x}{\cos^2 x} = \frac{\left(\left(1 - \dfrac{\sin x}{\cos x}\right)\cos x\right)^2}{\left(\left(1 + \dfrac{\sin x}{\cos x}\right)\cos x\right)^2} = \frac{(\cos x - \sin x)^2}{(\cos x + \sin x)^2} \\ &= \frac{\cos^2 x + \sin^2 x - 2\sin x \cos x}{\cos^2 x + \sin^2 x + 2\sin x \cos x} = \frac{1 - \sin 2x}{1 + \sin 2x} = \text{RHS} \end{aligned}$$

f) $\sin(30^\circ + \beta) + \sin(30^\circ - \beta) = \cos\beta$

$$\begin{aligned} \text{LHS} &= \sin(30^\circ + \beta) + \sin(30^\circ - \beta) = \sin 30^\circ \cos\beta + \cos 30^\circ \sin\beta + \sin 30^\circ \cos\beta - \cos 30^\circ \sin\beta \\ &= 2\sin 30^\circ \cos\beta = 2\cdot\frac{1}{2}\cos\beta = \cos\beta = \text{RHS} \end{aligned}$$

9. a) $\log_3\left(\frac{2\sqrt{6}}{3}\right)$ or $\log_3\left(2\sqrt{6}\right) - 1$ b) $\log_2\left(\frac{\sqrt{6}}{2}\right)$ or $\log_2\left(\sqrt{6}\right) - 1$ c) 3 10. $-\frac{40}{41}$ 11. $y = 4x + 16$

12. a) $-\frac{1}{2}$ b) $-\frac{\sqrt{3}}{2}$ 13. $\frac{-a + b + ab}{2b - 1}$ 14. a) $\cos\alpha$ b) $-\sin\alpha$ c) $\tan\alpha$ d) $-\cos\alpha$

15. only h is not one-to-one 16. a) 0 b) $\sqrt{3} - \sqrt{2}$ c) -1 d) $-\sqrt{3}$

17. $\{x \in \mathbb{R} : x \geq 2 \text{ and } x \neq 14.5\}$ in interval notation: $[2, 14.5) \cup (14.5, \infty)$

18. a) $\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \sin\frac{\pi}{2}\cos\alpha - \cos\frac{\pi}{2}\sin\alpha = 1\cdot\cos\alpha - 0\cdot\sin\alpha = \cos\alpha$$

b) $\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \cos\frac{\pi}{2}\cos\alpha + \sin\frac{\pi}{2}\sin\alpha = 0\cdot\cos\alpha + 1\cdot\sin\alpha = \sin\alpha$$

c) $\tan\left(\frac{\pi}{2}-\alpha\right)=\cot\alpha$ We can not use the difference formula for tangent here, because $\tan\frac{\pi}{2}$ is undefined. Instead, we need to use a compound angle formula separately for sine and cosine (see parts a and b).

$$\tan\left(\frac{\pi}{2}-\alpha\right)=\frac{\sin\left(\frac{\pi}{2}-\alpha\right)}{\cos\left(\frac{\pi}{2}-\alpha\right)}=\frac{\cos\alpha}{\sin\alpha}=\cot\alpha$$

19. a) $\frac{2\sqrt{2}}{3}$ b) $\frac{\sqrt{2}}{4}$ c) $\frac{12}{13}$ d) $\frac{12}{5}$ e) $\frac{4\sqrt{2}}{9}$ f) $-\frac{119}{169}$ g) $\frac{5+24\sqrt{2}}{39}$ h) $\frac{5+24\sqrt{2}}{39}$
i) $\frac{10\sqrt{2}-12}{39}$ j) $\frac{10\sqrt{2}+12}{39}$ k) $\frac{4\sqrt{2}}{7}$ l) $-\frac{120}{119}$ 20. a) $\pm\frac{24}{25}$ b) $\frac{7}{25}$

21. a) $-\frac{3\sqrt{2}}{4}$ b) $-\frac{4\sqrt{2}}{7}$ c) $\frac{8}{9}$ d) $-\frac{1}{2}$ 22. A 23. 5863 24. 1

25. a) $-\frac{\pi}{4}+k\pi$ and $k\pi$ where $k\in\mathbb{Z}$ b) $\frac{\pi}{4}+\frac{k\pi}{2}$ where $k\in\mathbb{Z}$ c) $k\pi$ or $\pm\frac{\pi}{3}+2k\pi$ where $k\in\mathbb{Z}$
d) $x=\frac{\pi}{2}+2k\pi$ or $x=-\frac{\pi}{6}+2k\pi$ or $x=-\frac{5\pi}{6}+2k\pi$ where $k\in\mathbb{Z}$ e) $x=2k\pi$ where $k\in\mathbb{Z}$

26. a) $(-\infty,-3)\cup(3,\infty)$ b) $(3,\infty)$ c) $(-\infty,-1)\cup(1,\infty)$ d) $(1,2)\cup(2,\infty)$

27. a) $60°$ b) $\sqrt{108}$ unit $=6\sqrt{3}$ unit 28. -4 or $\frac{1}{4}$

29. a) $y=\frac{5}{3}$ b) $y=\sqrt{2}x$ will only contain the origin c) there isn't such a line

30. about 284 years

Last revised: April 5, 2019