1. Solve each of the given triangles

   a) $b = 16\,\text{ft},\ \alpha = 38°,\ \beta = 83°$ b) $a = 4\,\text{cm},\ b = 7\,\text{cm},\ \alpha = 58°$ c) $a = 6\,\text{in},\ b = 4\sqrt{3}\,\text{in},\ \alpha = 60°$

2. Suppose that $\underline{v} = 2\underline{i} - 3\underline{j}$ and $\underline{w} = -\underline{i} + 2\underline{j}$. Compute each of the following.

   a) $\|\underline{v}\|$ b) $\|\underline{w}\|$ c) $\underline{v} + \underline{w}$ d) $\|\underline{v} + \underline{w}\|$ e) $5\underline{w} + 3\underline{v}$ f) $\underline{v} + 2\underline{w}$ g) $\|\underline{v} + 2\underline{w}\|$

   h) Can you find real numbers A and B such that $A\underline{v} + B\underline{w} = 5\underline{i} - \underline{j}$?

3. Compute each of the given sums.

   a) $32 + 35 + 38 + ... + 1829$ c) $\cos 1° + \cos 2° + \cos 3° + ... + \cos 180°$

   b) $\dfrac{1}{2019} + \dfrac{2}{2019} + ... + \dfrac{2018}{2019}$

4. Find $\tan x$ if we know that $\tan 2x = \dfrac{48}{55}$.

5. Solve each of the following inequalities.

   a) $\dfrac{2x-5}{3x+1} < -2$ b) $2x + \dfrac{1}{3}x^2 \le -3$ c) $\dfrac{3}{x-1} \le \dfrac{x}{2}$ d) $\dfrac{2}{x+1} \le \dfrac{3}{4}$ e) $\dfrac{1}{x} \ge 2$

6. Solve each of the following equations.

   a) $\sin 2x = \cos x$
   b) $\cos 2x = \sin x$
   c) $\sin 2x = \sin x$
   d) $\cos 2x = \cos x$
   e) $5 \cdot 2^{3x-2} = 7 \cdot 3^{2x-1}$
   f) $2^{x+2} - 5 = 2^{x+3} + 1$
   g) $3^{x+1} + 13 = 3^{x+2} - 17$
   h) $4^x - 2^{x+3} = -7$

7. Solve each of the given triangles.

   a) $a = 15,\ b = 13,\ \beta = 48°$ c) $a = 6,\ c = 5,\ \alpha = 83°$

   b) $b = 13,\ c = 8,\ \alpha = 57°$ d) $a = 32,\ b = 56,\ c = 35$

8. Find the domain for each of the following functions.

   a) $f(x) = \dfrac{1}{\log_2 x + \log_2 (x-4)}$ b) $g(x) = \sec x$ c) $f(x) = \dfrac{2x-3}{x^2+1}$ d) $g(x) = \sqrt{\dfrac{x}{x+1}}$

9. Suppose that C_1 is a circle with radius 15 units long, and C_2 is a circle with radius 7 long. Suppose that the centers of the two circle are at a distance of 10 units from each other. Find the **exact values** of the sine and cosine of the angle formed by the common tangent lines drawn to the circle.

10. Suppose that $f(x) = x^2 - 3x + 1$ and $g(x) = -2x + 1$. Compute each of the following.

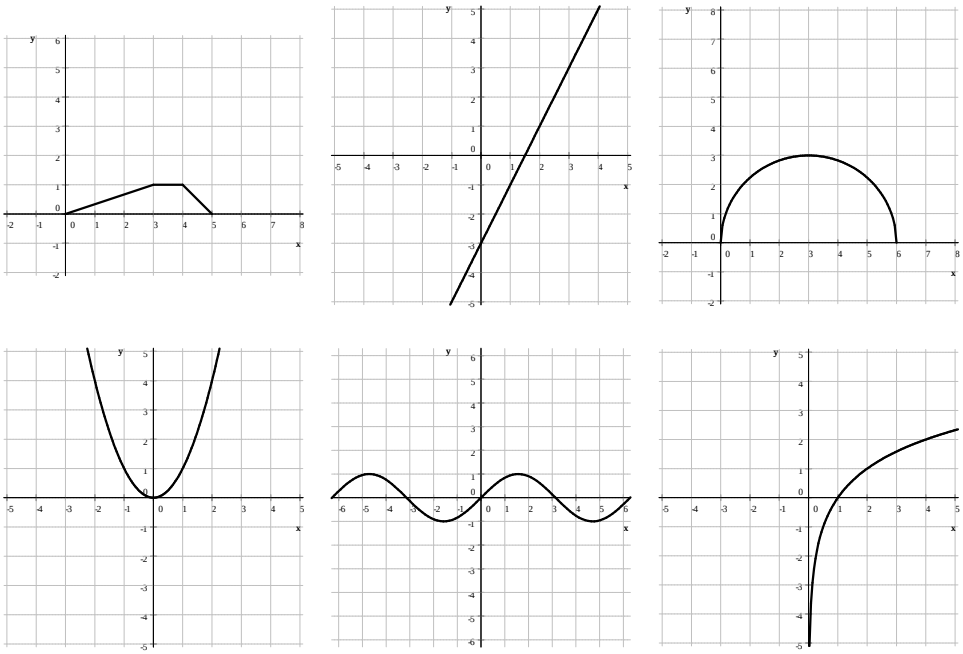
    a) $f(g(-1))$ b) $g(f(-1))$ c) $f(2a)$ d) $g(2a)$ e) $f(g(x))$ f) $g(f(x))$

    g) Is there any value of x for which $f(g(x)) = g(f(x))$?

11. Compute the inverse for each of the following functions.

    a) $f(x) = \dfrac{2}{3}x - 6$
    b) $f(x) = \sqrt[3]{2x+1}$
    c) $f(x) = (2x-3)^3 + 8$
    d) $f(x) = \dfrac{3x-1}{7x-5}$
    e) $f(x) = \dfrac{7x+10}{3x-7}$
    f) $f(x) = e^{4x-1} - 3$
    g) $f(x) = \log_3(5x-4)$

12. Given the graph of a function f, graph the inverse relation f^{-1} in the same coordinate system.



13. If we take Q amount of a certain medication, the amount of it in our system, t hours after intake is

$$A(t) = Q\left(\frac{3}{4}\right)^{0.8t}$$

a) Approximately what percent of the medication is in our system 2 hours after taking it?
b) How long until we have only 20% left in our system?
c) How long until we have only 1% left in our system?

14. The number of cells in a sample at time t (measured in hours) is $N(t) = 20\,000\left(1.2^{0.5t}\right)$. How long will it take for the sample to double

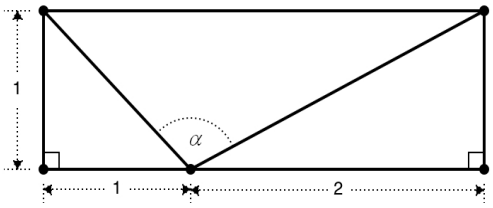
a) from the amount at $t = 0$ b) from the amount at $t = 3$

15. Prove that there is no real number x for which $\dfrac{1}{\sin^2 x - 1} = 1$ is true.

16. Find the exact value of each of the following.

a) $\log_5 15 + \log_5 35 - \log_5 21$ b) $3\log_3 6 + \log_3 35 - \log_3 20 - \log_3 42$ c) $\log 33 - \dfrac{1}{2}\log 44 - \log 15 - \log\sqrt{1100}$

17. Compute the exact value of $\tan\alpha$ if α is the angle shown on the picture.



18. Which is greater, $143^{\log_3 2018}$ or $2018^{\log_3 143}$?

19. Suppose that $x = \log_2 3$ and $y = \log_3 10$. Write $\log_6 20$ in terms of x and y.

20. Suppose that x is an angle belonging to the second quadrant and y is an angle belonging to the fourth quadrant. We also know that $\sin x = \dfrac{2}{3}$ and $\cos y = \dfrac{3}{5}$. Compute each of the following.

a) $\cos x$ c) $\sin 2x$ e) $\sin(x - y)$ g) $\tan 2x$
b) $\sin y$ d) $\cos 2x$ f) $\cos(x + y)$

21. Two sides of a parallelogram are 5 units and 7 units long. One angle in the parallelogram is 50°. How long are the diagonals? Present approximations.

22. a) Compute $\sin\alpha$ and $\cos\alpha$ in terms of M if we know that α is an acute angle and that $\tan\alpha = M$.
    b) Compute $\sin 2\alpha$ and $\cos 2\alpha$ in terms of M.

23. Redo the previous problem if α is in the second quadrant and $\tan\alpha = M$.

24. Compute $\sin\beta$ and $\cos\beta$ in terms of T if we know that β is an angle in the fourth quadrant and that $\tan\beta = T$.

25. Sketch the graph of each of the following functions.
    a) $f(x) = -(x+3)^2(x+1)$ b) $f(x) = (x+3)^2(x+1)(x-1)(x-3)^2$ c) $f(x) = 8x^2 - 2x^4$

26. Sketch the graph of each of the following.
    a) $f(x) = x - 2$
    b) $g(x) = (x-2)^2$
    c) $h(x) = (x-2)^3$
    d) $k(x) = (x-2)^4$
    e) $f(x) = \dfrac{1}{x-2}$
    f) $g(x) = \dfrac{1}{(x-2)^2}$
    g) $h(x) = \dfrac{1}{(x-2)^3}$
    h) $k(x) = \dfrac{1}{(x-2)^4}$

27. Prove by induction:
    a) For all natural numbers n, $\left(n^2+5\right)n$ is divisible by 6.
    b) For all natural numbers n, $4^n + 2$ is divisible by 6.
    c) For all natural numbers n, $2^{4n+1} + 3$ is divisible by 5.

28. *Find an equation for both tangent lines drawn to the graph of $(x-7)^2 + (y-4)^2 = 50$ from the external point $(-3, -16)$.

Answers

1. a) $\gamma = 59°$, $a \approx 9.924\,56$ ft, $c \approx 13.817\,671\,64$ ft b) no solution c) $\beta = 90°$, $\gamma = 30°$, $c = 2\sqrt{3}$ in
2. a) $\sqrt{13}$ b) $\sqrt{5}$ c) $\underline{i} - \underline{j}$ d) $\sqrt{2}$ e) $7\underline{i} - 9\underline{j}$ f) $\underline{j}$ g) 1 h) $A = 9$ and $B = 13$
3. a) 558 300 b) 1009 c) -1 4. $\dfrac{3}{8}$ or $-\dfrac{8}{3}$
5. a) $\left(-\dfrac{1}{3}, \dfrac{3}{8}\right)$ b) $\{-3\}$ c) $[-2, 1) \cup [3, \infty)$ d) $(-\infty, -1) \cup \left[\dfrac{5}{3}, \infty\right)$ e) $\left(0, \dfrac{1}{2}\right]$
6. a) $\dfrac{\pi}{2} + k\pi$, $\dfrac{\pi}{6} + 2k\pi$, $\dfrac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$ b) $-\dfrac{\pi}{2} + 2k\pi$, $\dfrac{\pi}{6} + 2k\pi$, $\dfrac{5\pi}{6} + 2k\pi$ where $k \in \mathbb{Z}$
   c) $k\pi$, $\pm\dfrac{\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$ d) $2k\pi$, $\pm\dfrac{2\pi}{3} + 2k\pi$ where $k \in \mathbb{Z}$ e) $\log_{9/8}\left(\dfrac{15}{28}\right)$
   f) no solution g) $\log_3 5$ h) $0, \log_2 7$
7. a) $\alpha_1 \approx 59.034\,225°$ $\gamma_1 \approx 72.965\,775°$ $c_1 \approx 16.725\,8$
   $\alpha_2 \approx 120.965\,775°$ $\gamma_2 \approx 11.034\,225°$ $c_2 \approx 3.348\,121\,6$
   b) $a \approx 10.941\,438\,7$ $\beta \approx 85.178\,262°$ $\gamma \approx 37.821\,738°$
   c) $b \approx 3.981\,483$ $\beta \approx 41.195\,798\,153°$ $\gamma \approx 55.804\,201\,847\,1°$
   d) $\alpha \approx 31.649\,327°$, $\beta \approx 113.327\,242°$, $\gamma \approx 35.023\,431°$

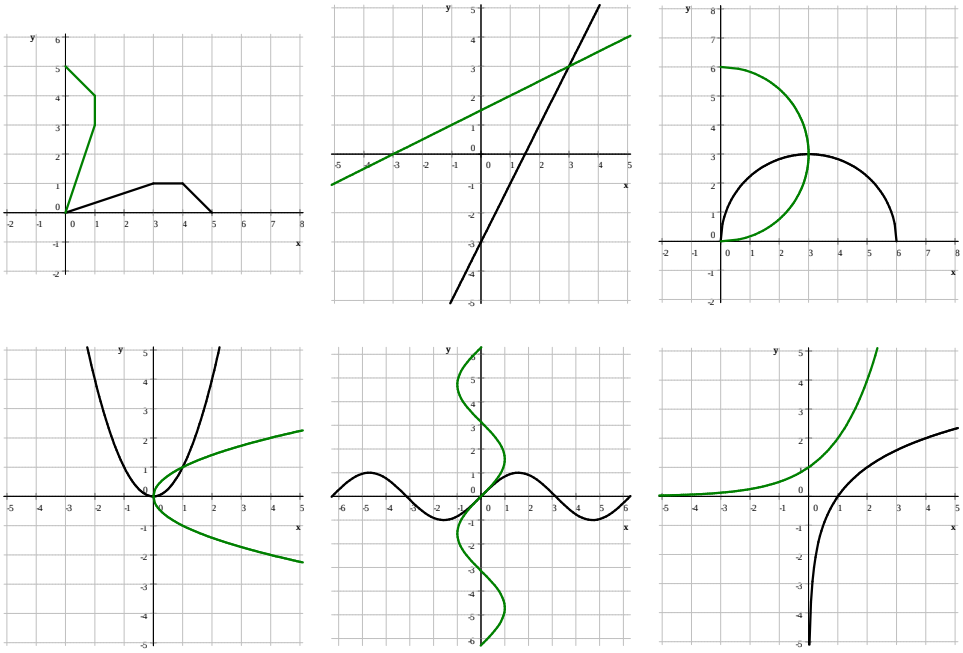
8. a) $\{x : x > 4 \text{ and } x \neq 2+\sqrt{5}\}$ b) $\left\{x : x \neq \frac{\pi}{2} + 2k\pi \text{ where } k \in \mathbb{Z}\right\}$ c) $\mathbb{R}$ d) $(-\infty, -1) \cup [0, \infty)$

9. $\sin\alpha = \frac{24}{25}$ $\cos\alpha = -\frac{7}{25}$

10. a) 1 b) -9 c) $4a^2 - 6a + 1$ d) $-4a + 1$ e) $4x^2 + 2x - 1$ f) $-2x^2 + 6x - 1$ g) 0 and $\frac{2}{3}$

11. a) $f^{-1}(x) = \frac{3}{2}x + 9$ b) $f^{-1}(x) = \frac{1}{2}x^3 - \frac{1}{2}$ c) $f^{-1}(x) = \frac{1}{2}\sqrt[3]{x-8} + \frac{3}{2}$ d) $f^{-1}(x) = \frac{5x-1}{7x-3}$
e) $f^{-1}(x) = \frac{7x+10}{3x-7}$ f) $f^{-1}(x) = \frac{1}{4}(\ln(x+3)+1)$ g) $f^{-1}(x) = \frac{1}{5}(3^x + 4)$

12. The green graph is the inverse.



13. a) 63.11% b) 6.99 hours c) 20 hours 14. a) 7.604 hours b) 7.604 hours

15. Proof: $\frac{1}{\sin^2 x - 1} = 1 \quad \Rightarrow \quad 1 = \sin^2 x - 1 \quad \Rightarrow \quad 2 = \sin^2 x \quad \Rightarrow \quad \sin x = \pm\sqrt{2}$
That's impossible $\sqrt{2} \approx 1.414\,21$ too big for sine of an angle

16. a) 2 b) 2 c) -2 17. -3 18. They are equal. Hint: take $\log_3$ of both expressions! 19. $\frac{xy+1}{x+1}$

20. a) $-\frac{\sqrt{5}}{3}$ b) $-\frac{4}{5}$ c) $-\frac{4\sqrt{5}}{9}$ d) $\frac{1}{9}$ e) $\frac{6-4\sqrt{5}}{15}$ f) $-\frac{3\sqrt{5}-8}{15}$ g) $4\sqrt{5}$

21. 5.385 616 7 unit and 10.908 489 unit

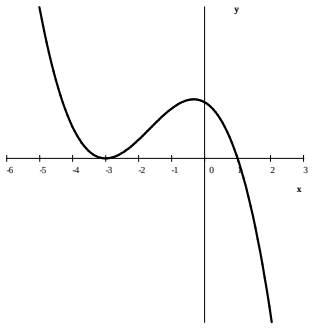
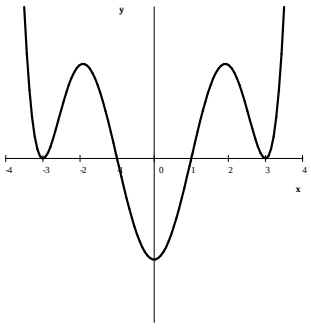
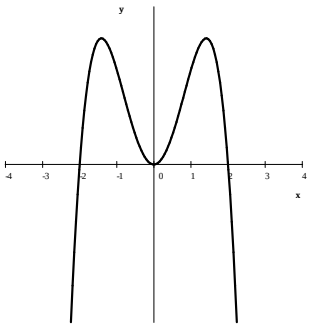
22. a) $\sin\alpha = \frac{M}{\sqrt{M^2+1}}$ and $\cos\alpha = \frac{1}{\sqrt{M^2+1}}$ b) $\sin 2\alpha = \frac{2M}{M^2+1}$ and $\cos 2\alpha = \frac{1-M^2}{M^2+1}$

23. If α is in the second quadrant, then $M < 0$. In the second quadrant, sine is positive and cosine is negative. Therefore,
a) $\sin\alpha = \frac{|M|}{\sqrt{M^2+1}}$ and $\cos\alpha = \frac{-1}{\sqrt{M^2+1}}$ b) $\sin 2\alpha = \frac{-|M|}{M^2+1} = \frac{2M}{M^2+1}$ and $\cos 2\alpha = \frac{1-M^2}{M^2+1}$

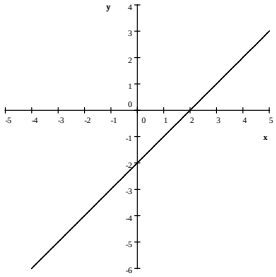
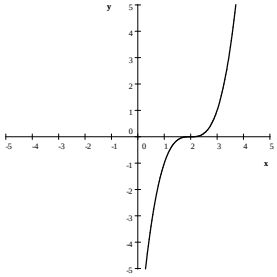
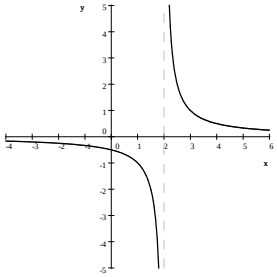
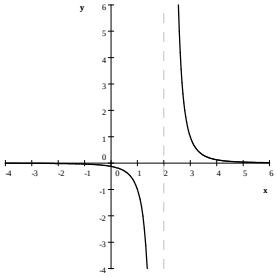
24. If β is in the fourth quadrant, then T is negative, $\sin\beta$ is negative, and $\cos\beta$ is positive.

$$\sin\beta = \frac{T}{\sqrt{T^2+1}} \text{ and } \cos\beta = \frac{1}{\sqrt{T^2+1}}$$

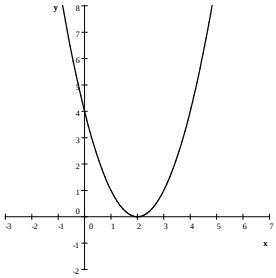
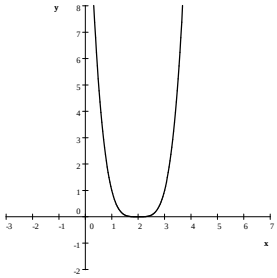
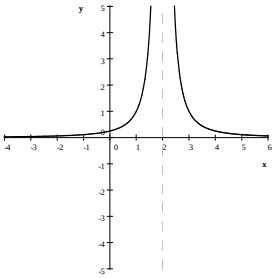
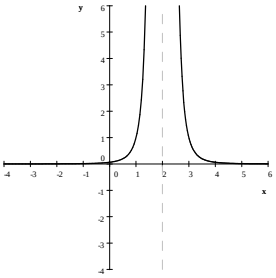
25. a) $f(x) = -(x+3)^2(x+1)$ b) $f(x) = (x+3)^2(x+1)(x-1)(x-3)^2$ c) $f(x) = -2x^2(x+2)(x-2)$

26. a) $f(x) = x-2$ c) $h(x) = (x-2)^3$ e) $f(x) = \dfrac{1}{x-2}$ g) $h(x) = \dfrac{1}{(x-2)^3}$

b) $g(x) = (x-2)^2$ d) $k(x) = (x-2)^4$ f) $g(x) = \dfrac{1}{(x-2)^2}$ h) $k(x) = \dfrac{1}{(x-2)^4}$

27. see handout 28. $y = 7x+5$ and $y = x-13$

Last revised: April 11, 2019