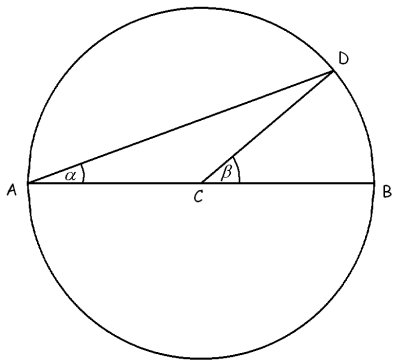
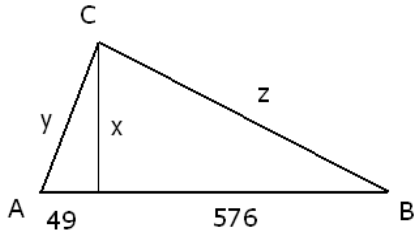
1. Suppose that $3^{400} = A$. Express each of the following in terms of A.
   a) 3^{402} b) 3^{399} c) 3^{200} d) 9^{400} e) $3^{400} + 3^{401} + 3^{402}$
2. List all three-digit numbers that can be formed using the numbers $1, 3, 6$ and 7 if repetition of digits is not allowed.
3. List all subsets of $A = \{1, 2, 3, 4\}$
4. List all two-element subsets of $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
5. Consider the circle shown on the picture. Prove that $\beta = 2\alpha$.

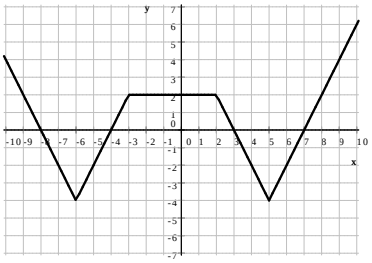


6. Rationalize the denominator in $\dfrac{\sqrt{x} - \sqrt{2}}{\sqrt{x} + \sqrt{2}}$.
7. Factor $\left(3x^2 - x + 5\right)^2 - \left(3x^2 - 3x - 5\right)^2$
8. Simplify each of the following. Present all answers using only positive exponents.
   a) $\dfrac{2^{-1} - 3^{-2}}{-2^{-2} + 1}$ b) $\dfrac{2b^{-2}\left(-a^3\right)^{-2} b^0}{\left(-b^2\right)^{-3} a^{-5}}$ c) $\dfrac{\sqrt{3} - 1}{\sqrt{3} + 1}$ d) $\left(\dfrac{1 - \sqrt{5}}{2}\right)^2 - 1$ e) $\dfrac{\sqrt{500} - \sqrt{20}}{\sqrt{45} - \sqrt{5}}$
9. Simplify each of the following.
   a) $2^{-3} + 5^{-2}$
   b) $\left(x^{-1}y^{-1}\right)^{-1}$
   c) $\left(x^{-1} + y^{-1}\right)^{-1}$
   d) $\left(\dfrac{2a^3b^{-2}\left(-ab^{-2}\right)^{-3} ba^0}{b^{-1}\left(-2b^2a^{-2}\right)^3 b}\right)$
   e) $\dfrac{2a^{-2}b^3}{a^5b^{-1}}$
   f) $\dfrac{2a^{-2} + b^3}{a^5 - b^{-1}}$
   g) $\left(\sqrt{3}\right)^6$
   h) $\left(\dfrac{1}{\sqrt{2}}\right)^{10}$
10. Solve the equation $5\left(x - 2\right)^2 - 3x + 2 = x - 2$. Check your solution(s) using exact values.
11. a) Solve $3x^2 + x = 3x + 2$.
    b) Check your solutions using exact values.
12. Find the exact value of x, y, and z based on the picture.



13. A person is standing 3 ft away from a street light that is 15.6 ft tall. How long is his shadow if he is 5.2 ft tall?
14. Solve each of the given inequalities.
    a) $\dfrac{2x + 3}{5} - \dfrac{x - 1}{2} \geq 11 - x$ b) $(x - 3)^2 > (x + 1)^2$ c) $\dfrac{2}{3}x - \dfrac{3}{10} < -\dfrac{1}{30}$
15. The conference was attended by 220 people.
    a) If everyone shook hands with everyone, how many handshakes took place?
    b) 120 of the attendees were men and 100 women. How many hadshakes took place if men only shook hands with men and woman only with women?
    c) 120 of the attendees were men and 100 women. How many hadshakes took place if men only shook hands with women?
    d) Out of the 220 people, 200 were married couples, the other 20 were single men. How many handshakes took place if everyone shook hands with everyone else, except for their spouse?
16. How many diagonals are there in a $20-$sided polygon?

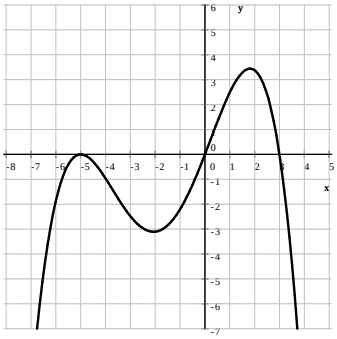
17. Find all values of x for which $P(x, y)$ is on the graph given and



a) $y \leq 0$ c) $y \geq 2$ e) $y < -2$

b) $y > 2$ d) $y \leq 2$ f) $y < -5$

18. Find all values of x for which $P(x, y)$ is on the graph given and



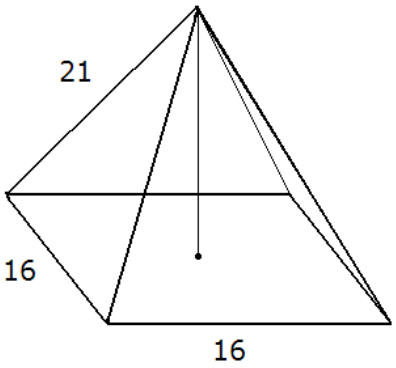
a) $y > 0$ c) $y \geq 0$

b) $y < 0$ d) $y \leq 0$

19. An arch is in the shape of a semicircle. At a point along the base 4 feet from an end of the arch, the height of the arch is 10 feet. Find the maximum height of the arch.

20. The hypotenuse of a right triangle is 50 feet long. Find the other two sides, given that the perimeter of the triangle is 112 feet.

21. Consider a square based straight pyramid as shown on the picture. The base is a square with sides 16 m long, and all other edges are 21 m long. Find the exact value of the height of the pyramid.



Answers

1. a) $9A$ b) $\frac{A}{3}$ c) $\sqrt{A}$ d) A^2 e) $13A$

2.

136	316	613	713
137	317	617	716
163	361	631	731
167	367	637	736
173	371	671	761
176	376	673	763

3. All subsets of $A = \{1, 2, 3, 4\}$

0-element subsets: $\varnothing$

1-element subsets: $\{1\},\{2\},\{3\},\{4\}$

2-element subsets: $\{1, 2\},\{1, 3\},\{1, 4\},\{2, 3\},\{2, 4\},\{3, 4\}$

3-element subsets: $\{1, 2, 3\},\{1, 2, 4\},\{1, 3, 4\},\{2, 3, 4\}$

4-element subsets: $\{1, 2, 3, 4\}$

4. All two-element subsets of $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$\{1,2\}$								
$\{1,3\}$	$\{2,3\}$							
$\{1,4\}$	$\{2,4\}$	$\{3,4\}$						
$\{1,5\}$	$\{2,5\}$	$\{3,5\}$	$\{4,5\}$					
$\{1,6\}$	$\{2,6\}$	$\{3,6\}$	$\{4,6\}$	$\{5,6\}$				
$\{1,7\}$	$\{2,7\}$	$\{3,7\}$	$\{4,7\}$	$\{5,7\}$	$\{6,7\}$			
$\{1,8\}$	$\{2,8\}$	$\{3,8\}$	$\{4,8\}$	$\{5,8\}$	$\{6,8\}$	$\{7,8\}$		
$\{1,9\}$	$\{2,9\}$	$\{3,9\}$	$\{4,9\}$	$\{5,9\}$	$\{6,9\}$	$\{7,9\}$	$\{8,9\}$	
$\{1,10\}$	$\{2,10\}$	$\{3,10\}$	$\{4,10\}$	$\{5,10\}$	$\{6,10\}$	$\{7,10\}$	$\{8,10\}$	$\{9,10\}$

5. Line segments AC, BC, and CD are all radii in the circle, and so they are equal. So ACD triangle is isosceles, and the angles opposite AC and CD are also equal to each other. Thus $\measuredangle ADC = \alpha$. The third angle in triangle ACD is $180^\circ - 2\alpha$. Angles ACD and DCB are supplementary angles beause together they form a straight angle. Thus

$$\begin{aligned} \measuredangle ACD + \measuredangle DCB &= 180^\circ \\ 180^\circ - 2\alpha + \beta &= 180^\circ \qquad \text{subtract } 180^\circ \\ -2\alpha + \beta &= 0 \qquad \text{add } 2\alpha \\ \beta &= 2\alpha \end{aligned}$$

6. $\dfrac{x + 2 - 2\sqrt{2x}}{x - 2}$ 7. $4x\,(x+5)\,(3x-2)$ 8. a) $\dfrac{14}{27}$ b) $-\dfrac{2b^4}{a}$ c) $2 - \sqrt{3}$ d) $\dfrac{1-\sqrt{5}}{2}$ e) 4

9. a) $\dfrac{33}{200}$ b) xy c) $\dfrac{xy}{x+y}$ d) $\dfrac{a^6}{4b}$ e) $\dfrac{2b^4}{a^7}$ f) $\dfrac{2b + a^2b^4}{a^7b - a^2}$ g) 27 h) $\dfrac{1}{32}$ 10. $x_{1,2} = \dfrac{12 \pm 2\sqrt{6}}{5}$

11. a) $\dfrac{1 \pm \sqrt{7}}{3}$ b) If $x = \dfrac{1-\sqrt{7}}{3}$, then

$$\text{LHS} = 3\left(\frac{1-\sqrt{7}}{3}\right)^2 + \frac{1-\sqrt{7}}{3} = 3\cdot\frac{8-2\sqrt{7}}{9} + \frac{1-\sqrt{7}}{3} = \frac{8-2\sqrt{7}}{3} + \frac{1-\sqrt{7}}{3} = \frac{9-3\sqrt{7}}{3} = \frac{3\left(3-\sqrt{7}\right)}{3} = 3 - \sqrt{7}$$

$$\text{RHS} = 3\left(\frac{1-\sqrt{7}}{3}\right) + 2 = 1 - \sqrt{7} + 2 = 3 - \sqrt{7}$$

Checking the other solution goes similarly.

12. $x = 168$, $y = 175$, $z = 600$ 13. $1.5\,\text{ft}$ 14. a) $[11, \infty)$ b) $(-\infty, 1)$ c) $\left(-\infty, \dfrac{2}{5}\right)$

15. a) $24\,090$ b) $12\,090$ c) $12\,000$ d) $23\,990$ 16. 170

17. a) $[-8, -4] \cup [3, 7]$ b) $(-\infty, -9) \cup (8, \infty)$ c) $(-\infty, -9] \cup [-3, 2] \cup [8, \infty)$ d) $[-9, 8]$ e) $(-7, -5) \cup (4, 6)$ f) no solution

18. a) $(0, 3)$ b) $(-\infty, -5) \cup (-5, 0) \cup (3, \infty)$ c) $[0, 3] \cup \{-5\}$ d) $(-\infty, 0] \cup [3, \infty)$

19. 14.5 feet 20. 14 and 48 feet 21. $\sqrt{313}\,\text{m}$