Students must be able to correctly state the following theorems: Intermediate Value Theorem (both forms), Extreme Value Theorem, Rolle's Theorem, Mean Value Theorem, the second derivative test.

Students must be able to prove the following theorems:

- Differentiating functions using the definition (limit of the differential quotient)
- Prove that $\frac{d}{dx}(\sin x) = \cos x$ and $\frac{d}{dx}(\cos x) = -\sin x$ and $\frac{d}{dx}(\ln x) = \frac{1}{x}$, $\frac{d}{dx}(e^x) = e^x$, and $\frac{d}{dx}(a^x) = a^x \cdot \ln a$
- If a function is differentiable at a number x, then it is continuous there.
- The product rule and quotient rule for derivatives.
- Rolle's Theorem.
- The Mean Value Theorem.

Review Problems

1. State and prove the mean value theorem.
2. Compute each of the given sums.

   a) $\sum_{n=1}^{50}(3n+2)$
   b) $\sum_{n=0}^{50}(3n+2)$
   c) $\sum_{k=0}^{100}\left(k^2+4k+4\right)$
   d) $\sum_{k=1}^{100}\left(k^2+4k+4\right)$
   e) $\sum_{j=0}^{150}\left((j+1)^2-j^2\right)$
   f) $\sum_{m=1}^{60}\left(m^3-m^2+1\right)$
   g) $\sum_{k=0}^{99}(2k+1)$
   h) $\sum_{k=0}^{99}(2k+1)^2$
   i) $\sum_{n=51}^{100}(2n+1)^2$
   j) $0.1^2+0.2^2+0.3^2+...+10^2$

3. The derivative of a function is given by $f'(x) = -2(x+3)(x+1)^2(x-2)^3(x-4)$.

   a) Sketch the graph of f'. b) Sketch the graph of f in the same coordinate system with f'.

4. Compute the inverse for each of the following functions.

   a) $f(x) = 3^{\sqrt{x}-1}$ b) $f(x) = \ln\left(\frac{1}{3}x-1\right)$ c) $f(x) = 5\sqrt[3]{2x-1}$ d) $f(x) = \frac{x-3}{5x+8}$

5. Differentiate each of the following.

   a) $f(x) = e^{\sin^2 x}e^{\cos^2 x}$
   b) $f(x) = \ln(\tan x)$
   c) $f(x) = \tan(\pi x)$
   d) $f(x) = \ln(\sec x)$
   e) $x^3y - x^2y^3 = x - y$
   f) $\ln x - \ln y = \sin xy$
   g) $f(x) = \sec x$
   h) $f(x) = \ln(\tan x)$
   i) $f(x) = \sin^2 x$
   j) $x^3 + y^3 = \ln(x+y)$
   k) $\sin x + \cos y = \tan xy$
   l) $\left(x^2+y^2\right)^4 = 5x^3y^4$

6. a) Find an equation of all tangent lines drawn to the graph of $2x^2 + y^2 = 5y - x$ at $x = -2$.
   b) Find an equation of the tangent line drawn to the graph of $x^2 - y^3 + 2xy = y^2 - 5x - 12$ to the point $(-2, 1)$.

7. Let g be a differentiable function with $g(5) = 10$ and $g'(5) = -2$. Compute the exact value of $f'(5)$ if f is defined as a) $f(x) = 3g(x) + 1$ b) $f(x) = (g(x))^3$ c) $f(x) = \ln(g(x))$ d) $f(x) = \cos(g(x))$

8. Find all relative maximums and minimums of $f(x) = (2x-1)^5 (7-x)^8$.

9. Compute each of the following indefinite integrals.
   a) $\int \sec^2 \theta d\theta$ b) $\int \sec\theta \tan\theta \, d\theta$ c) $\int \left(x - \frac{1}{x}\right) dx$ d) $\int \sin x dx$

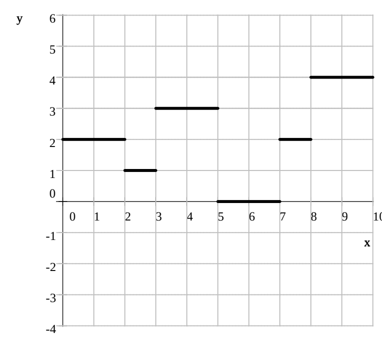
10. We would like to design a book. Each page should contain $60\,\mathrm{cm}^2$ of text. The upper margin needs to be $3\,\mathrm{cm}$ wide and all other margins need to be $2\,\mathrm{cm}$ wide. What dimensions for the book would minimize the amount of paper we need to use to produce this book?

11. Prove that if $f(x) = e^x$, then $f'(x) = e^x$.

12. A company finds that if they spend x dollars on research and y dollars on marketing, then they can make a profit of $\sqrt{x}y^2$ dollars. How much should the company spend on research and marketing if they can spend a total of $200\,000$ dollars on these things?

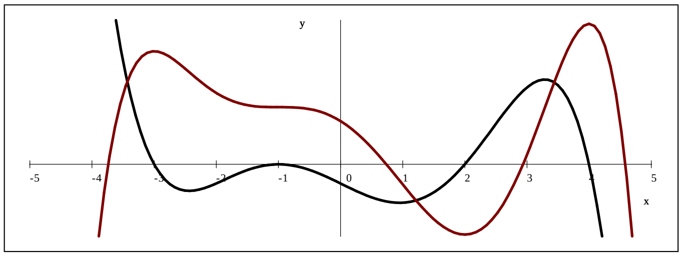
13. Find the number c that satisfies the conclusion of the Mean Value Theorem for $f(x) = x^3 - x + 2$ on $[1, 4]$.

14. An object starts from a location of $L(0) = 0$. Find the location of the object $t = 10$ if its velocity function is shown on the picture.



Answers

1. see handout 2. a) 3925 b) 3927 c) 358 954 d) 358 950 e) 22 801 f) 3275 150 g) 10 000 h) 1333 300 i) 1196 850 j) 3383. 5

3. 

4. a) $f^{-1}(x) = (1 + \log_3 x)^2$ b) $f^{-1}(x) = 3 + 3e^x$ c) $f^{-1}(x) = \dfrac{x^3}{250} + \dfrac{1}{2}$ d) $f^{-1}(x) = \dfrac{8x+3}{-5x+1}$

5. a) $f'(x) = 0$ b) $f'(x) = \dfrac{\tan^2 x + 1}{\tan x} = \dfrac{1}{\cos x \sin x}$ c) $f'(x) = \pi \sec^2(\pi x)$ d) $f'(x) = \tan x$

e) $y' = \dfrac{-3x^2 y + 2xy^3 + 1}{x^3 - 3x^2 y^2 + 1}$ f) $y' = \dfrac{-xy^2 \cos xy + y}{x^2 y \cos xy + x}$ g) $f'(x) = \sec x \tan x$

h) $f'(x) = \dfrac{\tan^2 x + 1}{\tan x} = \dfrac{1}{\sin x \cos x}$ i) $f'(x) = 2\cos x \sin x = \sin 2x$ j) $y' = \dfrac{-3x^2(x+y) + 1}{3y^2(x+y) - 1}$

k) $y' = \dfrac{-\cos x + y\left(\tan^2 xy + 1\right)}{-\sin y - x\left(\tan^2 xy + 1\right)}$ l) $y' = \dfrac{5y^4 - 8x\left(x^2+y^2\right)^3}{-20xy^3 + 8y\left(x^2+y^2\right)^3}$

6. a) $y = -7x - 12$ and $y = 7x + 17$ b) $\dfrac{1}{3}(x+2) = y - 1$ 7. a) -6 b) -600 c) $-\dfrac{1}{5}$ d) $2\sin 10$ e) $\dfrac{3}{5000}$

8. $f'(x) = -26(2x-1)^5(x-3)(7-x)^8$ maximum at $x = 3$, minimum at $x = 7$

9. a) $\tan\theta + C$ b) $\sec\theta + C$ c) $\dfrac{1}{2}x^2 - \ln|x| + C$ d) $-\cos x + C$

10. vertical: $\left(5 + 5\sqrt{3}\right)$ cm horizontal: $\left(4 + 4\sqrt{3}\right)$ cm 11. see handout

12. They should spend 40 000 on research and 160 000 on marketing 13. $\sqrt{7}$ 14. $L(10) = 21$