This problem set is not homework. Students can use this problem set as extra practice or study guide for quizzes.

1. Label each of the following statements as true or false.

   a) For any sets, A and B, $A \cap B \subseteq A$.
   b) For any sets A and B, $A \subseteq A \cup B$.
   c) For any sets A and B, $A \cup B \subseteq A \cap B$.
   d) For any sets A and B, if $A \subseteq B$ and $B \subseteq A$, then $A = B$.
   e) For any sets A, $\varnothing \subseteq A$.
   f) For any sets A, B, and C, if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.
   g) For any sets A, B, and C, $(A \cap B) \cup C = A \cap (B \cup C)$.
   h) For any sets A, B, and C, $(A \cap B) \cap C = A \cap (B \cap C)$.
   i) For any sets A, B, and C, $(A \cup B) \cup C = A \cup (B \cup C)$.

2. Suppose that $A = \{1, 2, 3, 4, 5\}$. List all two-element subsets of A.

3. Suppose that $P = \{1,\ 3,\ 6,\ 10\}$, and $S = \{1,\ 2,\ 5,\ 6,\ 9,\ 10\}$, and $T = \{3,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10\}$

   a) Draw a Venn-diagram depicting P, S, and T.
   b) Find each of the following.
      i) $S \cap P \cap T$ ii) $S \cup (P \cap T)$ iii) $(S \cup P) \cap T$ iv) $P \cup (S \cap T)$
   c) Find an operation or (operations) on P, S, and/or T so that the result is the set $\{1,\ 6,\ 10\}$.
   d) True or false? $P \subseteq S \cup T$
   e) True or false? $S \cap T \subseteq P$

4. Suppose that $T = \{n \in \mathbb{Z} : n \text{ is divisible by } 3\}$, $S = \{n \in \mathbb{Z} : n \text{ is divisible by } 6\}$, and $E = \{n \in \mathbb{Z} : n \text{ is divisible by } 2\}$. Label each of the statements as true or false.

   a) $T \cap E = S$ b) $E \subseteq S$ c) $S \subseteq T$ d) $T \cup E = S$

5. Find the prime factorization for each of the following numbers.

   a) 1200 b) 2016 c) 1820

6. Find the GCF (greatest common factor) and LCM (least common multiple) of each of the given pairs of numbers.

   a) 1200 and 990 b) 210 and 135

7. a) Find the last digit of 7^{2018}.
   b) Find the last digit of $2^{99} + 2^{100} + 2^{101} + 2^{102}$.

8. Evaluate each of the given numerical expressions.

   a) $5 - 2\left(20 - 12 \div 2\left(-3^2 + 15\right)\right)$ b) $\sqrt{2\sqrt{3 \cdot 7 - 5} - \sqrt{2^3 - (-1)^3} - 1}$ c) $\dfrac{-1^2 - 2\left(-3^2 - 2\left(3^2 - 7\right)\right)}{-2^2 - 1}$

9. Perform the given operations.

   a) $\dfrac{2}{3} - \dfrac{8}{5} \div \left(-3\dfrac{3}{7}\right)$ c) $\dfrac{1}{2} \cdot \dfrac{2}{3} \cdot \dfrac{3}{4} \cdot \dfrac{4}{5} \cdot \dfrac{5}{6}$ e) $\dfrac{5}{12} - \left(-\dfrac{3}{4}\right)^2$

   b) $\dfrac{\frac{5}{6} - \frac{3}{8}}{\frac{1}{3} - \frac{1}{2}}$ d) $\dfrac{3}{10} - \dfrac{12}{5}\left(\dfrac{1}{2} - \dfrac{2}{3}\right)$ f) $-\dfrac{3}{4} - \dfrac{5}{6} \div \left(-2\dfrac{1}{2}\right)$

10. Consider the expression $\dfrac{7x-2x^2-3}{2x-1}$. Evaluate the expression with the given value of x.

a) $x=-2$ b) $-\dfrac{1}{2}$ c) 3 d) $\dfrac{1}{2}$ e) $\dfrac{2}{3}$

11. Simplify each of the following. Note that each problem appears twice: once with integer coefficients, and then with fractions for coefficints. The steps are the same, only each step takes more work with fractions.

a) $2a-3-(5a-3)$

b) $\dfrac{2}{3}a-\dfrac{1}{2}-\left(\dfrac{3}{4}a-\dfrac{1}{2}\right)$

c) $-3(2x-5)-4(-x+2)$

d) $-\dfrac{2}{5}\left(3x-\dfrac{1}{3}\right)-\dfrac{3}{10}\left(-6x+\dfrac{1}{6}\right)$

e) $2(3m-4)-5(-2m+3)-2(m-7)$

f) $4\left(\dfrac{3}{2}m-\dfrac{1}{5}\right)-5\left(-\dfrac{2}{3}m+\dfrac{3}{2}\right)-2\left(\dfrac{1}{6}m-\dfrac{17}{4}\right)$

g) $-3a-2(5a-3(4a-b))$

h) $-3a-2\left(a-3\left(\dfrac{1}{2}a-\dfrac{1}{6}b\right)\right)$

i) $5x-(3(4-(x-3(2x-1))+x)-x)-1$

12. Solve each of the given equations.

a) $-3x+7=16$

b) $-\dfrac{2}{3}x+\dfrac{1}{2}=\dfrac{11}{6}$

c) $\dfrac{a}{-3}+7=-1$

d) $\dfrac{\frac{x}{3}}{8}+\dfrac{1}{2}=-\dfrac{1}{3}$

e) $\dfrac{a+7}{-3}=-1$

f) $\dfrac{x+\frac{4}{5}}{-\frac{2}{3}}=-\dfrac{7}{10}$

g) $5x-8=-8$

h) $\dfrac{2}{7}x-\dfrac{3}{2}=-\dfrac{5}{14}$

i) $3(2x-1)-2(5x-7)=-5$

j) $\dfrac{1}{2}(4x-10)-\dfrac{3}{2}(2x-8)=-\dfrac{1}{4}(12x-28)$

k) $2x-(3x-4(2x-1))=17$

g) $3x-8-5(2x-8)=-3$

h) $\dfrac{\frac{3x-1}{4}+9}{\frac{5}{2}}-7=-3$

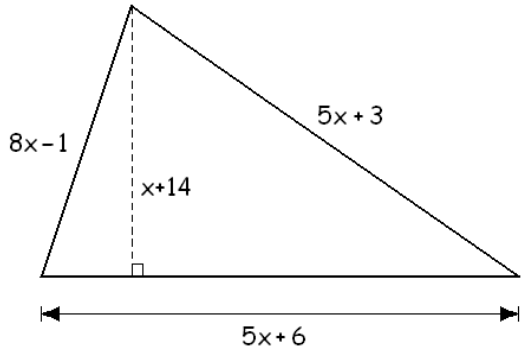
i) $5(3x-1)-4(4x-3)=7$

13. Solve each of the following application problems. In each case, set up and solve an equation.

a) The sum of three consecutive multiples of 4 is -24. Find these numbers.

b) One side of a rectangle is five inches shorter than four times another side. Find the sides of the rectangle if its perimeter is $110\,\text{in}$.

c) Consider the triangle shown on the picture. Find the value of x, given that the perimeter of the triangle is 98 unit.

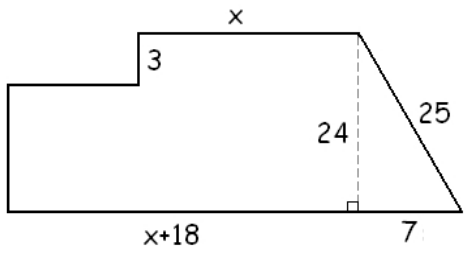


d) Samantha is asked about her age. She answers as follows. "*My age is ten less than twice the age of my younger brother. The sum of our ages is 32*". How old is Samantha?

e) The local library held a sale of their old books. Each softcover book was priced at 3 dollars, and hardcover books costed 5 dollars each. We purchased a whole lot of books! The number of softcover books we got was eight less than three times the number of hardcover books. How many softcover books did we buy if we paid a total of 74 dollars?

f) The first night in a hotel costs 38 dollars, and each additional night cost 25 dollars. How long did we stay in the hotel if the bill was 163 dollars?

g) Consider the object shown on the picture. Angles that look like right angles are right angles. Find the perimeter of the object, given that its area is 1182 unit^2.

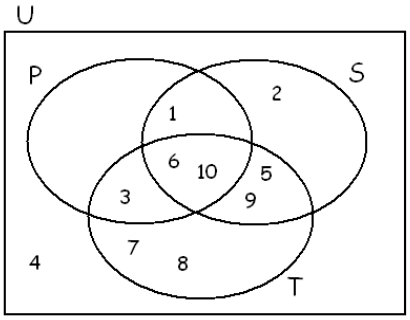


Answers

1. a) true b) true c) false d) true e) true f) true g) false h) true i) true

2. $\{1,2\}$ $\{2,3\}$ $\{3,4\}$ $\{4,5\}$ there are 10 such subsets
   $\{1,3\}$ $\{2,4\}$ $\{3,5\}$
   $\{1,4\}$ $\{2,5\}$
   $\{1,5\}$

3. a)



   b) i) $\{6,10\}$ ii) $\{1,2,3,5,6,9,10\}$ iii) $\{3,5,6,9,10\}$ iv) $\{1,3,5,6,9,10\}$ c) $P \cap S$ d) true e) false

4. a) true b) false c) true d) false

5. a) $2^4 \cdot 3 \cdot 5^2$ b) $2^5 \cdot 3^2 \cdot 7$ c) $2^2 \cdot 5 \cdot 7 \cdot 13$

6. a) $\gcd(1200, 990) = 30$ $\operatorname{lcm}(1200, 990) = 39\,600$
   b) $\gcd(210, 135) = 15$ $\operatorname{lcm}(210, 135) = 1890$

7. a) 9 b) 0

8. a) 37 b) 2 c) -5

9. a) $\frac{17}{15}$ b) $-\frac{11}{4}$ c) $\frac{1}{6}$ d) $\frac{7}{10}$ e) $-\frac{7}{48}$ f) $-\frac{5}{12}$

10. a) 5 b) $\frac{7}{2}$ c) 0 d) undefined e) $\frac{7}{3}$

11. a) $-3a$ b) $-\frac{1}{12}a$ c) $-2x+7$ d) $\frac{3}{5}x+\frac{1}{12}$ e) $14m-9$ f) $9m+\frac{1}{5}$ g) $11a-6b$ h) $-2a-b$ i) $-12x-4$

12. a) -3 b) -2 c) 24 d) $-\frac{5}{16}$ e) -4 f) $-\frac{1}{3}$ g) 0 h) 4 i) 4 j) 0 k) 3 g) 5 h) -5 i) 0

13. a) $-12, -8, -4$, and 0 b) $12\,\text{in}$ by $43\,\text{in}$ c) 5 d) 18 years old e) 13 f) 6 nights g) 145 unit

