1. Suppose that $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 4, 9\}$, and $B = \{2, 4, 6, 7\}$. Find each of the following.

    a) $A \cap B$ b) $A \cup B$ c) $P = \{x \in U : x > 4 \text{ or } x \leq 7\}$ d) $Q = \{x \in U : x > 4 \text{ and } x \leq 7\}$

2. Label each of the following statements as true or false.

    a) For all sets A, $\varnothing \subseteq A$.

    b) For all sets A, $A \cap \varnothing = A$.

    c) For all sets A and B, $A \cap B \subseteq A$.

    d) For all sets A and B, $A \subseteq A \cup B$.

    e) For all sets A and B, if $A \subseteq B$, then $A \cup B = B$.

    f) $\mathbb{N} \cup \mathbb{Z} = \mathbb{N}$

    g) $\mathbb{N} \cap \mathbb{Z} = \mathbb{N}$

3. Suppose that S is the set of all squares and R is the set of all rectangles. Label each of the following statements as true or false.

    a) $S \subseteq R$ c) $R \subseteq R$ e) $R \cup S = S$ g) $R \cap S = S$

    b) $R \subseteq S$ d) $\varnothing \subseteq S$ f) $R \cup S = R$ h) $R \cap S = R$

4. Suppose that F is the set of all integers divisible by four, S is the set of all integers divisible by six, and T is the set of all integers divisible by three. Label each of the following statements as true or false.

    a) $S \subseteq T$ b) $F \subseteq S$ c) $F \cap T = S$ d) $F \cap T \subseteq S$ e) $F \subseteq S \cup T$

5. Suppose that $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 4, 5, 7, 10\}$, $B = \{1, 4, 6, 7, 8\}$, and $C = \{1, 4, 5, 6, 9\}$.

    a) Draw a Venn diagram depicting these sets.

    b) Find each of the following sets.

         i) $(A \cup B) \cap C$ ii) $A \cup (B \cap C)$ iii) $(A \cap B) \cup C$ iv) $A \cap (B \cap C)$ v) $A \cap (B \cup C)$

6. The freshman class had 35 students. 22 students took Mathematics, 19 students took English, and 14 student took both of these subjects. How many students took neither Mathematics, nor English?

7. What can be the missing last digit A in the number $6040\,708\,45A$ if we know that the number is divisible by 6?

8. Perform the division with remainder: $2017 \div 17$

9. List all factors of 84.

10. Find the prime-factorization of each of the numbers given.

    a) 120 b) 156 c) 98 d) 2015

11. Consider the following numbers: 2011, $11\,060\,904$, 321, 3106. Select all the numbers from the list that are divisible
    a) by 2 b) by 3 c) by 6

12. Which of the following numbers is a prime? 2007, 143, 151, 91

13. Label each of the following statements as true or false.

    a) If an integer n is divisible by 6, then it is also divisible by 3.

    b) If integer a is divisible by 4 and integer b is divisible by 6, then the product ab is divisible by 24.

    c) If an integer n is divisible by 4 and also by 6, then it is also divisible by 24.

    d) If the product ab is divisible by 7, then a is divisible by 7 or b is divisible by 7.

    e) If the product ab is divisible by 6, then a is divisible by 6 or b is divisible by 6.

14. Find the smallest positive integer that is divisible by 2, 3, 4, 5, and 6.

15. Let $S = \{x \in \mathbb{N} : x \geq 2\}$ and $T = \{x \in \mathbb{N} : x < 10\}$. Find $S \cap T$ and $S \cup T$.

16. Let $A = \{n \in \mathbb{N} : n$ is divisible by $2\}$, $B = \{n \in \mathbb{N} : n$ is divisible by $6\}$. Which (if any) of the following is true?

   $A \subseteq B \qquad B \subseteq A$

17. Let E be the set of all even integers and O be the set of all odd integers. Find $E \cap O$ and $E \cup O$.

18. Let $P = \{n \in \mathbb{N} : n$ is divisible by $2\}$, $Q = \{n \in \mathbb{N} : n$ is divisible by $5\}$. What is $P \cap Q$?

19. Simplify each of the following.

   a) -3^2
   c) $12 \div 3 \cdot 2$
   e) $12 - 2\,(7 - 4 \cdot 3)$
   g) $-\sqrt{49}$
   i) -2^2

   b) $-|-6|$
   d) $15 - 3 + 2$
   f) $|-8 + 5|$
   h) $\sqrt{-49}$
   j) $(-2)^2$

20. Simplify each of the following.

   a) $\left|-3^3 - 2\,|-5 - 2\,(-4)|\right|$

   b) $\sqrt{-4^2 - (-1)^4 + 2 \cdot 3^2 \div 2 \cdot 6 - 1}$

   c) $-3^2 - |-12 + 2 \cdot 5| - 2 + 1$

   d) $-2^2 - 5\,(-2)$

   e) $-3^2 - 2\,(4 - 5^2 + 3\,(10 - 7 + 2))$

   f) $\sqrt{6^2 - 5\sqrt{16}}$

   g) $\dfrac{24 - (-2)^2 + 12 - 3 + 1}{-3^2 - 12 \div 3\,(-2)}$

   h) $|3 - 2 - |8 - 10||$

   i) $|3 - |2 - 8| - 10|$

   j) $|3 - 2\,|-8 - 10||$

21. Perform the indicated operations. Show all steps.

   a) $\sqrt{6 - \left(2 - 3\,(-4^2 - 3\,(-6))^2\right)}$

   b) $7 - 6\,(3^2 - 4\,(7 - 2\,(9 - 7)))$

   c) $\dfrac{40 \div (-2)\,5}{-3^2 + (-1)^3}$

   d) $\left(\left(\left(13 - (-3)^2\right)^2 - 12\right)^2 - 19\right)^2$

   e) $\dfrac{3 - 12\,(-4) \div (-6)}{(-2)^4 - 4^2}$

22. a) Re-write 35% as a reduced fraction.

   b) Re-write $\dfrac{7}{10}$ as a percent.

   c) Bring $\dfrac{3}{5}$ and $\dfrac{5}{8}$ to a common denominator to compare them.

23. Evaluate the algebraic expression $\dfrac{-x + 2x^2 - 1}{x - 1}$ if

   a) $x = 5$ b) $x = -5$ c) $x = 1$ d) $x = -1$

24. Let $p = 4$, $q = -3$, and $s = 1$. Evaluate each of the following expressions.

   a) $-q^2 - pq$ b) $\dfrac{2p - q}{p - (s - q)}$ c) $p^2 - 2s^2$ d) $p^2 - (2s)^2$ e) $|p - q| - 3s$ f) $p - |q - 3s|$

25. Suppose that $x = 4$ and $y = -3$. Evaluate each of the algebraic expressions.

   a) $2x - y + 1$ b) $-y^2 - 3x^2 y$ c) $(-x)^2 - 5y$ d) $5x - 2y + 2x + y = 31$ e) $\left|\dfrac{x^2 - y^2}{y^2 - x^2}\right|$

26. Consider the equation $x^3 - x^2 + 7 = x^2 + 5x + 1$. Which of the given numbers are solutions of the equation? The given numbers: $0, 1, -1, 2,$ and -2

27. Consider the equation $2x^3 - 10\left(x^2 - 2\right) + 4x = -x^2 + 5$. For each of the following numbers given, determine whether it is a solution of the equation or not. The given numbers: -2, -1, and 3

28. Consider the inequality $x^2 + 3x \leq x + 24$. Which of the given numbers are solutions of the inequality? The given numbers: 5, 6, 0, -10, 3, and 4

29. Compute the perimeter and area of the figure shown. Angles that look like right angles are right angles. Include units in your computation and answer.
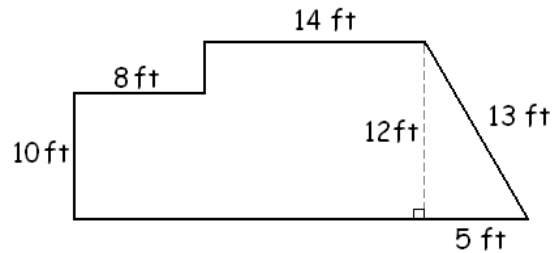
30. Compute the area of the shaded region shown on the picture. Units are in meters. Include units in your computation and answer.

31. What is the last digit of the number 2^{99}?
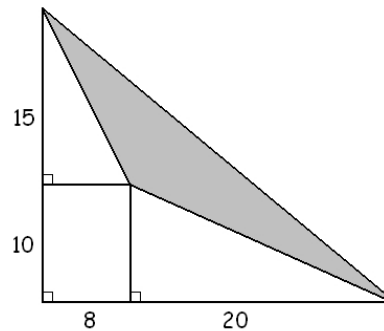
32. Translate each of the following sentences into algebraic statements.

   a) X is three less than twice Y.

   b) The opposite of x is five greater than the sum of y and half of z.

   c) The product of a and b is fourteen less than three times the sum of a and b.

   d) The sum of m and twice n is one greater than the quotient of m and n.

   e) Three times the difference of x and y is one less than the product of x and the opposite of y.

   f) Suppose that Peter's age is ten less than twice the age of his younger brother, David. If Peter's age is denoted by P and David's age is denoted by D, write an algebraic statement expressing P in terms of D.

   g) Suppose that a cab-fare for one person in Chicago is 2.50 dollars for the first mile and 1.5 dollars for each additional mile. Write an algebraic expression for the price of a cab ride in Chicago of a distance of x miles.

   h) We are thinking of three consecutive numbers. Express them in terms of x if x represents the smallest number.

   i) The longer side of a rectangle is three feet shorter than four times the shorter side. If the shorter side of the rectangle is denoted by x, express the area of the rectangle in terms of x.

   j) Suppose that Ann has A dollars and Beatrix has B dollars. The girls made a bet that Ann lost, so she must pay Beatrix 30 dollars. Express how much money do the girls have after Ann paid.

33. Given that $A \cap B = \{1, 2, 4, 5\}$ and $A \cup B = \{1, 2, 3, 4, 5, 6\}$, how many different sets are possible for A?

Answers

1. a) $\{4\}$ b) $\{1, 2, 4, 6, 7, 9\}$
   c) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ d) $\{5, 6, 7\}$

2. a) true b) false c) true d) true
   e) true f) false g) true

3. a) true b) false c) true d) true
   e) false f) true g) true h) false

4. a) true b) false c) false d) true e) false

5. a)
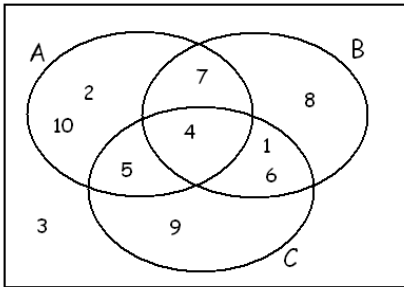
   b) i) $\{1, 4, 5, 6\}$ ii) $\{1, 2, 4, 5, 6, 7, 10\}$
   iii) $\{1, 4, 5, 6, 7, 9\}$ iv) $\{4\}$ v) $\{4, 5, 7\}$

6. 8

7. 2 or 8

8. 118 R 11

9. $1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84$

10. a) $120 = 2^3 \cdot 3 \cdot 5$ b) $156 = 2^2 \cdot 3 \cdot 13$
    c) $98 = 2 \cdot 7^2$ d) $2015 = 5 \cdot 13 \cdot 31$

11. a) $11\,060\,904$, 3106 b) $11\,060\,904$, 321
    c) $11\,060\,904$

12. 151

13. a) true b) true c) false d) true e) false

14. 60

15. $S \cap T = \{2, 3, 4, 5, 6, 7, 8, 9\}$ and
    $S \cup T = \mathbb{N}$ (set of all natural numbers)

16. $B \subseteq A$

17. $E \cap O = \varnothing$ and $E \cup O = \mathbb{Z}$

18. $\{n \in \mathbb{N} : n \text{ is divisible by } 10\} = \{10, 20, 30, 40, \ldots\}$

19. a) -9 b) -6 c) 8 d) 14 e) 22
    f) 3 g) -7 h) undefined i) -4 j) 4

20. a) 33 b) 6 c) -12 d) 6 e) 3
    f) 4 g) -30 h) 1 i) 13 j) 33

21. a) 4 b) 25 c) 10 d) 9 e) undefined

22. a) $\dfrac{7}{20}$ b) 70%
    c) $\dfrac{3}{5} = \dfrac{24}{40}$ and $\dfrac{5}{8} = \dfrac{25}{40}$ so $\dfrac{5}{8}$ is greater.

23. a) 11 b) -9 c) undefined d) -1

24. a) 3 b) undefined c) 14 d) 12 e) 4 f) -2

25. a) 12 b) 135 c) 31 d) 31 e) 1

26. -2 and 1

27. -1 and 3

28. 0, 3, and 4

29. $P = 74\,\text{ft}$ $A = 278\,\text{ft}^2$

30. $460\,\text{m}^2$

31. 8

32. a) $X = 2Y - 3$ b) $-x = \left(y + \dfrac{z}{2}\right) + 5$
    c) $ab = 3(a + b) - 14$ d) $m + 2n = \dfrac{m}{n} + 1$
    e) $3(x - y) = x(-y) - 1$ f) $P = 2D - 10$
    g) $2.5 + (x - 1)\,1.5$ h) x, $x + 1$, and $x + 2$
    i) $x(4x - 3)$
    j) Ann has $A - 30$ dollars and Beatrix has $B + 30$.

33. 4