

1. Suppose that $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 4, 9\}$, and $B = \{2, 4, 6, 7\}$. Find each of the following.

a) $A \cap B$ b) $A \cup B$ c) $P = \{x \in U : x > 4 \text{ or } x \leq 7\}$ d) $Q = \{x \in U : x > 4 \text{ and } x \leq 7\}$

2. Compute each of the following sets.

a) $(1, 5) \cup [2, 8]$ c) $(-\infty, 5) \cup (-2, \infty)$ e) $(3, 4) \cup [1, 7]$ g) $(-\infty, -3) \cup (-1, 12)$
 b) $(1, 5) \cap [2, 8]$ d) $(-\infty, 5) \cap (-2, \infty)$ f) $(3, 4) \cap [1, 7]$ h) $(-\infty, -3) \cap (-1, 12)$

3. Perform the division with remainder: $2017 \div 17$

4. Label each of the following statements as true or false.

- a) Every integer is a rational number.
 b) Every positive integer can be written as a product of primes.
 c) If a is divisible by 2 and b is divisible by 3, then the product ab is divisible by 6.
 d) If the product ab is divisible by 6, then a is divisible by 2 and b is divisible by 3 (or vica versa, i.e. a is divisible by 3 and b is divisible by 2.)
 e) If the product ab is divisible by 7, then a is divisible by 7 or b is divisible by 7.
 f) The sum of two consecutive integers is always an odd number.

5. List all factors of 84.

6. Which of the following numbers is a prime? 2007, 143, 151, 91

7. Find the prime factorization of 720.

8. Find the prime factorization for x if

a) $x = 12^{100}$ b) $x = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

Note that there is a shorter notation for the product above: $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 10!$

9. Suppose that $n = 2^{30} \cdot 3^{50} \cdot 5 \cdot 11^{100}$. What is the prime-factorization of n^3 ?

10. Compute the area of the right triangle determined by $A(-2, -1)$, $B(3, -1)$, and $C(-2, 7)$.

11. Re-write each of the following.

a) 60% as a reduced fraction c) $\frac{3}{8}$ with a denominator 32 e) $\frac{37}{5}$ as a mixed number
 b) $\frac{12}{25}$ as a percent d) $\frac{3}{8}$ with a numerator of 21 f) $\frac{37}{5}$ as a percent

12. Simplify each of the following.

a) $|11 - 3| - 5|$ c) $|11| - 3 - 5|$ e) $-3^2 - 12 \div 2 \cdot 3$ g) $|3 - |-7 + 2||$
 b) $|11 - |3 - 5||$ d) $12 - 2(5 - 3(-2))$ f) $\frac{18 - 5 + 3}{-2^2 - (-2)^2}$ h) $\frac{6 - 2(-3)}{-2^2 - (-1)}$

13. Simplify each of the following.

a) $-3^2 - 4(-5) + 24 \div 3 \cdot 2$ c) $\left(-\frac{2}{5}\right)^2 - \frac{1}{3} + \frac{1}{5}$ d) $2 - 5(8 - 3(2 - (-1)^3))$
 b) $\frac{3}{8} - \frac{1}{4} \cdot \frac{3}{5}$ e) $\frac{13 - 5 + 6}{-2^3 + 8}$

$$f) \frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{2} - \frac{1}{3}} \quad g) \frac{\frac{7}{5} - \left(-\frac{3}{4}\right)^2 + 1}{\frac{1}{10} - \frac{1}{20} + \frac{1}{40}} \quad h) \frac{3 - \frac{1}{5}}{2 + \frac{1}{3}} \quad i) \left(-\frac{1}{2}\right) - \left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^4$$

14. Evaluate the algebraic expression $\frac{-3x^2 - 2x + 1}{3x - 1}$ if

a) $x = 4$ b) $x = -3$ c) $x = \frac{1}{2}$ d) $x = \frac{1}{3}$ e) $x = -\frac{1}{3}$

15. Consider the equation $2x^3 - 10(x^2 - 2) + 4x = -x^2 + 5$. For each of the following numbers given, determine whether it is a solution of the equation or not.

a) $x = -2$ b) $x = -1$ c) $x = 3$ d) $x = \frac{5}{2}$

16. Add the algebraic expressions as indicated.

a) $(2a + 3b) + (-2a + 7b)$ b) $(3x - y + 2) + (-x + 6y - 2)$ c) $(3m - 4n) + (5m)$

17. Subtract the algebraic expressions as indicated.

a) $(2a + 3b) - (-2a + 7b)$ b) $(3x - y + 2) - (-x + 6y - 2)$ c) $(3m - 4n) - (5m)$

18. Simplify the algebraic expressions as indicated.

a) $-3(2a + 3b) + 5(-2a + 7b)$ d) $5 - 3(2x - y)$
 b) $2(3x - y + 2) - 6(-x + 6y - 2)$ e) $-5((3m - 2) - 4(m - 1) + 1)$
 c) $2(3x - 1) - 5(-x + 4)$ f) $-1 + 2(3x - 1) - (2x - 3) + 2(-2x + 1)$

19. Simplify each of the following.

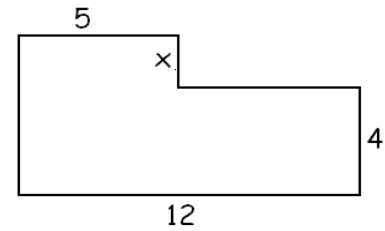
a) $x^3 \cdot x^8$ d) $(-x^8)^3$ g) $\frac{(-2ab^2)^3 (-3aba^2)^2}{(-2a^4b)^2}$
 b) $(-x)^3 (-x)^8$ e) $\frac{3x^2 (-2y)^3}{4xy}$ h) $\frac{(-a^3b^5)^2}{-a^5b^3a}$
 c) $(-x^3)^8$ f) $\frac{(2xy^2)^5 (-xy^2x^3)}{(-2x^3y^4)^3}$ i) $\sqrt{x^{36}}$

20. Solve each of the following equations. Make sure to check your solutions.

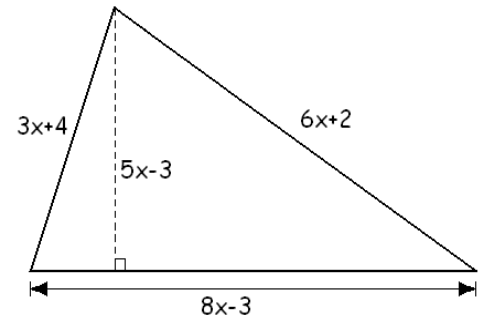
a) $-3x + 5 = 20$ d) $\frac{x}{3} - 8 = -2$ g) $\frac{3}{2}x - \frac{2}{3}\left(\frac{1}{4}x - 2\right) = -\frac{2}{3}$
 b) $-\frac{2}{3}x + \frac{3}{5} = \frac{11}{10}$ e) $5a - 3 = -3a + 21$ h) $x - (x - 2) - (-3x + 1) = x + 1$
 c) $\frac{x - 8}{3} = -2$ f) $5(x - 1) - 3(2x - 7) = 2x + 1$
 i) $3x - 4(2x - 8) = x - 6(x - 1)$ l) $5(y - 2) - 3(2y - 7) = 3y - 1 - 4(y - 3)$
 j) $3(y - 1) - 5(3y + 2) = -13(y + 1)$ m) $(3x - 1) - (5 - 2x) = 5(x - 3) - 2$
 k) $4(x - 3) - 2(x - 1) = x - 2(4 - x)$ n) $(2x - 3(4x + 5(-x + 2) - 3)) = 2(3(x - 5) + 1)$

21. Solve each of the following word problems.

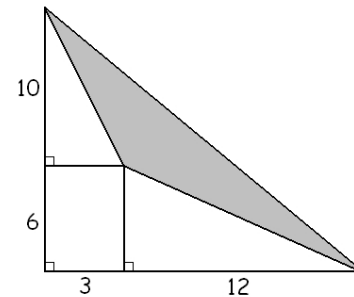
- a) One side of a rectangle is 4 ft shorter than three times the other side. Find the sides if the perimeter is 64 ft.
- b) A bank teller has 23 more five-dollar bills than ten-dollar bills. The total value of the money is \$610. How much of each denomination of bill does he have?
- c) Three times a number is one less than twice the difference of the number and three. Find this number.
- d) Amy's age is three less than five times her son's age. How old are they if the sum of their ages is 33?
- e) Consider the figure shown on the picture. Angles that look like right angles are right angles. Find the value of x if we know that the area of this object is 58 unit^2 .



- f) i) Find the value of x if the triangle shown on the picture has a perimeter 54 units.
ii) Given the value of x you found, compute the area of the triangle.
- g) There were a lot of coins in that jar, all quarters and dimes. The number of dimes was two less than five times the number of quarters. How many of each coins were there if all the coins in the jar were worth 8 dollars and 80 cents? (Hint: think in terms of cents)



22. What is the last digit of the number $2^{99} + 2^{100} + 2^{101} + 2^{102}$?
23. Suppose that 2^{100} is denoted by x . Express 2^{102} in terms of x .
24. Compute the area of the shaded region shown on the picture.



Answers

1. a) $\{4\}$ b) $\{1, 2, 4, 6, 7, 9\}$
 c) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ d) $\{5, 6, 7\}$
2. a) $(1, 8]$ b) $[2, 5)$ c) $(-\infty, \infty)$ d) $(-2, 5)$
 e) $[1, 7]$ f) $(3, 4)$
 g) $(-\infty, -3) \cup (-1, 12)$ cannot be simplified
 h) \emptyset
3. 118 R 11
4. a) true b) false (1 is the only exception)
 c) true d) false e) true f) true
5. 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84
6. 151
7. $720 = 2^4 \cdot 3^2 \cdot 5$
8. a) $2^{200} \cdot 3^{100}$ b) $2^8 \cdot 3^4 \cdot 5^2 \cdot 7$
9. $n^3 = 2^{90} \cdot 3^{150} \cdot 5^3 \cdot 11^{300}$
10. $A = 20 \text{ unit}^2$
11. a) $\frac{3}{5}$ b) 48% c) $\frac{12}{32}$ d) $\frac{21}{56}$ e) $7\frac{2}{5}$ f) 740%
12. a) 4 b) 9 c) 88 d) -10 e) -27
 f) -2 g) 2 h) -4
13. a) 27 b) $\frac{9}{40}$ c) $\frac{2}{75}$ d) 7 e) undefined f) 5
 g) $\frac{49}{2}$ h) $\frac{6}{5}$ i) $-\frac{11}{16}$
14. a) -5 b) 2 c) $-\frac{3}{2}$ d) undefined f) $-\frac{2}{3}$
15. a) no, $-44 \neq -2$ b) yes, $4 = 4$ c) yes, $-4 = -4$
 d) yes, $-\frac{5}{4} = -\frac{5}{4}$
16. a) $10b$ b) $2x + 5y$ c) $8m - 4n$
17. a) $4a - 4b$ b) $4x - 7y + 4$ c) $-2m - 4n$
18. a) $-16a + 26b$ b) $12x - 38y + 16$ c) $11x - 22$
 d) $-x + 3y$ e) $5m - 15$ f) 2
19. a) x^{11} b) $-x^{11}$ c) x^{24} d) $-x^{24}$ e) $-6xy^2$
 f) 4 g) $-18ab^6$ h) $-b^7$ i) x^{18}
20. a) -5 b) $-\frac{3}{4}$ c) 2 d) 18 e) 3
 f) 5 g) $-\frac{3}{2}$ h) 0 i) no solution
 j) 0 k) -2 l) all real numbers
 m) no solution n) 7
21. a) 9 ft and 23 ft b) 33 ten-dollar bills and 56 five-dollar bills
 c) -7 d) 6 and 27 e) 2 units f) g) 12 quarters, 58 dimes
 i) 3 units ii) 126 unit^2
22. 0
23. $4x$
24. $A = 51 \text{ unit}^2$