1. Let $U = \{1,2,3,4,5,6,7,8,9,10,11,12\}$, $A = \{2,5,6,7,9,11\}$, $B = \{1,5,6,8,9,10,12\}$, and $C = \{2,4,5,7,9,12\}$
   a) Draw a Venn diagramm depicting these sets.
   b) Find each of the following.
   i) $A \cap (B \cup C)$ ii) $(A \cap B) \cup (A \cap C)$ iii) $A \cup (B \cap C)$ iv) $(A \cap B) \cap C$

2. Simplify each of the following.
   a) $(-\infty, 5) \cap [-2, \infty)$ c) $[3,10] \cup (7,12)$ e) $(-\infty, 7] \cap (9, \infty)$ g) $(2, \infty) \cup [5, \infty]$
   b) $(-\infty, 5) \cup [-2, \infty)$ d) $[3,10] \cap (7,12)$ f) $(-\infty, 7] \cup (9, \infty)$ h) $(2, \infty) \cap [5, \infty]$

3. Label each of the following statements as true or false.
   a) If n is a positive integer with $n \geq 2$, then the prime factorization of n^2 will have only even exponents.
   b) If n is a positive integer with $n \geq 2$, then all exponents in the prime factorization of n^3 are divisible by 3.
   c) For any pairs of numbers a and b, if x is the greatest common factor of a and b, and y is the least common multiple of a and b, then x is a factor of y.
   d) There is no prime number that is divisible by 5.

4. a) List all factors of 150.
   b) List all prime numbers between 50 and 70.
   c) Which of the following numbers is not a prime? $29, 79, 89, 109, 119$

5. a) Find the prime factorization of each of the following. i) 180 ii) 1575 iii) 80^{100}
   b) Compute the greatest common factor and least common multiple of 180 and 1575.
   c*) The greatest common factor of 48 and x is 6. The least common multiple of 48 and x is 720. What values are possible for x?
   d*) Is it possible for two numbers to have their greatest common factor be equal to their least common multiple?

6. Perform the operations as indicated.
   a) $\dfrac{\frac{3}{4} + \frac{8}{15} \div \left(-\frac{2}{5}\right)}{2\frac{1}{3}}$
   b) $4 - \left|\left|2^2 - (-2)^4\right| - 5^2\right|$
   c) $\dfrac{2\left((-2)^2 - (3^2 - 3)\right)}{-2^2} - \dfrac{(-3^2 + 2)\,3}{3 - (-4)}$
   d) $\dfrac{\left||-2^2 + 2| - 5\right| - 1}{-2^2 - \left((-3)^3 + 5^2\right)}$
   e) $\sqrt{(-1)^4 - 2 \cdot 3^2 \div (-2) \cdot 6 + (-3)^2}$
   f) $\dfrac{3}{10} + \left(3 + \dfrac{3}{5}\right) \div \left(1 + \dfrac{1}{3}\right)$

7. Compute each of the following.
   a) $2^{-1} - 3^{-2}$ b) $\dfrac{5^{-1} + 2^{-2}}{5^{-1} - 2^{-2}}$ c) $\dfrac{2}{3} - \dfrac{4}{5} \div \left(-\dfrac{3}{10}\right)$ d) $\left(\dfrac{-3a^{-2}b}{(-2ab^3)^5}\right)^0$

8. Evaluate the expression $\dfrac{11a - 2a^2 - 15}{2a - 5}$ if
   a) $a = 0$ b) $a = 2$ c) $a = \frac{1}{3}$ d) $a = 2\frac{1}{2}$ e) $a = -\frac{1}{2}$ f) $a = 3$ g) $a = 1\frac{1}{2}$

9. Evaluate the expression $\dfrac{-x^2+4}{x-2}+x$ if

   a) $x=0$ b) $x=-2$ c) $x=10$ d) $x=-7$ e) $x=\dfrac{3}{5}$ f) $x=-\dfrac{2}{3}$

10. Simplify each of the following.

    a) $2a^{-3}\left(-2ab^{-2}\right)^3 a^{-2}b^7$

    b) $\dfrac{\left(-2xy^{-2}\right)^{-3} y^{-4}\left(-4x^3\right)^2}{-2x^{-2}y^2}$

    c) $\left(-2xy^{-3}\right)^2 x\,(-y)^5\,x^{-2}$

    d) $\dfrac{\left(3ba^{-4}b^{-2}\right)^{-2}\left(-a^3b^{-1}\right)^{-2}}{(-2)^2\left(3a^{-3}b^2\right)^{-2}}$

    e) $\dfrac{\left(-2x^{-2}y^3\right)^{-4} x\,(-y)^{-5}\left(8x^{-2}y^5\right)^2}{(2x)^2\,y^{-9}\left(2x^{-1}y\right)^4}$

    f) $\left(\dfrac{2a^{-2}b^3}{-3ab^{-5}}\right)^{-3}\left(\dfrac{-2ab^{-3}}{3a^2b^{-8}}\right)^4$

    g) $(5x+2)(x-3)$

    h) $(5a-1)^2$

    i) $(a-b)\left(a^3+a^2b+ab^2+b^3\right)$

    j) $\left(3a^5-1\right)\left(3a^5+1\right)$

    k) $(2x-3)^3$

11. Suppose that $A=12\,500\,000\,000$ and $B=0.000\,000\,064$. Write each of the following in scientific notation.

    a) A b) B c) AB d) A^2B e) $\dfrac{1}{B}$ f) $\sqrt{10B}$

12. Factor out the greatest common factor in each of the following.

    a) $10a^2b^3-5ab^4c+5ab^3$

    b) $2x^5-12x^4+6x^3$

    c) $6a^2bc-2ab^3c+12a^4b^3c^2$

    d) $4p^2q-6pq+9pq^2$

    e) $x^4-5x^3+x^2$

    f) $3x(a-3)-6x^2(a-3)+12(a-3)$

    g) $8x(5x-2)-(5x-2)$

13. Completely factor each of the following.

    a) $2x^2-98$

    b) $-20a^2b+5a^2bc^2$

    c) $4x^2+36$

    d) $2x^3-x^4$

    e) $6x^4-6x^2$

    f) $-200+2m^6$

    g) $5p^5-45p^3$

    h) $10x^2(x-5)-20x(x-5)$

14. Solve each of the following equations. Make sure to check your solutions.

    a) $\dfrac{2}{3}x+\dfrac{3}{5}=-\dfrac{1}{15}$

    b) $2x-3(x-1)=7-x$

    c) $x^2+x=0$

    d) $\dfrac{2x+1}{5}-\dfrac{5-x}{2}=x-1$

    e) $-3(2x-1)=2(x+1)-(x-1)$

    f) $36m^4=4m^2$

    g) $36m^4=4m^3$

    h) $\dfrac{x-1}{2}-3+x=\dfrac{3x-7}{2}$

    i) $2-(3-x)(2x+5)=(x-1)(2x-1)$

    j) $(2x-1)^2+x(x-6)=(x+1)^2$

    k) $\dfrac{\dfrac{3x-1}{5}-1}{2}=3$

15. Solve each of the following inequalities. Graph the solution set.

    a) $\dfrac{3-4x}{3}-\dfrac{2x-3}{7}\le -x+7$ b) $3(2x-3)-(5x+4)<-14$

16. Graph each of the following.

    a) $y = -\frac{1}{2}x + 3$ b) $3x + 2y = -12$ c) $x + y = 4$

17. Consider the equations $2x + y = -1$ and $x - y = -5$.

    a) Graph these lines in the same coordinate system. Use your graph to find the coordinates where the points intersect.

    b) Use algebraic methods to check your answer for part a).

18. A TV is priced at $\$500$. How much would it cost if it went on a 18% sale?

19. The difference between two numbers is 7, their sum is 37. Find these numbers.

20. A total of $\$20\,000$ is to be invested in bonds and stocks. If the amount invested in bonds is to be $\$4500$ more than the amount invested in stocks, how much money is invested in each category?

21. The tickets for the field trip were purchased yesterday for both students and instructors. Children tickets cost $\$5$, adult tickets cost $\$12$. The number of children ticket purchased was three less than four times the number of adults tickets purchased. How many of each were purchased if all of the tickets cost a total of $\$209$ dollars?

22. Ann and Betty are roommates. The monthly rent is $\$950$. The amount paid by Ann is $\$310$ less than twice the amount paid by Betty. How much do they each pay for rent?

23. One side of a rectangle is $4\,\text{ft}$ shorter than three times the other side. Find the sides if the perimeter is $64\,\text{ft}$.

24. If we raise a number to the third power, we get nine times the original number. Find all numbers with this property.

25. Ann took four exams. Her scores on the first three exams were 68, 72, and 81. How many points did she earn on the fourth exam if her average is 76?

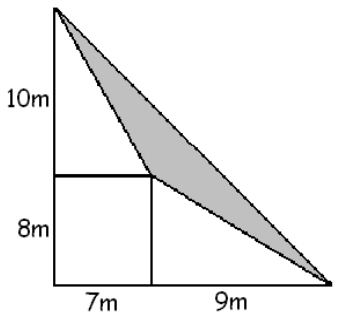
26. A number is four less than the sum of -8 and twice the opposite of the number. Find this number.

27. Consider the answers for problem #9. What do you notice? Can you explain why this is happening?

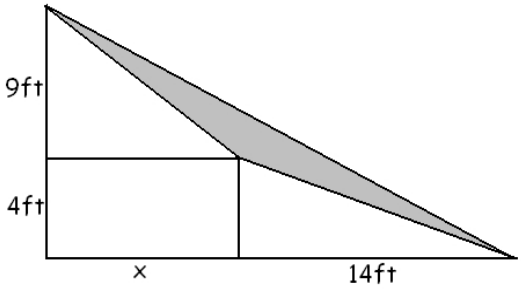
28. Find the area of the triangle determined by the points $A\,(-5, -2)$, $B\,(7, -2)$, and $C\,(3, 6)$.

29. a) Compute the area of the shaded region shown on the picture. Angles that look like right angles are right angles.

    b) Find the value of x if we know that the shaded region shown on the picture has area $43\,\text{ft}^2$. Angles that look like right angles are right angles.

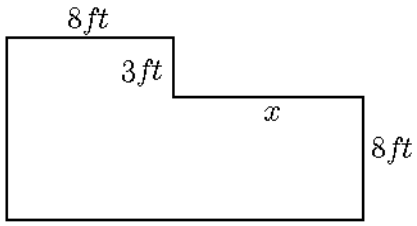


a)

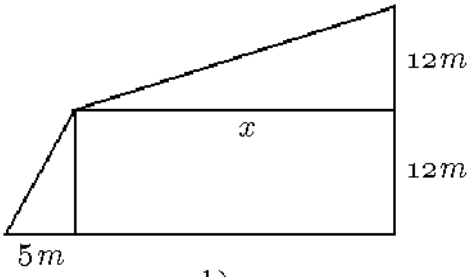


b)

30. a) Find the value of x if the area of the object shown on the picture below is $168\,\text{ft}^2$.
    b) Find the value of x if the area of the object shown on the picture below is $534\,\text{m}^2$.



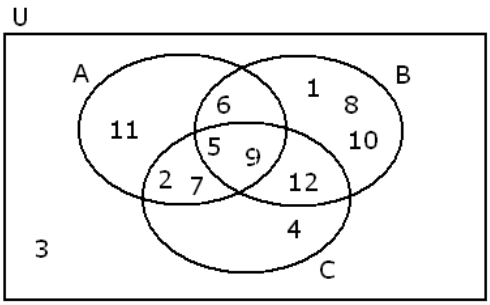
a)



b)

Answers

1. a) see below b) i) $\{2,5,6,7,9\}$ ii) $\{2,5,6,7,9\}$ iii) $\{2,5,6,7,9,11,12\}$ iv) $\{5,9\}$



2. a) $[-2,5)$ b) $(-\infty,\infty)$ c) $[3,12)$ d) $(7,10]$ e) $\varnothing$
   f) cannot be simplified g) $(2,\infty)$ h) $[5,\infty]$
3. a) true b) true c) true d) false
4. a) $1,2,3,5,6,10,15,25,30,50,75,150$ b) $53,59,61,67$
   c) $119 = 7\cdot 17$

5. a) i) $180 = 2^2\cdot 3^2\cdot 5$ ii) $1575 = 3^2\cdot 5^2\cdot 7$ iii) $80^{100} = 2^{400}\cdot 5^{100}$ b) 45 and 6300
   c) Only 90 is possible d) Yes, but only if the two numbers are equal.

6. a) $-\frac{1}{4}$ b) -9 c) 4 d) -1 e) 8 f) 3 7. a) $\frac{7}{18}$ b) -9 c) $\frac{10}{3}$ d) 1

8. a) 3 b) 1 c) $\frac{8}{3}$ d) undefined e) $\frac{7}{2}$ f) 0 g) $\frac{3}{2}$ 9. a) -2 b) -2 c) -2 d) -2 e) -2 f) -2

10. a) $-\frac{16b}{a^2}$ b) x^5 c) $-\frac{4x}{y}$ d) $\frac{b^8}{4a^4}$ e) $-\frac{x^7}{16y^2}$ f) $-\frac{2a^5}{3b^4}$ g) $5x^2-13x-6$ h) $25a^2-10a+1$
    i) a^4-b^4 j) $9a^{10}-1$ k) $8x^3-36x^2+54x-27$

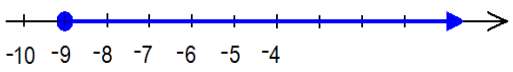
11. a) $1.25\cdot 10^{10}$ b) $6.4\cdot 10^{-8}$ c) $8\cdot 10^2$ d) $1\cdot 10^{13}$ e) $1.5625\cdot 10^7$ f) $8\cdot 10^{-4}$

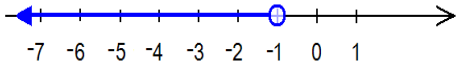
12. a) $5ab^3(2a-bc+1)$ b) $2x^3\left(x^2-6x+3\right)$ c) $2abc\left(3a-b^2+6a^3b^2c\right)$ d) $pq(4p+9q-6)$
    e) $x^2\left(x^2-5x+1\right)$ f) $3(a-3)\left(x-2x^2+4\right)$ g) $(5x-2)(8x-1)$

13. a) $2(x+7)(x-7)$ b) $5a^2b(c+2)(c-2)$ c) $4\left(x^2+9\right)$ d) $x^3(2-x)$ e) $6x^2(x-1)(x+1)$
    f) $2\left(m^3+10\right)\left(m^3-10\right)$ g) $5p^3(p+3)(p-3)$ h) $10x(x-2)(x-5)$

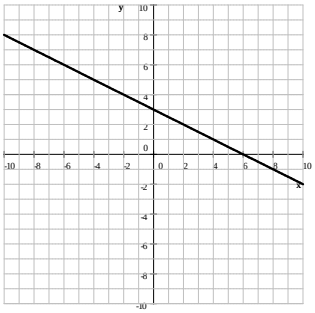
14. a) -1 b) no solution c) $0,-1$ d) -13 e) 0 f) $-\frac{1}{3},0,\frac{1}{3}$ g) $0,\frac{1}{9}$
    h) identity, all numbers are solution i) 7 j) $0,3$ k) 12

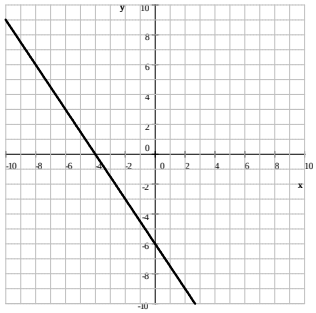
15. a) $-9 \le x$ b) $x < -1$

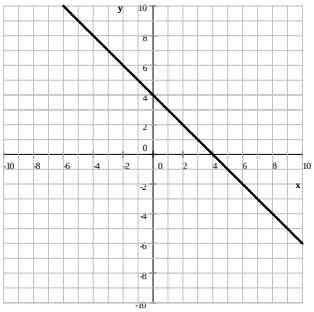




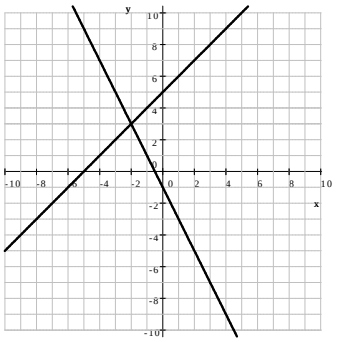
16. a) $y = -\frac{1}{2}x + 3$ b) $3x + 2y = -12$ c) $x + y = 4$







17. a) $(-2, 3)$ b) Is the point $(-2, 3)$ on the line $2x + y = -1$?



$$\begin{aligned} \text{LHS} &= 2x + y = 2(-2) + 3 = -4 + 3 = -1 \quad \text{and} \quad \text{RHS} = -1 \\ \text{LHS} &= \text{RHS} \checkmark \end{aligned}$$

Thus $(-2, 3)$ is on the line $2x + y = -1$.

Is the point $(-2, 3)$ on the line $x - y = -5$?

$$\begin{aligned} \text{LHS} &= x - y = -2 - 3 = -5 \quad \text{and} \quad \text{RHS} = -5 \\ \text{RHS} &= \text{LHS} \checkmark \end{aligned}$$

Thus $(-2, 3)$ is on the line $x - y = -5$. Since it is on both line, it is the intersection point.

18. \$410 19. 15 and 22 20. \$7750 in stocks and \$12 250 in bonds 21. 7 adult tickets and 25 children tickets

22. \$420 and \$530 23. 9 ft and 23 ft 24. $-3, 0, 3$ 25. 83 26. -4

27. We keep getting the same number, -2. We can factor the numerator by factoring out first -1 and then factor the difference of squares.

$$-x^2 + 4 = -1\left(x^2 - 4\right) = -1(x + 2)(x - 2)$$

Now we can re-write the expression

$$\frac{-x^2 + 4}{x - 2} + x = \frac{-1(x + 2)(x - 2)}{x - 2} + x$$

Notice that the fraction now has the same factor, $x - 2$ in its numerator and denominator Thus we can cancel it out. Then

$$\frac{-1(x + 2)(x - 2)}{x - 2} + x = \frac{-1(x + 2)}{1} + x = -(x + 2) + x = -x - 2 + x = -2$$

Thus, no matter what the value of x is (with the exception of $x = 2$), the expression will always have a value of -2.

28. 48 unit^2 29. a) $A = 17\,\text{m}^2$ b) 10 ft 30. a) 10 ft b) 28 m

Last revised: November 7, 2018