

- Suppose that  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ . Find each of the following sets.
  - $A = \{x \in U : x > 6 \text{ or } x \text{ is odd}\}$
  - $B = \{x \in U : x > 6 \text{ and } x \text{ is odd}\}$
  - $C = \{x \in U : x \text{ is divisible by 2 or by 3}\}$
  - $D = \{x \in U : x \text{ is divisible by 2 and by 3}\}$
- Suppose that  $A = \{1, 2, 4, 8, 9\}$ ,  $B = \{1, 4, 5, 7, 9, 10\}$ , and  $C = \{2, 3, 4, 7, 9\}$ . Find each of the following sets.
  - $A \cap B$
  - $A \cap C$
  - $B \cap C$
  - $A \cap (B \cup C)$
  - $(A \cap B) \cup C$
  - $A \cup (B \cap C)$
- Assume that  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ . Draw a Venn diagram depicting  $A$ ,  $B$ , and  $C$  in the previous problem.
- Perform the division with remainder.  $2018 \div 99$ .
- List all factors of 54.
  - What is the greatest factor of 54?
  - What is the greatest prime factor of 54?
- Find the prime factorization of 528.
- Find the prime factorization of  $60^{100}$ .
- Use prime factorization to find the least common multiple and greatest common factor of 270 and 600.
- Label each of the following as true or false.
  - If the integer  $n$  is divisible by 6 and the integer  $m$  is divisible by 9, then  $nm$  is divisible by 54.
  - If an integer  $n$  is divisible by 6 and by 9, then it is also is divisible by 54.
  - If a right triangle's sides are integers, then at least one of them must be even.
  - If  $n$  is an integer such that  $n^2$  is divisible by 20, then  $n$  is divisible by 20.

Let  $T$  be the set of all triangles,  $R$  the set of all right triangles, and  $S$  the set of all isosceles triangles. Then

  - $R \cup S = T$
  - $R \subseteq T$
  - $R \cap S = \emptyset$
- Compute the exact value of  $-2x^2 + 3x - 8$  if
  - $x = 3$
  - $x = -2$
  - $x = \frac{2}{3}$
  - $x = -\frac{5}{2}$
- Suppose that  $A = 160\,000\,000\,000$  and  $B = 0.000\,000\,012\,5$ . Write each of the following in scientific notation.
  - $A$
  - $B$
  - $A^2$
  - $\sqrt{A}$
  - $AB$
  - $A^2B^3$
- Simplify each of the given expressions. Assume that all variables represent positive numbers. Express your answer using only positive integer exponents.
  - $\frac{(-2p^{-1}q^2)^3(2pq^4)^{-1}}{-4p(-p^4q^{-1})^{-2}}$
  - $\left(\frac{2x^2}{y^{-2}z^{-1}}\right)^2 \left(\frac{x^{-2}(-z)^2}{4y}\right)$
  - $\frac{(-4x^4y^4)xy^{-1}}{(-2x)^2y^2}$
- Expand each of the following.
  - $(3n + m)(-2a + 5b)$
  - $(3x^3 + 1)(-2x^3 + 5)$
  - $(3x - 2)^2$
  - $(3x - 2)(3x + 2)$
  - $(a + b)(a^2 - ab + b^2)$
  - $(a - b)(a^2 + ab + b^2)$
- Simplify each of the following. Show all steps.
  - $\frac{-5^2 - 60 \div (-4) \cdot 3}{|-21 \div (-7)| - (-1)^6}$
  - $\frac{\frac{1}{5} + \left(-\frac{1}{2}\right)^2 \cdot \left(3\frac{2}{5}\right)}{\frac{1}{5} + \frac{1}{2}} - 2^{-1}$
  - $2^{-1} - (3 - 2^{-1})^{-1}$

15. Simplify each of the following.

a)  $\frac{x^3 - x}{x + 1}$       b)  $\frac{x^2 - 10x + 25}{x^2 - 5x + 4} \left( \frac{x^2 - 2x - 8}{x^2 - 6x + 5} \div \frac{x - 5}{x - 1} \right)$

16. Completely factor each of the following expressions.

a)  $3a^4x - 48x$       e)  $x^2 - 3x + 2$       i)  $(a + 2b - 5)^2 - (3a - 2b + 5)^2$   
 b)  $21x^2 - 18ax^2 - 3a^2x^2$       f)  $m^2(3x - 1) - 3x + 1$       j)  $30xa^2y^2 - 27xay^2 + 6xy^2$   
 c)  $15ax - 5ay - 3bx + by$       g)  $2x^7 + x^5 - 6x^2 - 3$   
 d)  $x^2 - 3x - 54$       h)  $6x^2 - 19x + 15$

17. Solve each of the following equations. Make sure to check your solution(s).

a)  $\frac{3x + 17}{2} = x - 1 + \frac{x + 19}{2}$       d)  $\frac{2}{3}(x - 7) = \frac{4}{5}(x + 1)$       g)  $3(2x - 1) - 2(x + 5) = 4(x - 3)$   
 b)  $(2x - 1)^2 = 3x^2 + 1$       e)  $x^5 = x^4$       h)  $4x(x + 5) = (x + 2)^2 + 8$   
 c)  $3x^3 + 10x^2 = 8x$       f)  $x^5 = x^3$       i)  $x^2 + 7 = 3$   
 k)  $4 - (2x - 5)(x + 1) = 18 - 2x^2$       m)  $\frac{3}{4}(x + 1) - \frac{2}{5}(x - 1) = \frac{1}{6}(x - 3)$       j)  $(3x - 5) = 7(x - 15)$   
 l)  $4x^2 + 6x - 1 = (x + 2)(x - 2)$       n)  $7x^2 + (x + 3)(2x - 1) = (3x + 1)^2$

18. Solve each of the following system of linear equations.

a)  $\begin{cases} 2x - y = 11 \\ 3x + 2y = 6 \end{cases}$       b)  $\begin{cases} x - y = -3 \\ (y - 1)^2 + 2x = y^2 \end{cases}$       c)  $\begin{cases} \frac{1}{2}x - \frac{1}{5}y = -5 \\ \frac{1}{3}x + \frac{1}{2}y = 3 \end{cases}$

19. Solve each of the following the inequalities.

a)  $\frac{1 - 3x}{4} - \frac{2x + 1}{3} \geq x - 17$       b)  $-2(3x - 1) - (x - 3) < 12$       c)  $(x - 2)^2 > x^2$

20. a) Compute the perimeter and area of the parallelogram determined by the points  $A(-5, -3)$ ,  $B(2, -3)$ ,  $C(-2, 1)$  and  $C(5, 1)$ .

21. Compute the distance between the points  $A(3, 8)$  and  $B(-2, 4)$ .

22. Graph  $2x - 3y = -9$

23. The hypotenuse of a right triangle is 74 ft. The difference between the other two sides is 46 ft. Find the sides of the triangle.

24. There is an animal farm where chickens and cows live. All together, there are 53 heads and 174 legs. How many chickens, how many cows?

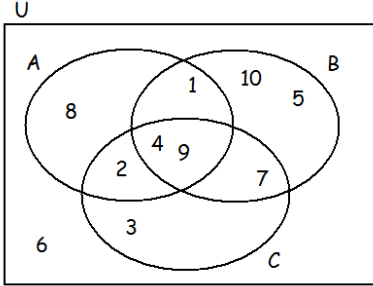
25. The area of a rectangle is  $1260 \text{ m}^2$ . Find the dimensions of the rectangle if we know that one side is 48 m longer than three times the other side.

26. The measure of one angle in a rectangle is one degree less than the greatest angle in the triangle, and two degrees more than twice the smallest angle. Find the angles in this triangle.

27. Suppose that  $a$ ,  $b$ , and  $c$  are sides of a right triangle. Side  $a$  is 3 units longer than side  $b$ , and side  $c$  is 3 units longer than side  $a$ . Find the sides of this triangle.

28. We invested \$10 000 into two bank accounts. One account earns 14% per year, the other account earns 8% per year. How much did we invest into each account if the combined interest from the two accounts is \$1238 after the first year?
29. We have 150 coins in a jar, all dimes and quarters. How many of each do we have if the value of all coins is \$27.30?
30. Ann invested \$20,000 in two accounts. She took a 4% loss on one of the accounts and made a 12% profit on the other investment, but ended up breaking even. How much money did she lose in the first investment?

## Answers

1. a)  $\{1, 3, 5, 7, 8, 9, 10\}$  b)  $\{7, 9\}$   
c)  $\{2, 3, 4, 6, 8, 9\}$  d)  $\{6\}$
2. a)  $\{1, 4, 9\}$  b)  $\{2, 4, 9\}$  c)  $\{4, 7, 9\}$   
d)  $\{1, 2, 4, 9\}$  e)  $\{1, 2, 3, 4, 7, 9\}$   
f)  $\{1, 2, 4, 7, 8, 9\}$
3. 
4. 20 R 38
5. a) 1, 2, 3, 6, 9, 18, 27, 54 b) 54 c) 3
6.  $528 = 2^4 \cdot 3 \cdot 11$  7.  $60^{100} = 2^{200} \cdot 3^{100} \cdot 5^{100}$
8.  $\gcd(270, 600) = 30$   $\text{lcm}(270, 600) = 5400$
9. a) true b) false c) true d) false  
e) false f) true g) false
10. a) -17 b) -22 c)  $-\frac{62}{9}$  d) -28
11. a)  $1.6 \cdot 10^{11}$  b)  $1.25 \cdot 10^{-8}$  c)  $2.56 \cdot 10^{22}$   
d)  $4 \cdot 10^5$  e)  $2 \cdot 10^3$  f)  $5 \cdot 10^{-2}$
12. a)  $p^3$  b)  $x^2y^3z^4$  c)  $-x^3y$
13. a)  $5bm - 6an - 2am + 15bn$  b)  $-6x^6 + 13x^3 + 5$   
c)  $9x^2 - 12x + 4$  d)  $9x^2 - 4$  e)  $a^3 + b^3$   
f)  $a^3 - b^3$  14. a) 10 b) 1 c)  $\frac{1}{10}$
15. a)  $x(x - 1)$  b)  $\frac{x + 2}{x - 1}$
16. a)  $3x(a^2 + 4)(a + 2)(a - 2)$   
b)  $-3x^2(a + 7)(a - 1)$  c)  $(3x - y)(5a - b)$   
d)  $(x + 6)(x - 9)$  e)  $(x - 1)(x - 2)$   
f)  $(3x - 1)(m + 1)(m - 1)$   
g)  $(x^5 - 3)(2x^2 + 1)$  h)  $(2x - 3)(3x - 5)$   
i)  $-8a(a - 2b + 5)$  j)  $3xy^2(2a - 1)(5a - 2)$
17. a) all real numbers b) 0, 4 c)  $-4, 0, \frac{2}{3}$   
d) -41 e) 0, 1 f) -1, 0, 1 g) no solution  
h)  $-6, \frac{2}{3}$  i) no solution j) 25 k) 3 l) -1  
m) -9 n) -4
18. a) (4, -3) b) no solution c) (-6, 10)
19. a)  $(-\infty, 7]$  b)  $(-1, \infty)$  c)  $(-\infty, 1)$
20.  $P = 24$  unit  $A = 28$  unit<sup>2</sup>
21. 13 units
22. see on right
23. 24 ft and 70 ft
24. 19 chickens and 34 cows
25. 14 m by 90 m
26.  $35^\circ, 72^\circ$  and  $73^\circ$
27. 9, 12, and 15 units
28. \$7300 at 14% and \$2700 at 8%
29. 82 quarters and 68 dimes 30. \$600

