

This problem set is not homework. Students can use this problem set as extra practice or study guide for quizzes.

1. Label each of the following statements as true or false.

- |                                    |  |   |
|------------------------------------|--|---|
| a) $\{1, 2\} \subseteq \mathbb{N}$ | d) $\mathbb{N} \subseteq \mathbb{Z}$         | g) $-3 \in \mathbb{N}$ and $7 \in \mathbb{Z}$   |
| b) $4 < 4$                         | e) $-5 \in \mathbb{Z}$                       | h) For all sets $S$ , $\emptyset \subseteq S$ . |
| c) $5 \geq 5$                      | f) $-3 \in \mathbb{N}$ or $7 \in \mathbb{Z}$ | i) $\mathbb{Z} \subseteq \mathbb{N}$            |

2. Suppose that  $A = \{1, 2, 3, 4, 5\}$ . Label each of the following statements as true or false.

- |                    |                     |                                 |                               |
|--------------------|---------------------|---------------------------------|-------------------------------|
| a) $1 \in A$       | c) $7 \notin A$     | e) $\{1, 3, 5, 6\} \subseteq A$ | g) $A \subseteq A$            |
| b) $1 \subseteq A$ | d) $\{1, 2\} \in A$ | f) $A \subseteq \mathbb{N}$     | h) $\{3, 4\} \not\subseteq A$ |

3. Suppose  $A = \{2, 3, 4, 5, 7, 8\}$ , and  $B = \{1, 2, 4, 6, 8, 10\}$ . Draw a Venn-diagram depicting  $A$  and  $B$ .

4. Suppose that  $U = \{0, 1, 2, 3, \dots, 19, 20\}$ . Find each of the following sets.

- |  |   |
|--|---|
| a) $A = \{x \in U : x \text{ is divisible by } 3\}$                    | f) $F = \{x \in U : x < 4 \text{ or } x < 8\}$  |
| b) $B = \{x \in U : x \text{ is divisible by } 5 \text{ or } x < 8\}$  | g) $G = \{x \in U : x < 4 \text{ and } x < 8\}$   |
| c) $C = \{x \in U : x \text{ is divisible by } 5 \text{ and } x < 8\}$ | h) $H = \{x \in U : x \text{ is divisible by } 4\}$   |
| d) $D = \{x \in U : x < 12 \text{ or } x \geq 7\}$                     | i) $I = \{x \in U : x \text{ is divisible by } 3 \text{ or } x \text{ is divisible by } 4\}$  |
| e) $E = \{x \in U : x < 12 \text{ and } x \geq 7\}$                    | j) $J = \{x \in U : x \text{ is divisible by } 3 \text{ and } x \text{ is divisible by } 4\}$ |

5. Recall the following definitions. A **rectangle** is a four-sided polygon with four right angles. A **square** is a rectangle with four equal sides. Let  $R$  be the set of all rectangles and  $S$  the set of all squares.

- a) Label each of the following statements as true or false.
- |                                  |                      |
|----------------------------------|----------------------|
| i) Every square is a rectangle.  | iii) $R \subseteq S$ |
| ii) Every rectangle is a square. | iv) $S \subseteq R$  |
- b) Describe  $x$  if we know that  $x \in R$  and  $x \notin S$ .

6. Find each of the following sets and if possible, present them by listing their elements.

- |   |   |
|---|---|
| a) $A = \{a \in \mathbb{N} \mid a < 6\}$                    | c) $C = \{c \in \mathbb{N} \mid c < 7 \text{ or } c > 3\}$                  |
| b) $B = \{b \in \mathbb{N} \mid b < 7 \text{ and } b > 3\}$ | d) $D = \{x \in \mathbb{N} \mid x \leq 10 \text{ and } x \text{ is even}\}$ |

7. Evaluate each of the given numerical expressions.

- |               |             |           |                |                   |                    |
|---------------|-------------|-----------|----------------|-------------------|--------------------|
| a) $-2 - 3$   | e) $(-2)^2$ | i) $-2^2$ | m) $3^2 - 7^2$ | q) $10 - 3(-8)$   | t) $(10 - 3)(-8)$  |
| b) $-2(-3)$   | f) $(-2)^3$ | j) $-2^3$ | n) $(3 - 7)^2$ | r) $10 - (3 - 8)$ | u) $10(- (3 - 8))$ |
| c) $-(2 - 3)$ | g) $(-2)^4$ | k) $-2^4$ | o) $3^2 + 7^2$ | s) $10(-3 - 8)$   |                    |
| d) $(-2) - 3$ | h) $(-2)^5$ | l) $-2^5$ | p) $(3 + 7)^2$ |                   |                    |

8. Evaluate each of the given numerical expressions.

- |             |                |                |                |                   |                   |
|-------------|----------------|----------------|----------------|-------------------|-------------------|
| a) $(-2)^2$ | e) $5(-2)^2$   | i) $2^2 + 5^2$ | m) $(2 - 5)^2$ | q) $(2^2 - 5)^2$  | u) $(2 - (-5)^2)$ |
| b) $-2^2$   | f) $5 - 2^2$   | j) $(2 + 5)^2$ | n) $(2 - 5^2)$ | r) $2^2 - (-5)^2$ | v) $2^2(-(-5^2))$ |
| c) $(-2^2)$ | g) $5(-2^2)$   | k) $(2 + 5^2)$ | o) $2^2(-5)^2$ | s) $2^2 - (-5^2)$ | w) $2^2(-(-5)^2)$ |
| d) $-(2)^2$ | h) $5 - (2)^2$ | l) $2^2 - 5^2$ | p) $2^2(-5^2)$ | t) $(2 - (-5))^2$ | x) $2^2(-(-5))^2$ |

9. Simplify each of the following expressions by applying the order of operations agreement. **Show all steps. Perform only one operation in each step.**

a)  $7 \cdot 3^2 - (3 - 2^2 \cdot 5 - 1) \div 2$

f)  $\left(\left(\left(8 - 5\right)^2 - 7\right)^2 - 2\right)^2 - 1$

k)  $-2^2 - 3\left(5 - (-2)^2\right) - (-1)^3$

b)  $\frac{5 - 1 + 2}{-1^2 + (-1)^2}$

g)  $\frac{4^2 + 5^2 - 6 \div 2 \cdot 3}{4^2 - 8 \cdot 2}$

l)  $-2 - 5(-3^2 - 2(-7))$

c)  $\frac{(-2)^3 - 5(-3) - (-1)^4 + (-3)^2}{-2^2 - (-1)}$

h)  $3 + 2(5 + 3(15 - 2^3) - 2^2 - 1)$

m)  $|-10 - 7| - |1 - 4|$

d)  $|3 - 8| - (|3| - |8|)$

i)  $4(3(2(2^2 - 1) - 1) - 1) + 5$

n)  $|-10 - 7 - |1 - 4||$

o)  $|-10 - 7||1 - 4||$

e)  $2^3 - 2(5 - (-3)^2)^2$

j)  $-3^2 - (-24) \div (5 - (-1)^3) \cdot 2$

p)  $|-10 - |7 - 1 - 4||$

q)  $|-10||-7 - 1 - 4||$

10. Perform the indicated operations. Show all steps.

a)  $8 - 2(7 - 3^2 + 1) + 5$

b)  $\left((3^2 - 13)^2 - 11\right)^2 - 1^3$

c)  $(-4^2 - 2(-5^2 - 12 \div (-3) \cdot 2^2))^2$

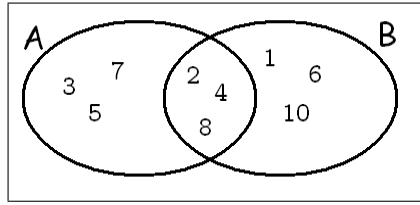
11. Compute each of the following.

a)  $-100 + (-99) + (-98) + \dots + 98 + 99 + 100 + 101 + 102$

b)  $-100(-99)(-98)\dots \cdot 98 \cdot 99 \cdot 100 \cdot 101 \cdot 102$

## Answers

1. a) true b) false c) true d) true e) true f) true g) false h) true i) false  
 2. a) true b) false c) true d) false e) false f) true g) true h) false



4. a)  $\{0, 3, 6, 9, 12, 15, 18\}$  b)  $\{0, 1, 2, 3, 4, 5, 6, 7, 10, 15, 20\}$  c)  $\{0, 5\}$  d)  $U$  or  $\{0, 1, 2, 3, \dots, 19, 20\}$   
 e)  $\{7, 8, 9, 10, 11\}$  f)  $\{0, 1, 2, 3, 4, 5, 6, 7\}$  g)  $\{0, 1, 2, 3\}$  h)  $\{0, 4, 8, 12, 16, 20\}$   
 i)  $\{0, 3, 4, 6, 8, 9, 12, 15, 16, 18, 20\}$  j)  $\{0, 12\}$
5. a) i) true ii) false iii) false iv) true  
 b)  $x$  is a rectangle that is NOT a square, i.e. a rectangle that has two sides with different lengths.
6. a)  $\{1, 2, 3, 4, 5\}$  b)  $\{4, 5, 6\}$  c)  $\mathbb{N}$  (all natural numbers) d)  $\{2, 4, 6, 8, 10\}$
7. a)  $-5$  b)  $6$  c)  $1$  d)  $-5$  e)  $4$  f)  $-8$  g)  $16$  h)  $-32$  i)  $-4$  j)  $-8$  k)  $-16$   
 l)  $-32$  m)  $-40$  n)  $16$  o)  $58$  p)  $100$  q)  $34$  r)  $15$  s)  $-110$  t)  $-56$  u)  $50$
8. a)  $4$  b)  $-4$  c)  $-4$  d)  $-4$  e)  $20$  f)  $1$  g)  $-20$  h)  $1$  i)  $29$  j)  $49$  k)  $27$  l)  $-21$  m)  $9$   
 n)  $-23$  o)  $100$  p)  $-100$  q)  $1$  r)  $-21$  s)  $29$  t)  $49$  u)  $-23$  v)  $100$  w)  $-100$  x)  $100$
9. a)  $72$  b) undefined c)  $5$  d)  $10$  e)  $-24$  f)  $3$  g) undefined h)  $45$  i)  $61$  j)  $-1$  k)  $-6$   
 l)  $-27$  m)  $14$  n)  $20$  o)  $31$  p)  $12$  q)  $120$
10. a)  $15$  b)  $24$  c)  $4$
11. a)  $203$  b)  $0$