This problem set is not homework. Students can use this problem set as extra practice or study guide for quizzes.

1. Label each of the following statements as true or false.

   a) For any sets, A and B, $A \cap B \subseteq A$.
   b) For any sets A and B, $A \subseteq A \cup B$.
   c) For any sets A and B, $A \cup B \subseteq A \cap B$.
   d) For any sets A and B, if $A \subseteq B$ and $B \subseteq A$, then $A = B$.
   e) For any sets A, $\varnothing \subseteq A$.
   f) For any sets A, B, and C, if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.
   g) For any sets A, B, and C, $(A \cap B) \cup C = A \cap (B \cup C)$.
   h) For any sets A, B, and C, $(A \cap B) \cap C = A \cap (B \cap C)$.
   i) For any sets A, B, and C, $(A \cup B) \cup C = A \cup (B \cup C)$.

2. Suppose that $A = \{1, 2, 3, 4, 5\}$. List all two-element subsets of A.

3. Suppose that $P = \{1,\ 3,\ 6,\ 10\}$, and $S = \{1,\ 2,\ 5,\ 6,\ 9,\ 10\}$, and $T = \{3,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10\}$
   a) Draw a Venn-diagram depicting P, S, and T.
   b) Find each of the following.
      i) $S \cap P \cap T$ ii) $S \cup (P \cap T)$ iii) $(S \cup P) \cap T$ iv) $P \cup (S \cap T)$
   c) Find an operation or (operations) on P, S, and/or T so that the result is the set $\{1,\ 6,\ 10\}$.
   d) True or false? $P \subseteq S \cup T$
   e) True or false? $S \cap T \subseteq P$

4. Suppose that $U = \{1,\ 2,\ 3,\ \ldots,\ 14,\ 15\}$. Find each of the following sets.
   a) $A = \{x \in U : x \text{ is divisible by 3 or } x \geq 11\}$ b) $A = \{x \in U : x \text{ is divisible by 3 and } x \geq 11\}$

5. Suppose that $T = \{n \in \mathbb{Z} : n \text{ is divisible by } 3\}$, $S = \{n \in \mathbb{Z} : n \text{ is divisible by } 6\}$, and $E = \{n \in \mathbb{Z} : n \text{ is divisible by } 2\}$. Label each of the statements as true or false.
   a) $T \cap E = S$ b) $E \subseteq S$ c) $S \subseteq T$ d) $T \cup E = S$

6. Find the prime factorization for each of the following numbers.
   a) 1200 b) 2016 c) 1820

7. Find the last digit of 7^{2018}.

8. Evaluate each of the given numerical expressions.
   a) $5 - 2\left(20 - 12 \div 2\left(-3^2 + 15\right)\right)$ b) $\sqrt{2\sqrt{3 \cdot 7 - 5} - \sqrt{2^3 - (-1)^3} - 1}$ c) $\dfrac{-1^2 - 2\left(-3^2 - 2\left(3^2 - 7\right)\right)}{-2^2 - 1}$

9. Evaluate each of the given algebraic expressions if $x = -3$, $y = 4$, and $z = -5$.

   a) xyz
   b) $x - y - z$
   c) $\dfrac{-x - 2y - 5z}{x + 1}$
   d) $x\,|-y - z|$
   e) $x - |y - z|$
   f) $x - y\,|-z|$
   g) $-x^2 + y^2$
   h) $-x^3 + y^3$
   i) $(-x + y)^2$
   j) $\dfrac{-x + y + z}{x + 2y + z}$
   k) $-z^2 + x^2 + (-y)^2$
   l) $(z - x)^y$

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10. a) Consider the equation $-2x\left(3x-x^2\right)+5=(x+2)(x-2)$. Find all the numbers from $-2,-1,0,1,2,3$ that are solutions of the equation.

    b) Consider the inequality $-2x\left(3x-x^2\right)+5<(x+2)(x-2)$ .Find all the numbers from $-2,-1,0,1,2,3$ that are solutions of the inequality.

11. Simplify each of the following.

    a) $2a-3-(5a-3)$

    b) $-3(2x-5)-4(-x+2)$

    c) $2(3m-4)-5(-2m+3)-2(m-7)$

    d) $-3a-2(5a-3(4a-b))$

    e) $5x-(3(4-(x-3(2x-1))+x)-x)-1$

    f) $5(x-(3(4-(x-3(2x-1))+x)-x))-1$

12. Solve each of the given equations.

    a) $-3x+7=16$

    b) $\dfrac{a}{-3}+7=-1$

    c) $\dfrac{a+7}{-3}=-1$

    d) $5x-8=-8$

    e) $3(2x-1)-2(5x-7)=-5$

    f) $2x-(3x-4(2x-1))=17$

    g) $3x-8-5(2x-8)=-3$

    h) $\dfrac{\dfrac{3x-1}{4}+9}{\dfrac{5}{2}}-7=-3$

    i) $5(3x-1)-4(4x-3)=7$

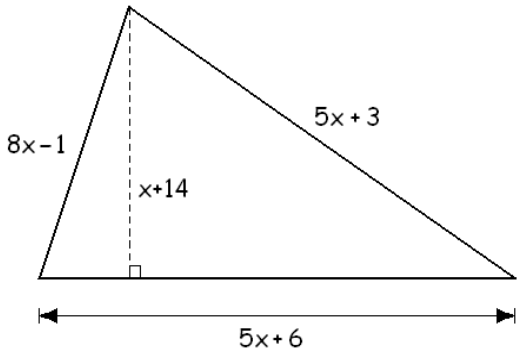
13. Solve each of the following application problems. In each case, set up and solve an equation.

    a) The sum of three consecutive multiples of 4 is -24. Find these numbers.

    b) One side of a rectangle is five inches shorter than four times another side. Find the sides of the rectangle if its perimeter is $110\,\text{in}$.

    c) Consider the triangle shown on the picture. Find the value of x, given that the perimeter of the triangle is 98 unit.

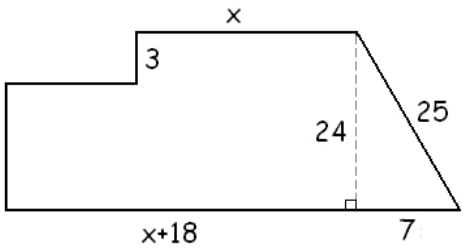
    d) Samantha is asked about her age. She answers as follows. "*My age is ten less than twice the age of my younger brother. The sum of our ages is 32*". How old is Samantha?



    e) The local library held a sale of their old books. Each softcover book was priced at 3 dollars, and hardcover books costed 5 dollars each. We purchased a whole lot of books! The number of softcover books we got was eight less than three times the number of hardcover books. How many softcover books did we buy if we paid a total of 74 dollars?

    f) The first night in a hotel costs 38 dollars, and each additional night cost 25 dollars. How long did we stay in the hotel if the bill was 163 dollars?

    g) Consider the object shown on the picture. Angles that look like right angles are right angles. Find the perimeter of the object, given that its area is $1182\ \text{unit}^2$.



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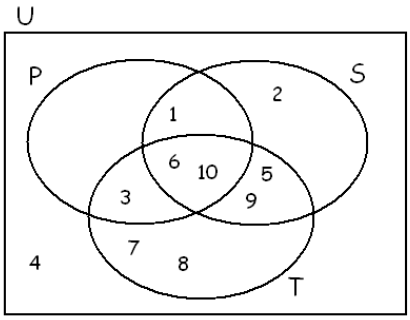
Answers

1. a) true b) true c) false d) true e) true f) true g) false h) true i) true

2. $\{1,2\}$ $\{2,3\}$ $\{3,4\}$ $\{4,5\}$ there are 10 such subsets
   $\{1,3\}$ $\{2,4\}$ $\{3,5\}$
   $\{1,4\}$ $\{2,5\}$
   $\{1,5\}$

3. a)



   b) i) $\{6,10\}$ ii) $\{1,2,3,5,6,9,10\}$ iii) $\{3,5,6,9,10\}$ iv) $\{1,3,5,6,9,10\}$ c) $P \cap S$ d) true e) false

4. a) $\{3,6,9,11,12,13,14,15\}$ b) $\{12,15\}$

5. a) true b) false c) true d) false

6. a) $2^4 \cdot 3 \cdot 5^2$ b) $2^5 \cdot 3^2 \cdot 7$ c) $2^2 \cdot 5 \cdot 7 \cdot 13$

7. 9

8. a) 37 b) 2 c) -5

9. a) 60 b) -2 c) -10 d) -3 e) -12 f) -23 g) 7 h) 91 i) 49 j) undefined k) 0 l) 16

10. a) -1 and 3 b) -2 and 2

11. a) $-3a$ b) $-2x+7$ c) $14m-9$ d) $11a-6b$ e) $-12x-4$ f) $-80x-16$

12. a) -3 b) 24 c) -4 d) 0 e) 4 f) 3 g) 5 h) -5 i) 0

13. a) $-12, -8, -4$, and 0 b) $12\,\text{in}$ by $43\,\text{in}$ c) 5 d) 18 years old e) 13 f) 6 nights g) 145 unit

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