

This problem set is not homework. Students can use this problem set as extra practice or study guide for quizzes.

1. Simplify each of the following.

$$\text{a) } \frac{2^{-1} + 5^{-1} \cdot 3}{2^{-1} - 5^{-1} \cdot 3} \quad \text{b) } -\frac{2}{5} + \frac{1}{4} \left(-\frac{3}{2}\right)^2 \quad \text{c) } 1 - \frac{2}{3 - \frac{4}{5 - \frac{1}{6}}} \quad \text{d) } \frac{1}{3} - \left(\left(-\frac{2}{3}\right)^2 - \frac{5}{6}\right)$$

2. Estimate each of the given radicals by placing them between two consecutive integers.

$$\text{a) } \sqrt{30} \quad \text{b) } \sqrt{60} \quad \text{c) } \sqrt{120}$$

3. Simplify each of the following. Use exact values in each step.

$$\begin{array}{llll} \text{a) } 3\sqrt{5} - \sqrt{5} & \text{d) } \sqrt{2}(3\sqrt{2} - 1) & \text{g) } (2\sqrt{5} - 1)^2 & \text{j) } (\sqrt{5} + 1)^2 - (\sqrt{5} - 1)^2 \\ \text{b) } \sqrt{18} - \sqrt{50} & \text{e) } (2\sqrt{3} - 1)(\sqrt{3} + 2) & \text{h) } (\sqrt{3} - \sqrt{2})^2 & \text{k) } (\sqrt{5} - 2)^4 (\sqrt{5} + 2)^4 \\ \text{c) } \frac{\sqrt{45}}{\sqrt{5}} & \text{f) } (\sqrt{10} - 3)(\sqrt{10} + 3) & \text{i) } (2 - \sqrt{3})^3 & \end{array}$$

4. Compute the exact value of the expression $-x^2 + 2x + 5$ if

$$\text{a) } x = \sqrt{2} \quad \text{b) } x = \sqrt{3} - 1 \quad \text{c) } x = 1 - \sqrt{3} \quad \text{d) } x = 1 - \sqrt{6}$$

5. Compute each of the following sums.

$$\text{a) } 20 + 23 + 26 + \dots + 152 \quad \text{b) } 137 + 142 + 147 + \dots + 632 \quad \text{c) } 387 + 398 + 409 + \dots + 2587$$

6. Prove that each of the given decimal represents a rational number by converting each of them to a quotient of two integers. You do not need to reduce the fractions.

$$\text{a) } 3.407 \quad \text{b) } 0.3\overline{20} \quad \text{c) } 0.7\overline{819} \quad \text{d) } 0.33\overline{4}$$

7. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Find each of the given sets.

$$\begin{array}{lll} \text{a) } \{x \in U : x \geq 8 \text{ and } x \text{ is even}\} & \text{c) } \{x \in U : x < 3 \text{ and } x > 7\} & \text{e) } \{x \in U : x \geq 4 \text{ and } x > 6\} \\ \text{b) } \{x \in U : x \geq 8 \text{ or } x \text{ is even}\} & \text{d) } \{x \in U : x < 3 \text{ or } x > 7\} & \text{f) } \{x \in U : x \geq 4 \text{ or } x > 6\} \end{array}$$

8. Compute the prime factorization of each of the following.

$$\text{a) } 2500 \quad \text{b) } 72^{50}$$

9. Use the prime factorization to find the greatest common factor and least common multiple of 72 and 960.

10. Label each of the following statements as true or false.

- Every positive integer has an even number of factors.
- The product of two consecutive integers is always even.
- A degree three equation can have up to three different solutions.
- If we square an odd number and then subtract one, the result is always divisible by 4.
- There is no prime number that is divisible by 5.
- If n is a perfect square, then all exponents in the prime-factorization of n are even.
- If A and B are any sets such that $A \cup B = A$, then $A = B$.
- If A and B are any sets such that $A \cup B = A$, then $B \subseteq A$.

11. Simplify each of the following.

a) $x^6 \cdot x^7$

e) $(2xy^3)^3$

h) $\sqrt{x^8}$

k) $\frac{(2x^7y^{-3})^{-2}(-2xy^{-2}x^4)^3}{(-x^3y^{-2})^{-4}}$

b) $(-x)^6 \cdot (-x)^7$

f) $\frac{(x^3)^8}{x^3 \cdot x^8}$

i) $(-5a^{-2}b)^2$

l) $\frac{2a^{-3}(-2a^{-1}b)^3(-a^2b^5)^{-2}}{4b^{-5}(-2a^{-6}b)^2}$

c) $(-x^6) \cdot (-x^7)$

d) $(x^6)^7$

g) $(x^3)^2$

j) $(-5a^{-2}b)^3$

12. Suppose that $x = 2\,500\,000\,000$ and $y = 0.000\,004$. Write each of the following in scientific notation.

a) x

b) y

c) xy^2

d) x^2y^3

e) $\frac{1}{y}$

f) \sqrt{y}

13. Graph each of the following.

a) $y = \frac{2}{3}x - 1$

b) $2x + 3y = -5$

c) $y = -2$

14. Simplify each of the following. Assume that x is positive.

a) $(\sqrt{2})^8$

b) $(\sqrt[4]{3})^{12}$

c) $(\sqrt{x})^8$

d) $\sqrt{x^{60}}$

e) $\sqrt[3]{x^{60}}$

f) $\frac{5^{102} - 5^{101}}{5^{100}}$

15. Simplify each of the following.

a) $(2x + 3) + (5x - 1)$

d) $(2x + 3)(5x - 1)$

g) $(2(x + 3) - 5)(x - 1)$

b) $(2x + 3) - (5x - 1)$

e) $2x + 3(5x - 1)$

c) $-2(2x + 3) - 8(5x - 1)$

f) $(2x + 3)5x - 1$

h) $(2x + 3)^2 - (5x - 1)^2$

16. Completely factor each of the following over the integers.

a) $3x^2 - 48$

e) $x^2 + 6x$

i) $(3x^5 - x + 5)^2 - (3x^5 + x - 5)^2$

b) $A^{10} - 9$

f) $x^2 - 1$

j) $18a^2bx^2 - 2a^2by^2$

c) $-2x^2 - 50$

g) $x^4 - 1$

d) $100 - x^2$

h) $(5a + 3b)^2 - (3a - b)^2$

17. Completely factor each of the following over the real numbers by completing the square.

a) $-2x^2 - 8x + 24$

b) $10x^3 - 60x^2 + 100x$

c) $-x^2 + 12x - 36$

18. Solve each of the following equations. Make sure to check your solutions.

a) $2x - 5(x - 3) = (x + 1)^2 - (x - 2)^2$

f) $\frac{2}{3}(x - 1) - \frac{1}{2}(x + 5) = \frac{1}{6}(x - 2)$

b) $6x + x^2 = 7$

g) $\frac{3}{4}x - \frac{2}{5} - \left(\frac{1}{2} - \frac{x}{4}\right) = x - \frac{9}{10}$

c) $x(x + 1)(3x - 7)(x + 5)^2 = 0$

h) $x^2 = 36$

d) $-x(x - 2) + 3(x - 1)^2 = 3$

i) $x^5 = 36x^4$

e) $\frac{1}{2}(x - 3) + \frac{1}{2}(x + 1) = 3x - 1$

j) $x^5 = 36x^3$

19. Solve each of the following inequalities.

a) $\frac{2x - 1}{3} - \frac{x - 1}{2} \geq -x + 6$

b) $(x - 3)^2 - (2x + 1)^2 \geq 8 - 3x^2$

c) $-\frac{3}{5}x + \frac{1}{2} < \frac{2}{5}$

20. Expand each of the following.

a) $(a - b)(a + b)$

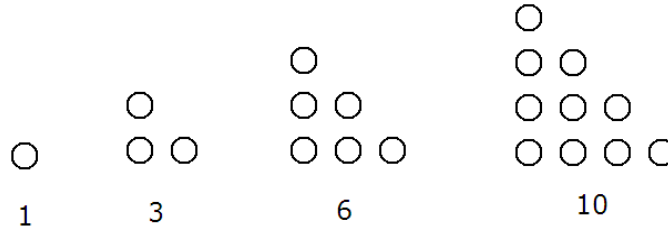
b) $(a - b)(a^2 + ab + b^2)$

c) $(a - b)(a^3 + a^2b + ab^2 + b^3)$

21. In a class of 30, students discuss their summer activities. 20 students report that they traveled to other cities or countries in the summer. 18 students report that they visited local beaches or waterparks. If 7 students claimed to do none of the above, how many students did both traveling and waterparks? (Hint: draw a Venn diagram!)

22. $A(-3, -1)$, $B(-5, 4)$, and $C(5, 8)$ are three vertices of a rectangle. Find the fourth vertex of the rectangle.

23. The picture shows the first few triangular numbers. Find the hundredth triangular number.



24. Apply the difference of squares theorem to evaluate the given expressions mentally.

a) $99^2 - 98^2$

b) $60^2 - 40^2$

c) $35^2 - 15^2$

d) $21^2 - 19^2$

25. One side of a rectangle is ten feet shorter than five times another side. Find the sides of the rectangle if we also know that its area is 120 ft^2 .

26. Express a 30% increase and a subsequent 20% decrease as a single change. Is it an increase or decrease? What percent?

27. We throw a small object upward from the top of a 720 ft tall building. The vertical position (or height) h of the object, (measured in feet) t seconds after we threw it is

$$h = -16t^2 + 192t + 720$$

How long does it take for the object to hit the ground?

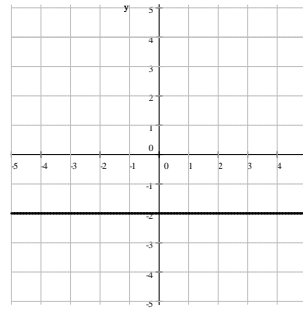
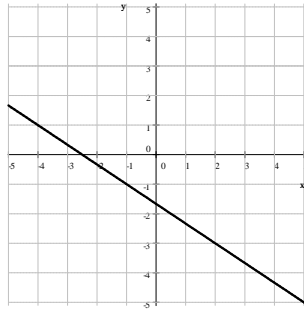
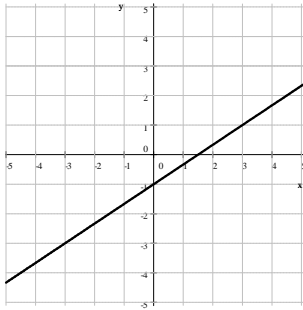
28. The first row in a theater has 31 seats in it. The second row has two more seats than the first row. The third row has two more seats than the second row. And so on, each row has two more seats than the row before. If the last row has 115 seats in it, how many seats are there in the entire theater?

29. If we increase the side of a square by 2 units, its area will increase by 12 unit^2 . How long are the sides of the original square?

30. If we raise a number to the third power, we get sixteen times the number. Find this number. $x^3 = 16x$.

Answers

1. a) -11 b) $\frac{13}{80}$ c) $\frac{5}{63}$ d) $\frac{13}{18}$ 2. a) $5 < \sqrt{30} < 6$ b) $7 < \sqrt{60} < 8$ c) $10 < \sqrt{120} < 11$
3. a) $2\sqrt{5}$ b) $-2\sqrt{2}$ c) 3 d) $6 - \sqrt{2}$ e) $4 + 3\sqrt{3}$ f) 1 g) $21 - 4\sqrt{5}$ h) $5 - 2\sqrt{6}$ i) $26 - 15\sqrt{3}$ j) $4\sqrt{5}$
k) -1 4. a) $3 + 2\sqrt{2}$ b) $4\sqrt{3} - 1$ c) 3 d) 0 5. a) 3870 b) 38450 c) 298887
6. a) $\frac{3407}{1000}$ b) $\frac{317}{990}$ c) $\frac{7812}{9990}$ d) $\frac{301}{900}$
7. a) $\{8, 10\}$ b) $\{2, 4, 6, 8, 9, 10\}$ c) \emptyset d) $\{1, 2, 8, 9, 10\}$ e) $\{7, 8, 9, 10\}$ f) $\{4, 5, 6, 7, 8, 9, 10\}$
8. a) $2^2 \cdot 5^4$ b) $2^{150} \cdot 3^{100}$ 9. 24 and 2880
10. a) false b) true c) true d) true e) false f) true g) false h) true
11. a) x^{13} b) $-x^{13}$ c) x^{13} d) x^{42} e) $8x^3y^9$ f) x^{13} g) x^6 h) x^4 i) $\frac{25b^2}{a^4}$ j) $-\frac{125b^3}{a^6}$ k) $\frac{-2x^{13}}{y^8}$ l) $-\frac{a^2}{b^4}$
12. a) $2.5 \cdot 10^9$ b) $4 \cdot 10^{-6}$ c) $4 \cdot 10^{-2}$ d) $4 \cdot 10^2$ e) $2.5 \cdot 10^5$ f) $2 \cdot 10^{-3}$
13. a) $y = \frac{2}{3}x - 1$ b) $2x + 3y = -5$ c) $y = -2$



14. a) 16 b) 27 c) x^4 d) x^{30} e) x^{20} f) 20 15. a) $7x + 2$ b) $-3x + 4$ c) $-44x + 2$ d) $10x^2 + 13x - 3$
e) $17x - 3$ f) $10x^2 + 15x - 1$ g) $2x^2 - x - 1$ h) $-21x^2 + 22x + 8$
16. a) $3(x + 4)(x - 4)$ b) $(A^5 + 3)(A^5 - 3)$ c) $-2(x^2 + 25)$ d) $-(x + 10)(x - 10)$ e) $x(x + 6)$
f) $(x + 1)(x - 1)$ g) $(x^2 + 1)(x + 1)(x - 1)$ h) $4(a + 2b)(4a + b)$ i) $-12x^5(x - 5)$ j) $2a^2b(3x + y)(3x - y)$
17. a) $-2(x + 6)(x - 2)$ b) $10x(x^2 - 6x + 10)$ c) $-(x - 6)^2$
18. a) 2 b) $-7, 1$ c) $0, -1, -5, \frac{7}{3}$ d) $0, 2$ e) 0 f) There is no solution. g) All numbers are solution. h) $-6, 6$
i) $0, 36$ j) $-6, 0, 6$
19. a) $[5, \infty)$ b) $(-\infty, 0]$ c) $(\frac{1}{6}, \infty)$ 20. a) $a^2 - b^2$ b) $a^3 - b^3$ c) $a^4 - b^4$ 21. 15 22. $(7, 3)$
23. 5050 24. a) 197 b) 2000 c) 1000 d) 80 25. 6 ft by 20 ft 26. 4% increase 27. 15 seconds
28. 3139 29. 2 units 30. $-4, 0, 4$