

This problem set is not homework. Students can use this problem set as extra practice or study guide for quizzes.

- Which number has more divisors, 24 or 36?
- Suppose that $A = 2^{99}$. Express each of the following in terms of A .
 - 2^{100}
 - $2^{99} + 2^{100} - 3 \cdot 2^{101}$
 - 8^{99}
- Find the smallest value of each of the following expressions.
 - $x^2 - 6x + 4$
 - $x^2 + 10x + 7$
 - $x^2 + 6x$
- Find the prime factorization for each of the following.
 - 1200
 - 10^{100}
 - $75 \cdot 80$
- Use the prime factorization to find the greatest common factor and least common multiple of 90 and 135.
- Simplify each of the given expressions.
 - $\frac{3}{8} \cdot (-3)^{-2}$
 - $\frac{2^{-1} + 5^{-1}}{2^{-2} - 5^{-2}}$
 - $\frac{2^{-1} \cdot 5^{-1}}{2^{-2} \cdot 5^{-2}}$
 - $\frac{(2x^{-1}y^3)^{-5}}{(2xy^{-2})^{-3}}$
 - $\frac{(-2xy^{-2})^3 (-2^{-2}x^{-3}y)^4}{(-4x^3y)^{-2}}$
- Compute each of the following sums.
 - $42 + 50 + 58 + \dots + 482$
 - $-35 + (-25) + \dots + 755$
 - $31 + 40 + 49 + \dots + 1102$
- Perform the indicated operations and simplify.
 - $(2\sqrt{7} - 1)(\sqrt{7} + 3)$
 - $(3\sqrt{2} - 1)^2$
 - $(\sqrt{2} - 1)^4$
 - $(3\sqrt{7} - 8)(3\sqrt{7} + 8)$
 - $\sqrt{50} - \sqrt{18} + 2\sqrt{72}$
 - $(2\sqrt{5} - 4)^3 (2\sqrt{5} + 4)^3$
 - $\frac{12 - \sqrt{20}}{2}$
 - $\frac{\sqrt{80}}{\sqrt{45}}$
 - $(-\sqrt{2})^8$
 - $(-\sqrt{2})^9$
- Simplify each of the following.
 - $(1, 4) \cup (2, 7)$
 - $(1, 4) \cap (2, 7)$
 - $(-\infty, 3) \cup (-\infty, 5]$
 - $(-\infty, 3) \cap (-\infty, 5]$
 - $(-\infty, 10] \cup (7, \infty)$
 - $(-\infty, 10] \cap (7, \infty)$
 - $(1, 5) \cup [12, \infty)$
 - $(1, 5) \cap [12, \infty)$
- Evaluate each of the given expressions with the values given for the variable(s).
 - $2x^2 - x - 3$ if $x = 2\sqrt{3} - 5$
 - $-a^2 + 5a - 1$ if $a = 4 - \sqrt{3}$
 - $x^2 - 8x + 6$ if $x = 4 - \sqrt{10}$
- Graph each of the following.
 - $y = -\frac{2}{3}x + 1$
 - $x + y = 3$
 - $2x - 3y = -1$
- Graph $y = x^2 - 4x - 5$. State the coordinates of at least five points, including vertex and intercepts.
- Find the perimeter and area of the quadrilateral determined by the points $A(-3, 5)$, $B(-3, -2)$, $C(2, -2)$, and $D(2, 5)$.

14. Perform the following operations. Show all steps.

a) $18 - 2(-5) - 2(11 - 2(-5))$

d) $\left| -\frac{1}{2} - 2 \right| - \left| \frac{3}{4} - 3 \right|$

g) $\left| -\frac{1}{2} - \left| 2 - \frac{3}{4} - 3 \right| \right|$

b) $\frac{-3^2 + (-1)^3}{7 - 3(-1)^3}$

e) $\left| -\frac{1}{2} - 2 - \left| \frac{3}{4} - 3 \right| \right|$

h) $\left| -\frac{1}{2} \right| - 2 - \frac{3}{4} - 3 \left| \right|$

c) $-2^2(24 - 2(-3) - 5(-2)^2) - 12$

f) $\left| -\frac{1}{2} - 2 \left| \frac{3}{4} - 3 \right| \right|$

i) $\sqrt{3\sqrt{49} - \sqrt{25}}$

15. Completely factor each of the following over the integers.

a) $(3a + 1)^2 - 49$

d) $x^{10} - 49$

g) $(2x^2 - x + 2)^2 - (2x^2 + x - 4)^2$

b) $200x + 2x^3$

e) $405 - 5x^4$

c) $5x^6 - 80x^2$

f) $12x^2 - 110x + 2x^3$

16. If A is the set of all integers divisible by 5 and B is the set of all integers divisible by 3, what is $A \cap B$?

17. Evaluate $\frac{3xy + 2x^2 - 2y^2}{x + 2y}$ if

a) $x = \frac{1}{2}$ and $y = -3$

b) $x = -1$ and $y = -2$

c) $x = -6$ and $y = 3$

18. Simplify each of the following algebraic expressions.

a) $(3x^2 - 5x + 8) + (-3x^2 - 5x + 12)$

f) $(2x - 3)(2x + 3)$

b) $(3x^2 - 5x + 8) - (-3x^2 - 5x + 12)$

g) $2(m - 3)^2 - (2m + 5)(m - 3)$

c) $3(x - 5) - 4(x - 2)$

h) $(a - 4)(2a + 1) - (a - 3)^2$

d) $5(2a + 1) - 2(2 - a) - 7(a + 8)$

i) $(3x - 1)^3$

e) $(x - 5)(x - 2)$

19. Solve each of the following equations.

a) $-\frac{2}{5}x + \frac{1}{2} = \frac{1}{5}$

f) $20x^5 = 5x^3$

b) $\frac{3x + 1}{5} - \frac{2x - 4}{3} = 2x - 15$

g) $(2x - 5)^2 - (2x + 6)^2 = 4x - 11$

c) $x(3x - 1) - 2x - 3 = 3(x - 1)^2$

h) $60x - 20x^2 = 5x^3$

d) $\frac{2}{3}\left(x - \frac{1}{2}\right) - \frac{1}{2}\left(x + \frac{2}{3}\right) = \frac{1}{6}(x + 2)$

i) $x^4(x^2 + 1)(x^4 + 9) = 0$

e) $20x^5 = 5x^4$

j) $4(x - 3) - 3(x + 1) = 2x - 10 - (x + 5)$

k) $(2x - 1)^2 - (x - 2)^2 = 2x^2 + 1$

20. Solve each of the following inequalities.

h) $3(x - 2) - 5(6x - 1) \geq 3x - 1$

j) $\frac{2x - 3}{7} - \frac{3x - 1}{2} \leq x - 11$

i) $\frac{3x - 8}{4} < \frac{5x - 17}{3}$

k) $(3x - 1)^2 + (x - 1)^2 \geq 10x^2 + 6$

21. Solve each of the given system of linear equations.

a)
$$\begin{cases} 2x + 3y = -4 \\ 4x - 5y = -30 \end{cases}$$

b)
$$\begin{cases} -x + 2y = 19 \\ 3x + y = -1 \end{cases}$$

c)
$$\begin{cases} (x - 2)^2 + (y - 1)^2 = x^2 + y^2 - 1 \\ (x - 3)^2 - x^2 = (y + 3)^2 - y^2 \end{cases}$$

22. Prove that each of the following numbers is rational by re-writing them as a fraction of two integers.
a) $0.\overline{304} = 0.304040404\dots$ b) $7.\overline{2673} = 7.2673673673673$ c) $0.\overline{9} = 0.99999\dots$
23. The opposite of a number is thirty-five more than the sum of -5 and the number. Find this number.
24. The sum of three consecutive numbers is 63. Find these numbers.
25. There is a farm where chickens and cows live. All together, there are 72 heads and 200 legs. How many chickens, how many cows are there?
26. Find the sides of the square if we know the following. If we increased the length of each side by 1 feet, the area of the square would increase by 17 ft^2 .
27. We have some \$5 bills and some \$10 bills. All together, there are 74 bills. How many \$5 bills do we have if the value of all bills is \$430?
28. A number is 42 less than its own square. Find this number.
29. Find all numbers with the following property: if we multiply the number by seven, the result is the original number.
30. The first row in a theater has 15 seats in it. The second row has three more seats than the first row. The third row has three more seats than the second row. And so on, each row has three more seats than the row before. If the last row has 81 seats in it, how many seats are there in the entire theater?
31. Find all numbers with the following property: if we square the number, the result is the original number.
32. We have a jar full of coins, all dimes and quarters. How many of each coins do we have if all together, we have 154 coins in the total value of \$24.70?
33. Find all numbers with the following property: if we raise the number, to the third power, the result is the original number.
34. We throw a small object upward from the top of a 432 ft tall building. The vertical position (or height) h of the object, (measured in feet) t seconds after we threw it is

$$h = -16t^2 + 96t + 432$$

How long does it take for the object to hit the ground?

35. One number is twelve less than another. Find these numbers if their product is 160.
36. There are 120 people in a room. If everyone shakes hands with everyone else, how many handshakes took place?
37. One number is twelve less than twice another. Find these numbers if their product is 320.
38. List all subsets of $A = \{1, 2, 3, 4\}$.
39. List all two-element subsets of $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Answers

1. 36 (24 has 8 divisors, 36 has 9)

2. a) $2A$ b) $-9A$ c) A^3

3. a) -5 b) -18 c) -9

4. a) $1200 = 2^4 \cdot 3 \cdot 5^2$ b) $10^{100} = 2^{100} \cdot 5^{100}$
c) $75 \cdot 80 = 2^4 \cdot 3 \cdot 5^3$

5. $\text{lcm}(90, 135) = 270$ $\text{gcd}(90, 135) = 45$

6. a) $-\frac{1}{2}$ b) $\frac{10}{3}$ c) 10 d) $\frac{x^8}{4y^{21}}$ e) $-\frac{1}{2x^3}$

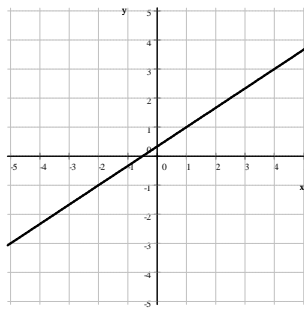
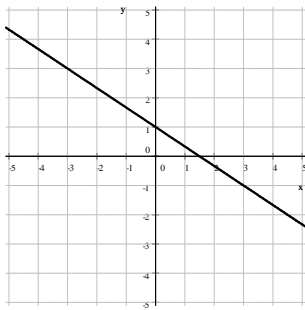
7. a) 14 672 b) 28 800 c) 67 980

8. a) $11 + 5\sqrt{7}$ b) $19 - 6\sqrt{2}$ c) $17 - 12\sqrt{2}$
d) -1 e) $14\sqrt{2}$ f) 64 g) $6 - \sqrt{5}$
h) $\frac{4}{3}$ i) 16 j) $-16\sqrt{2}$

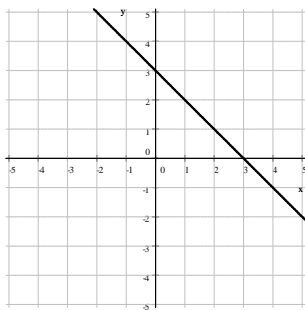
9. a) (1, 7) b) (2, 4) c) $(-\infty, 5]$ d) $(-\infty, 3)$
e) \mathbb{R} same as $(-\infty, \infty)$ f) (7, 10]
g) can not be simplified h) \emptyset

10. a) $76 - 42\sqrt{3}$ b) $3\sqrt{3}$ c) 0

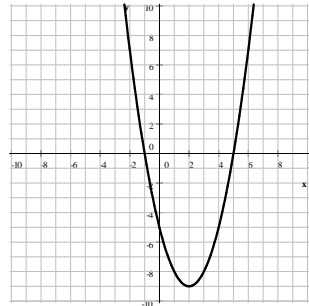
11. a) $y = -\frac{2}{3}x + 1$ c) $2x - 3y = -1$



b) $x + y = 3$



12. $y = x^2 - 4x - 5 = (x - 2)^2 - 9$ vertex: (2, -9)
x-intercepts: (-1, 0) and (5, 0) y-intercept: (0, -5)



13. $P = 24$ unit, $A = 35$ unit²

14. a) -14 b) -1 c) -40 d) $\frac{1}{4}$ e) $\frac{19}{4}$
f) 5 g) $\frac{9}{4}$ h) $\frac{23}{8}$ i) 4

15. a) $3(3a + 8)(a - 2)$ b) $2x(x^2 + 100)$
c) $5x^2(x^2 + 4)(x + 2)(x - 2)$ d) $(x^5 + 7)(x^5 - 7)$
e) $-5(x^2 + 9)(x + 3)(x - 3)$ f) $2x(x + 11)(x - 5)$
g) $-4(2x^2 - 1)(x - 3)$

16. the set of all integers divisible by 15

17. a) 4 b) 0 c) undefined

18. a) $-10x + 20$ b) $6x^2 - 4$ c) $-x - 7$
d) $5a - 55$ e) $x^2 - 7x + 10$ f) $4x^2 - 9$
g) $-11m + 33$ h) $a^2 - a - 13$
i) $27x^3 - 27x^2 + 9x - 1$

19. a) $\frac{3}{4}$ b) 8 c) 2 d) There is no solution.

e) $0, \frac{1}{4}$ f) $-\frac{1}{2}, 0, \frac{1}{2}$ g) 0 h) 2, 0, -6

i) 0 j) \mathbb{R} (all real numbers are solution)

k) 2, -2

20. a) $(-\infty, 0]$ b) $(4, \infty)$ c) $[5, \infty)$ d) $(-\infty, -\frac{1}{2}]$

21. a) (-5, 2) b) (-3, 8) c) (3, -3)

22. a) $\frac{301}{990}$ b) $\frac{72601}{9990}$ c) 1

23. -15 24. 20, 21, 22 25. 44 chickens and 28 cows

26. 8 ft 27. 12 ten-dollar bills and 62 five-dollar bills

28. -6 and 7 29. 0 30. 1104 31. 0, 1

32. 62 quarters and 92 dimes 33. 0, 1, -1

34. 9 seconds 35. -8 with -20 or 8 with 20 36. 7140 37. -10 with -32 and 16 with 20

38. All subsets of $A = \{1, 2, 3, 4\}$

0-element subsets: \emptyset

1-elt subsets: $\{1\}, \{2\}, \{3\}, \{4\}$

2-elt subsets: $\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}$

3-elt subsets: $\{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$

4-elt subsets: $\{1, 2, 3, 4\}$

39. List all two-element subsets of $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

$\{1, 2\}$

$\{1, 3\}$ $\{2, 3\}$

$\{1, 4\}$ $\{2, 4\}$ $\{3, 4\}$

$\{1, 5\}$ $\{2, 5\}$ $\{3, 5\}$ $\{4, 5\}$

$\{1, 6\}$ $\{2, 6\}$ $\{3, 6\}$ $\{4, 6\}$ $\{5, 6\}$

$\{1, 7\}$ $\{2, 7\}$ $\{3, 7\}$ $\{4, 7\}$ $\{5, 7\}$ $\{6, 7\}$

$\{1, 8\}$ $\{2, 8\}$ $\{3, 8\}$ $\{4, 8\}$ $\{5, 8\}$ $\{6, 8\}$ $\{7, 8\}$

$\{1, 9\}$ $\{2, 9\}$ $\{3, 9\}$ $\{4, 9\}$ $\{5, 9\}$ $\{6, 9\}$ $\{7, 9\}$ $\{8, 9\}$

$\{1, 10\}$ $\{2, 10\}$ $\{3, 10\}$ $\{4, 10\}$ $\{5, 10\}$ $\{6, 10\}$ $\{7, 10\}$ $\{8, 10\}$ $\{9, 10\}$