Review Problems

1. Simplify each of the following.

   a) $(3x-7)^2$

   b) $(2a^3-5b)(2a^3+5b)$

   c) $\dfrac{2x-7}{7-2x}$

   d) $\left(\dfrac{2a^{-2}b^3}{-2^2(a^{-1}b)^{-3}}\right)^{-2}$

   e) $\dfrac{x^{-1}+y^{-1}}{x^{-2}-y^{-2}}$

   f) $\dfrac{2x+6}{18x-2x^3}$

   g) $\dfrac{6xy-2y-3x+1}{9x^2-1}$

   h) $\dfrac{3a-12}{3a+15}\cdot\dfrac{5a+20}{a^2-16}$

   i) $\dfrac{7x-13}{2x-5}-\dfrac{7-x}{2x-5}$

   j) $\dfrac{5m+6}{4m+m^2-12}-\dfrac{3}{m+6}$

   k) $\dfrac{5}{a-1}-\dfrac{6a-1}{a^2-a}$

   l) $(a+b)(a^4-a^3b+a^2b^2-ab^3+b^4)$

   m) $\sqrt{300}-2\sqrt{75}+\sqrt{12}$

   n) $\left(3\sqrt{2}-1\right)^2$

   o) $\left(3\sqrt{2}-1\right)^3$

2. Rationalize the denominator in each of the following expressions.

   a) $\dfrac{4}{\sqrt{7}}$ b) $\dfrac{1}{\sqrt{7}-3}$ c) $\dfrac{1}{\sqrt{10}+3}$ d) $\dfrac{2}{3\sqrt{5}-8}$

3. Find the exact value of $-x^2-6x+1$ if $x=3-\sqrt{10}$.

4. Factor $2x^2-13x+15$ by completing the square.

5. Factor completely each of the following.

   a) $2bnxy-4anxy+12anx^2-6bnx^2$

   b) $75bm^3-150am^3+24am^5-12bm^5$

   c) $240a^5p-160a^5q-15apx^4+10aqx^4$

   d) $8b^2-42b+2b^3$

   e) $28a^2bp^2-2a^2bp-6a^2b$

   f) $14m+5m^2-3$

6. In each case, use completing the square to decide whether the expression factors of not. Explain your answer.

   a) $x^2-14x+53$ b) $14x+x^2+46$ c) $x^2-16x+64$

7. Solve each of the following equations. Make sure to check your solutions.

   a) $\dfrac{3x-1}{5}-\dfrac{7-x}{3}=x-2$

   b) $5(x-2)-(3-4x)=8(x-2)-(5-x)$

   c) $5p^7=20p^6$

   d) $\dfrac{2a+1}{5}-\dfrac{7-a}{2}=-a-9$

   e) $\left|\dfrac{1}{3}x-2\right|-5=11$

   h) $x^2-6x=1$

   i) $5p^7=20p^5$

   j) $(-1-2x)-(3x+5)(2x-1)=3(1-2x)(x-1)+7$

   k) $m^2+55=16m$

   l) $7p+15p^2=4$

f) $(3x-8)-(4x-5)=x-3$

g) $\left|\frac{1}{3}x-2\right|+11=5$

m) $14-(2x-5)^2=2x-x\,(4x-7)$

n) $|2x-5|=|3x+1|$

8. Graph the straight lines $2x+y=5$ and $y=-x+1$ in the same coordinate system.

   (a) Use your graph to find th coordinates of the point where the lines intersect.

   (b) Use algebraic methods to check your solution for part a).

9. Find an equation of the straight line that is parallel to $7x-y=10$ and passes through the point $(-3,-4)$.

10. Find an equation of the line that is perpendicular to $7x-y=10$ and passses through the point $(14,1)$

11. Find an equation of the line that passes through the points $(5,-2)$ and $(7,4)$.

12. Graph the parabola $y=12x-2x^2-10$. Clearly label the coordinates of five points on the parabola, including vertex and intercepts.

13. Word Problems.

   (a) The population of a town has decreased by 10%. Now there are 7650 residents. Find the original population.

   (b) The difference between two numbers is 34, their sum is 20. Find these numbers.

   (c) Ann and Betty are roommates. The monthly rent is \$ 980. The amount paid by Ann is \$ 130 less than twice the amount paid by Betty. How much do they each pay for rent?

   (d) The price of a TV is \$ 680. If this price was to be changed to \$ 442, what percent of a change does this represent?

   (e) One side of a rectangle is 5 ft shorter than twice the other side. Find the sides if the perimeter is 32 ft.

   (f) One side of a rectangle is 5 ft shorter than twice the other side. Find the sides if the area is $150\,\text{ft}^2$.

   (g) The hypotenuse of a right triangle is 82 cm. The difference between the other two sides is 62 cm. Find the sides of the triangle.

   (h) We have invested \$8000 in two bank accounts. One account earns an annual interest rate of 6%, the other account earns an annual interest rate of 9%. How much money was invested at each rate if after one year, the combined interest from these accounts was \$624?

14. Find the distance between the points $A\,(3,-8)$ and $B\,(-5,7)$.

Answers

1. a) $9x^2 - 42x + 49$ b) $4a^6 - 25b^2$ c) -1 d) $\dfrac{4a^{10}}{b^{12}}$ e) $\dfrac{xy}{y-x}$ f) $\dfrac{-1}{x(x-3)}$
   g) $\dfrac{2y-1}{3x+1}$ h) $\dfrac{5}{a+5}$ i) 4 j) $\dfrac{2}{m-2}$ k) $-\dfrac{1}{a}$ l) $a^5 + b^5$ m) $2\sqrt{3}$
   n) $19 - 6\sqrt{2}$ o) $63\sqrt{2} - 55$

2. a) $\dfrac{4\sqrt{7}}{7}$ b) $-\dfrac{\sqrt{7}+3}{2}$ c) $\sqrt{10} - 3$ d) $-\dfrac{6\sqrt{5}+16}{19}$

3. $12\sqrt{10} - 36$

4. $\dfrac{1}{2}\left(x - \dfrac{3}{2}\right)(x-5) = (2x-3)(x-5)$

5. a) $2nx(2a-b)(3x-y)$ b) $3m^3(2a-b)(2m-5)(2m+5)$
   c) $5a(2q-3p)(x^2+4a^2)(x-2a)(x+2a)$ d) $2b(b-3)(b+7)$
   e) $2a^2b(2p-1)(7p+3)$ f) $(5m-1)(m+3)$

6. a) does not factor $(x-7)^2 + 4$ b) factors $\left(x+7+\sqrt{3}\right)\left(x+7-\sqrt{3}\right)$
   c) factors $(x+8)^2$

7. a) -8 b) identity, all numbers are solution c) $0, 4$ d) -3 e) $-42, 54$
   f) 0 g) no solution h) $3 - \sqrt{10}, 3 + \sqrt{10}$ i) $-2, 0, 2$ j) 0 k) $5, 11$
   l) $-\dfrac{4}{5}, \dfrac{1}{3}$ m) 1 n) $-6, \dfrac{4}{5}$

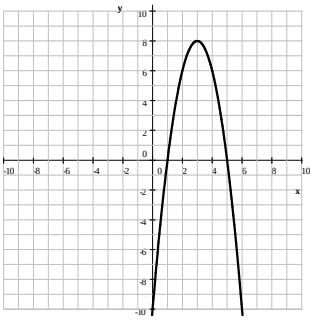
8. a) $(4, -3)$ b) The coordinates of the point $(-4, 3)$ form a solution to both equations.

9. $y = 7x + 17$

10. $y = -\dfrac{1}{7}x + 3$

11. $y = 3x - 17$

12. Vertex: $(3, 8)$ $y-$intercept: $(0, -10)$ $x-$intercepts: $(1, 0)$ and $(5, 0)$
    additional points: $(1, 0)$ $(2, 6)$ $(3, 8)$ $(4, 6)$ $(5, 0)$



13. a) 8500 b) $-7,\ 27$ c) \$370, \$610 d) 35% e) 7 ft, 9 ft f) 10 ft, 15 ft
    g) 18 cm and 80 cm h) \$3200 at 6% and \$4800 at 8%

14. 17 units