1. Suppose that $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 4, 9\}$, and $B = \{2, 4, 6, 7\}$. Find each of the following.
   a) $A \cap B$ b) $A \cup B$ c) $P = \{x \in U : x > 4 \text{ or } x \leq 7\}$ d) $Q = \{x \in U : x > 4 \text{ and } x \leq 7\}$

2. Suppose that S is the set of all squares and R is the set of all rectangles. Label each of the following statements as true or false.

   a) $S \subseteq R$ c) $R \subseteq R$ e) $R \cup S = S$ g) $R \cap S = S$
   b) $R \subseteq S$ d) $\varnothing \subseteq S$ f) $R \cup S = R$ h) $R \cap S = R$

3. a) List all three-digit numbers that can be formed using only the digits $2, 5, 7$ and 9.

   b) List all three-digit numbers that can be formed using only the digits $2, 5, 7$ and 9 and repetition of digits is not allowed. (For example, 225 is not allowed.)

   c) List all three-digit numbers that can be formed using only the digits $2, 5, 7$ and 9, repetition of digits is not allowed, and are divisible by 3.

4. Perform the division with remainder: $2017 \div 17$

5. List all factors of 84.

6. Consider the following numbers: 2011, 11 060 904, 321, 3106. Select all the numbers from the list that are divisible
   a) by 2 b) by 3 c) by 6

7. Which of the following numbers is a prime? 2007, 143, 151, 91

8. Find the prime factorization of 720.

9. Find the prime factorization for x if
   a) $x = 12^{100}$ b) $x = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

   Note that there is a shorter notation for the product above: $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 10!$

10. Find the least common multiple and least common divisor of 630 and 144.

11. Label each of the following statements as true or false.

    a) Every integer is a rational number.

    b) Every positive integer can be written as a product of primes.

    c) If a is divisible by 2 and b is divisible by 3, then the product ab is divisible by 6.

    d) If the product ab is divisible by 6, then a is divisible by 2 and b is divisible by 3 (or vica versa, i.e. a is divisible by 3 and b is divisible by 2.)

    e) If the product ab is divisible by 7, then a is divisible by 7 or b is divisible by 7.

    f) The sum of two consecutive integers is always an odd number.

    g) If a perfect square is divisible by 8, then it is also divisible by 16.

12. Find the smallest positive integer that is divisible by 2, 3, 4, 5, and 6.

13. Consider a rectangle with sides 15 m and 12 m long.
    a) Compute the perimeter of the rectangle. Include units in your computation and answer.
    b) Compute the area of the rectangle. Include units in your computation and answer.

14. a) We would like to buy a bed that is priced at \$700. Next week, the bed is going on a 12% off sale. What will be the sale price?

    b) Paul had a 15% raise. Until now, his monthly pay was \$1600. How much will be his monthly pay after the raise?

15. Simplify each of the following.

    a) -3^2 c) $12 \div 3 \cdot 2$ e) $12 - 2(7 - 4 \cdot 3)$ g) $-\sqrt{49}$ i) -2^2

    b) $-|-6|$ d) $15 - 3 + 2$ f) $|-8 + 5|$ h) $\sqrt{-49}$ j) $(-2)^2$

16. Simplify each of the following.

    a) $\left|-3^3 - 2\left|-5 - 2(-4)\right|\right|$

    b) $\sqrt{-4^2 - (-1)^4 + 2 \cdot 3^2 \div 2 \cdot 6 - 1}$

    c) $-3^2 - |-12 + 2 \cdot 5| - 2 + 1$

    d) $-2^2 - 5(-2)$

    e) $-3^2 - 2\left(4 - 5^2 + 3(10 - 7 + 2)\right)$

    f) $\sqrt{6^2 - 5\sqrt{16}}$

    g) $\frac{1}{2} - \frac{3}{5} \cdot \left(-\frac{4}{7}\right)$

    h) $\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^4$

    i) $\dfrac{24 - (-2)^2 + 12 - 3 + 1}{-3^2 - 12 \div 3(-2)}$

    j) $|3 - 2 - |8 - 10||$

    k) $|3 - |2 - 8| - 10|$

    l) $|3 - 2|-8 - 10||$

17. Add the algebraic expressions as indicated.

    a) $(2a + 3b) + (-2a + 7b)$ b) $(3x - y + 2) + (-x + 6y - 2)$ c) $(3m - 4n) + (5m)$

18. Subtract the algebraic expressions as indicated.

    a) $(2a + 3b) - (-2a + 7b)$ b) $(3x - y + 2) - (-x + 6y - 2)$ c) $(3m - 4n) - (5m)$

19. Multiply the algebraic expressions as indicated.

    a) $(2a + 3b)(-2a + 7b)$ b) $(3x - y)(-x + 6y)$ c) $(3m - 4n)(5m)$

20. Simplify the algebraic expressions as indicated.

    a) $-3(2a + 3b) + 5(-2a + 7b)$

    b) $2(3x - y + 2) - 6(-x + 6y - 2)$

    c) $2x(3x - y) - 5y(-x + 4y)$

    d) $5x - 3(2x - y)$

    e) $-3m(3m - 4n) + 4n(5m)$

    f) $-y + 2(3x - y) - (2x - y) + 2(-2x + y)$

21. Multiply the algebraic expressions as indicated.

    a) $(x + 1)(3x - 5)$

    b) $(3x - y)^2$

    c) $(3a - 4b)(5ab)$

    d) $\left(3x^5 - 2\right)\left(3x^5 + 2\right)$

    e) $(5x + 2)(5x - 2)$

    f) $(5x - 2)^2$

    g) $\left(3a^3 - 1\right)^2$

    h) $(3m - 2)\left(9m^2 + 6m + 4\right)$

    i) $(3x - 1)^3$

22. Simplify each of the following.

    a) $(2x - 1)^2 - 3x(x - 5)$

    b) $(-x + 2)^2 - (2x - 1)(x + 8)$

    c) $(2x - 1)(x - 3) - 2(x - 4)^2$

    d) $x(2x - 5) - 3(2x + 1) - (x - 4)^2$

23. Evaluate the algebraic expression $\dfrac{-x + 2x^2 - 1}{x - 1}$ if

    a) $x = 5$ b) $x = -5$ c) $x = 1$ d) $x = -1$ e) $x = -\frac{1}{2}$

24. Consider the equation $2x^3 - 10\left(x^2 - 2\right) + 4x = -x^2 + 5$. For each of the following numbers given, determine whether it is a solution of the equation or not.

a) $x = -2$ b) $x = -1$ c) $x = 3$ d) $\frac{1}{2}$ e) $\frac{5}{2}$

25. Solve each of the following equations. Make sure to check your solutions.

a) $-3x + 5 = 20$

b) $\frac{3}{8}y + \frac{1}{2} = \frac{1}{4}$

c) $\frac{x - 8}{3} = -2$

d) $\frac{x}{3} - 8 = -2$

e) $5a - 3 = -3a + 21$

f) $\frac{1}{3}x - \frac{4}{5} = \frac{1}{2}x - \frac{9}{5}$

g) $0.04x + 0.79 = 0.3\,(0.5x - 2.5)$

h) $5\,(x - 1) - 3\,(2x - 7) = 2x + 1$

i) $5x - 3\,(2x - 8) = 24$

j) $x - (x - 2) - (-3x + 1) = x - 7$

k) $(3x - 1)\,(2x - 8) = 6\,(x - 1)^2 - 12$

l) $3\,(y - 1) - 5\,(3y + 2) = -13\,(y + 1)$

m) $4\,(x - 3) - 2\,(x - 1) = x - 2\,(4 - x)$

n) $(2x + 1)\,(2x - 5) = (x - 2)\,(4x - 1)$

o) $(3x - 1)^2 - (5 - 2x)^2 = 5\,(x - 3)^2 - 25$

p*) $\dfrac{\dfrac{2x - 1}{5} + 2}{3} - 1 = -4$

26. What is the last digit of the number 2^{99}?

27*. Suppose that A is a real number such that $A^3 + \frac{1}{A^3} = 7$. Compute the exact value of $A^6 + \frac{1}{A^6}$. (Hint: do not try to find the value of A.)

Answers

1. a) $\{4\}$ b) $\{1,2,4,6,7,9\}$
   c) $\{1,2,3,4,5,6,7,8,9,10\}$ d) $\{5,6,7\}$

2. a) true b) false c) true d) true
   e) false f) true g) true h) false

3. a) There are 64 numbers as listed below.

222	252	272	292	522	552	572	592
225	255	275	295	525	555	575	595
227	257	277	297	527	557	577	597
229	259	279	299	529	559	579	599
722	752	772	792	922	952	972	992
725	755	775	795	925	955	975	995
727	757	777	797	927	957	977	997
729	759	779	799	929	959	979	999

   b) There are 24 such numbers.

257	527	725	925
259	529	729	927
275	572	752	952
279	579	759	957
295	592	792	972
297	597	795	975

   c) There are 12 such numbers

279	579	729	927
297	597	759	957
		792	972
		795	975

4. 118 R 11

5. $1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84$

6. a) 11 060 904, 3106 b) 11 060 904, 321
   c) 11 060 904

7. 151

8. 360

9. $\text{lcm}\,(144, 630) = 5040$ $\gcd(144, 630) = 18$

10. a) $2^{200} \cdot 3^{100}$ b) $2^8 \cdot 3^4 \cdot 5^2 \cdot 7$

11. a) true b) false (1 is the only exception, though)
    c) true d) false e) true f) true g) true

12. $720 = 2^4 \cdot 3^2 \cdot 5$

13. a) $P = 54\,\text{m}$ b) $A = 180\,\text{m}^2$

14. a) \$616 b) \$1840

15. a) -9 b) -6 c) 8 d) 14 e) 22
    f) 3 g) -7 h) undefined i) -4 j) 4

16. a) 33 b) 6 c) -12 d) 6 e) 3
    f) 4 g) $\frac{59}{70}$ h) $\frac{5}{16}$ i) -30 j) 1
    k) 13 l) 33

17. a) $10b$ b) $2x + 5y$ c) $8m - 4n$

18. a) $4a - 4b$ b) $4x - 7y + 4$ c) $-2m - 4n$

19. a) $-4a^2 + 8ab + 21b^2$ b) $-3x^2 + 19xy - 6y^2$
    c) $-20mn + 15m^2$

20. a) $-16a + 26b$ b) $12x - 38y + 16$
    c) $6x^2 + 3xy - 20y^2$ d) $-x + 3y$ e) $-9m^2 + 32mn$
    f) 0

21. a) $3x^2 - 2x - 5$ b) $9x^2 - 6xy + y^2$
    c) $15a^2b - 20ab^2$ d) $9x^{10} - 4$ e) $25x^2 - 4$
    f) $25x^2 - 20x + 4$ g) $9a^6 - 6a^3 + 1$ h) $27m^3 - 8$
    i) $27x^3 - 27x^2 + 9x - 1$

22. a) $x^2 + 11x + 1$ b) $-x^2 - 19x + 12$ c) $9x - 29$
    d) $x^2 - 3x - 19$

23. a) 11 b) -9 c) undefined d) -1 e) 0

24. a) no, $-44 \neq -2$ b) yes, $4 = 4$
    c) yes, $-4 = -4$ d) no, $\frac{79}{4} \neq \frac{19}{4}$
    e) yes, $-\frac{5}{4} = -\frac{5}{4}$

25. a) -5 b) $-\frac{2}{3}$ c) 2 d) 18 e) 3 f) 6
    g) 14 h) 5 i) 0 j) -4 k) 1 l) 0
    m) -2 n) 7 o) 1 p) -27

26. 8

Last revised: June 15, 2017