

1. Perform each of the divisions with remainder.

a) $2017 \div 31$ b) $99 \div 7$ c) $1355 \div 24$

2. Find the last digit of each of the following.

a) 7^{2018} b) 5^{99} c) 9^{2017} d) 2^{123}

3. Suppose that $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 3, 5, 7, 9\}$. Find each of the following.

a) $A \cup B$ b) $A \cap B$

4. Suppose that $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$. Let $P = \{x \in U : x \text{ is divisible by } 2\}$ and $Q = \{x \in U : x \text{ is divisible by } 3\}$.

a) Draw a Venn-diagram depicting U , P , and Q . (U is the universal set, depicted as the rectangle containing the sets P and Q).

Find each of the following. d) $\{x \in U : x \text{ is divisible by } 5\}$ g) $\{w \in U : w < 2 \text{ and } w > 10\}$

b) $P \cap Q$ e) $\{y \in U : y \text{ is a prime number}\}$ h) $\{u \in U : u > 5 \text{ or } u \geq 8\}$

c) $P \cup Q$ f) $\{z \in U : z < 2 \text{ or } z > 10\}$ i) $\{u \in U : u > 5 \text{ and } u \geq 8\}$

5. List all two-element subsets of $A = \{1, 2, 3, 4, 5\}$.

6. List all factors of 60.

7. Which of the following numbers is a prime number?

39, 91, 71, 45, 81, 201

8. Consider the following numbers: 72, 99, 40, 150, 135, 190, 360

a) List all the numbers from the given list that are divisible by 2.

b) List all the numbers from the given list that are divisible by 3.

c) List all the numbers from the given list that are divisible by 5.

d) List all the numbers from the given list that are divisible by 9.

9. Find the prime factorization of each of the given numbers.

a) 135 b) 825 c) 3600 d) x where $x = 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9$

10. Label each of the following statements as true or false.

a) 3 is a multiple of 3.

f) Every rectangle is a square.

b) $4 < 4$

g) Every square is a rectangle.

c) $5 \geq 5$

h) Every number divisible by 12 is also divisible by 6.

d) 14 is a multiple of 4 or 7 is a prime number

e) 14 is a multiple of 4 and 7 is a prime number

i) $\mathbb{N} \subseteq \mathbb{Z}$

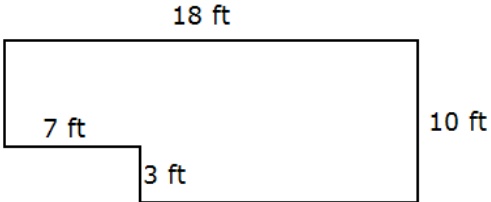
11. Perform the operations as indicated. Show all steps.

a) $(-2)^2$ d) $-2(-5)$ g) $20 - 3(-8)$

b) -2^2 e) $-2 - (-5)$ h) $\frac{(-2)^3 - 5(-3) - (-1)^4 + (-3)^2}{-2^2 - (-1)}$

c) $-2 - 5$ f) $-2(-(-5))$ i) $-3^2 - (-24) \div (5 - (-1)^3) \cdot 2$

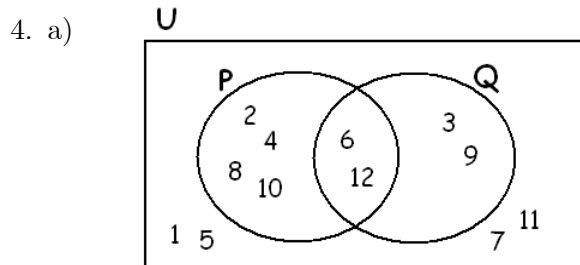
12. Compute the sum $-1 + (-1)^2 + (-1)^3 + (-1)^4 + \dots + (-1)^{100}$

13. Simplify each of the following expressions by applying the order of operations agreement. **Show all steps. Perform only one operation in each step.**
- a) $7 \cdot 3^2 - (3 - 2^2 \cdot 5 - 1) \div 2$ g) $\frac{4^2 + 5^2 - 6 \div 2 \cdot 3}{4^2 - 8 \cdot 2}$ m) $|-10 - 7| - |1 - 4|$
- b) $\frac{5 - 1 + 2}{-1^2 + (-1)^2}$ h) $3 + 2(5 + 3(15 - 2^3) - 2^2 - 1)$ n) $|-10 - 7 - |1 - 4||$
- c) $\sqrt{169 - 144}$ i) $4(3(2(2^2 - 1) - 1) - 1) + 5$ o) $|-10 - 7|1 - 4||$
- d) $\sqrt{169} - \sqrt{144}$ j) $\sqrt{\sqrt{36} + 5\sqrt{9} - \sqrt{25}}$ p) $|-10 - |7 - 1 - 4||$
- e) $2^3 - 2(5 - (-3)^2)^2$ k) $-2^2 - 3(5 - (-2)^2) - (-1)^3$ q) $|-10||-7 - 1 - 4||$
- f) $\left(\left((8 - 5)^2 - 7\right)^2 - 2\right)^2 - 1$ l) $-2 - 5(-3^2 - 2(-7))$ r) $\sqrt{4\sqrt{64} - \sqrt{49}}$
14. Let $p = 4$, $q = -3$, and $s = 1$. Evaluate each of the following expressions.
- a) $\frac{p - q - s}{p + q + s}$ b) $\frac{2p - q}{p - (s - q)}$ c) $p^2 - 2s^2$ d) $p^2 - (2s)^2$ e) $2pq^2$
15. Suppose that $x = 4$ and $y = -3$. Evaluate each of the algebraic expressions.
- a) $2x - y + 1$ b) $-y^2 - 3x^2y$ c) $\sqrt{x^2 + y^2}$ d) $5x - 2y + 2x + y$ e) $\left|\frac{x^2 - y^2}{y^2 - x^2}\right|$
16. Consider a rectangle with sides 18 cm and 40 cm long.
- a) Compute the perimeter of the rectangle in centimeters. Include units in your computation and answer.
- b) Compute the area of the rectangle in square centimeters. Include units in your computation and answer.
17. Consider the object shown on the picture.
- a) Compute the perimeter of the object in feet. Include units in your computation and answer.
- b) Compute the area of the object in square feet. Include units in your computation and answer.
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18. Consider the equation $x^3 - x^2 + 7 = x^2 + 5x + 1$. Which of the given numbers are solutions of the equation? The given numbers: 0, 1, -1, 2, and -2
19. Consider the inequality $x^2 + 3x \leq x + 24$. Which of the given numbers are solutions of the inequality? The given numbers: 5, 6, 0, -10, 3, and 4
- 20*. (Enrichment) Two mathematicians are having a conversation. Mathematician A asks B about his kids. B answers: "I have three children, the product of their ages is 36." A says: "I still don't know how old your children are." Then B tells A the sum of his three kids' ages. A answers: "I still don't know how old they are. Then B adds: "The youngest one has red hair." Now A knows how old the kids are. Do you?
- 21*. (Enrichment) How many diagonals are there in a regular polygon of n sides?

Answers

1. a) 65 R 2 b) 14 R 1 c) 56 R 11

2. a) 9 b) 5 c) 9 d) 8

3. a) $\{1, 2, 3, 4, 5, 7, 9\}$ b) $\{1, 3, 5\}$ 

b) $\{6, 12\}$ c) $\{2, 3, 4, 6, 8, 9, 10, 12\}$
 d) $\{5, 10\}$ e) $\{2, 3, 5, 7, 11\}$ f) $\{1, 11, 12\}$
 g) \emptyset h) $\{6, 7, 8, 9, 10, 11, 12\}$
 i) $\{8, 9, 10, 11, 12\}$

5. There are 10 two-element subsets as shown below.

$\{1, 2\}$
 $\{1, 3\}$ $\{2, 3\}$
 $\{1, 4\}$ $\{2, 4\}$ $\{3, 4\}$
 $\{1, 5\}$ $\{2, 5\}$ $\{3, 5\}$ $\{4, 5\}$

6. 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

7. 71

8. a) 72, 40, 150, 190, 360

b) 72, 99, 150, 135, 360

c) 40, 150, 135, 190, 360

d) 72, 99, 135, 190, 360

9. a) $3^3 \cdot 5$ b) $3 \cdot 5^2 \cdot 11$ c) $2^4 \cdot 3^2 \cdot 5^2$ d) $2^4 \cdot 3^3 \cdot 5 \cdot 7$

10. a) true b) false c) true d) true

e) false f) false g) true h) true i) true

11. a) 4 b) -4 c) -7 d) 10 e) 3f) -10 g) 44 h) -5 i) -1

12. 0

13. a) 72 b) undefined c) 5 d) 1 e) -24

f) 3 g) undefined h) 45 i) 61 j) 4

k) -6 l) -27 m) 14 n) 20 o) 31

p) 12 q) 120 r) 5

14. a) 3 b) undefined c) 14 d) 12 e) 72

15. a) 12 b) 135 c) 5 d) 31 e) 1

16. a) $P = 116 \text{ cm}$ b) $P = 720 \text{ cm}^2$ 17. a) $P = 56 \text{ ft}$ b) $A = 159 \text{ ft}^2$ 18. -2 and 1

19. 0, 3, and 4