

1. Simplify each of the following expressions.

a) $(\sqrt{3})^4$ b) $(\sqrt[6]{5})^{12}$ c) $(-27)^0$ d) -27^0 e) -2^{-3} f) -3^{-2} g) $\frac{1}{7^{-2}}$

2. Perform the given set of operations on the intervals.

a) $(-1, 5) \cup [3, 8]$ c) $(-\infty, 14] \cup (-3, \infty)$ e) $(-\infty, -2) \cup [5, \infty)$
 b) $(-1, 5) \cap [3, 8]$ d) $(-\infty, 14] \cap (-3, \infty)$ f) $(-\infty, -2) \cap [5, \infty)$

3. Simplify each of the following expressions. Assume that all variables represent positive numbers.

a) $\frac{x^3 y^0 (-x^4 y^{-2} x^{-2})^{-3}}{(2x)^{-2} y x^0}$ d) $\frac{1}{x^{-1}}$ g) $\frac{x^{-1} - y^{-1}}{x^{-2} - y^{-2}}$ j) $(-16)^{3/2}$
 b) $(x^{-3})^{-5} x^{-6}$ e) $\frac{x}{\sqrt{x}}$ h) $\left(\frac{1 - x^{-3}}{3x^{-2}}\right)^0$ k) $\sqrt{x^3 \sqrt{x^4 \sqrt{x}}}$
 c) $\sqrt{\frac{x^3}{x^2 (x^{-3})^5}}$ f) $2^{-1} - 5^{-1}$ i) $16^{-3/4}$ l) $\frac{a^{3/4} b^{-1/2}}{-a^{-1/4} b^{3/2}}$

4. a) Solve the given equation over the real numbers by completing the square. $3x^2 = 4x + 1$

b) Use exact values to check your solution(s).

5. Simplify each of the following expressions.

a) $\frac{(x-5)^2}{x^2-25}$ b) $\frac{x^2-5x}{x^2-2x-15} \cdot \frac{x^2-9}{x^2-3x}$ c) $\frac{x^2-4x-12}{x^2-2x-8} \div \frac{x^2-6x}{x^2-16}$

6. Simplify each of the following. If the denominator is irrational, rationalize it.

a) $(7 - 3\sqrt{2})^2$ d) $\frac{4x-9}{2\sqrt{x}-3}$ f) $\frac{-6 + \sqrt{24}}{-10}$ h) $\frac{1}{1 + \sqrt{3}} - \frac{1}{1 - \sqrt{3}}$
 b) $(2 - \sqrt{5})^3 (2 + \sqrt{5})^3$
 c) $\frac{2}{\sqrt{29} + 5}$ e) $3\sqrt{8} + 2\sqrt{18} - \sqrt{50}$ g) $\frac{\sqrt{5}-1}{-\sqrt{5}+1}$ i) $\left(\sqrt{6 - \sqrt{11}} - \sqrt{6 + \sqrt{11}}\right)^2$

7. Use rational exponents to simplify each of the following. Present your answer using only positive integer exponents. Assume that all variables represent positive numbers.

a) $\frac{\sqrt{x}}{\sqrt[3]{x}}$ b) $\sqrt[3]{x^2} \sqrt{x^3}$ c) $\sqrt[5]{\sqrt[4]{\sqrt[3]{x}}}$ d) $\sqrt[3]{\frac{x^2 \sqrt[4]{x^7}}{x\sqrt{x}}}$ e) $\sqrt[5]{x \sqrt[4]{x^2} \sqrt[3]{x^7}}$

8. Evaluate the expression $-x^2 + 3x + 8$ if

a) $x = -3$ b) $x = -\frac{1}{2}$ c) $x = 3\sqrt{2} - 1$

9. Find the greatest or smallest value of each of the given expressions. State which (smallest or greatest) in each expression.

a) $3x^2 - 24x + 45$ b) $-2x^2 - 4x + 16$ c) $5x^2 - 30x + 55$ d) $-\frac{1}{2}x^2 + 4x - 8$

10. Completely factor each of the following binomials over the real numbers. If an expression cannot be factored, state so.

a) $x^2 + x$ b) $x^3 - x$ c) $x^4 + x^3$ d) $x^5 + x^3$ e) $x^6 - x^4$ f) $x^7 - x^3$

11. Completely factor each of the following trinomials over the real numbers. If an expression cannot be factored, state so.

a) $x^2 - 6x + 5$ c) $x^2 - 6x + 13$ e) $3x^2 - 14x - 5$
 b) $x^2 - 6x + 9$ d) $x^2 - 6x + 7$ f) $x^2 - x - 1$

12. Completely factor $x^2 - 9$ a) over \mathbb{Z} b) over \mathbb{R}

13. Completely factor $x^2 - 5$ a) over \mathbb{Z} b) over \mathbb{R}

14. Completely factor $x^2 + 4$ a) over \mathbb{Z} b) over \mathbb{R}

15. Solve each of the following system of equations.

a)
$$\begin{cases} (x-2)^2 + (y-3)^2 = x^2 + (1-y)^2 \\ x = 3y - 1 \end{cases}$$

b)
$$\begin{cases} \frac{1}{2}(x+3) + \frac{1}{3}(y-2) = x - 4 \\ \frac{1}{6}(x-1) - \frac{1}{2}(y+8) = y + 3 \end{cases}$$

16. Solve each of the given system of linear equations.

a)
$$\begin{cases} 5x - 2y - z = 29 \\ -2x + 3y - 4z = 6 \\ \frac{2}{3}x - \frac{1}{2}y - \frac{1}{3}z = 4 \end{cases}$$

b)
$$\begin{cases} -2x + y - 3z = 7 \\ x - 3y + z = -4 \\ 4x + 2y - 2z = -22 \end{cases}$$

17. Solve each of the following compound inequalities.

a) $-\frac{1}{3}x + 1 < -8$ and $(x+1)^2 - (x-1)^2 \leq -16$ c) $(2x-5)^2 > 4x^2 - 15x$ and $\frac{1}{3}x + 1 > \frac{5}{6}$
 b) $-\frac{1}{3}x + 1 < -8$ or $(x+1)^2 - (x-1)^2 \leq -16$ d) $(2x-5)^2 > 4x^2 - 15x$ or $\frac{1}{3}x + 1 > \frac{5}{6}$

18. Solve each of the following equations over the real numbers. Make sure to check your solutions.

a) $\frac{5-4x}{3} - \frac{2x-7}{5} = -2x + 2$ e) $2x^2 - 32x = 0$ i) $5 - (2-x)(x+3) = (x-2)^2$
 b) $x^3 - 2x^2 - 35x = 0$ f) $4x + x^3 = 6x^2$ j) $1 + 4\sqrt{x-2} = -x$
 c) $(3x-2)^2 = (x-2)^2$ g) $x^2 + 134 = 22x$ k) $\sqrt{10x-1} = 2x + 1$
 d) $2(x-3) - \frac{x}{2} = \frac{3}{2}(x-4)$ h) $2 = x - \sqrt{3x+4}$ l) $\sqrt{x+3} = 3-x$

19. In each case, graph the parabola given. State the coordinates of at least five points, including vertex and intercepts.

a) $y = x^2 + 2x - 3$ b) $y = -2x^2 + 4x + 6$

20. Re-write each of the following decimals as a fraction of two integers. You do NOT have to bring the fraction to lowest terms.

a) $0.65\bar{2} = 0.65222222\dots$ b) $0.8\bar{79} = 0.87979797979\dots$ c) $0.99\bar{2016} = 0.99201620162016\dots$

21. Compute the sum $91 + 103 + 115 + \dots + 715$

22. A 6 ft tall person is standing 20 ft away from a street light that is 21 ft tall. How long is her shadow?

23. Express two consecutive 30% increases as a single change. What percentage of a change does this represent?

24. Compute the distance between the points $A(-3, -7)$ and $B(2, 5)$.

25. Suppose that a triangle is determined by the points $A(-5, -2)$, $B(3, 8)$, and $C(-1, 14)$. Find an equation for each of the following.

- a) The perpendicular bisector of line segment AB .
 b) The altitude belonging to side AC

26. In each of the cases given, compute the slope of the line.

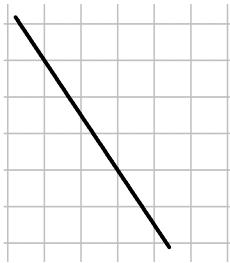
a) a horizontal line

c) the line passing through the points $A(3, -1)$ and $B(-5, 7)$

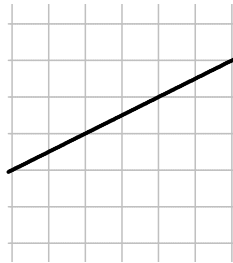
b) a vertical line

d) the line passing through the points $N(-3, 4)$ and $M(-3, 7)$

e)



f)



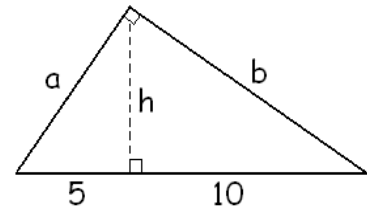
g)



27. Find the value of k if the slope of the line connecting the points $A(3, -1)$ and $B(8, k)$ is 2.

28. Find the exact values of a , b , and h in the triangle seen on the picture.

29. One third of a rope is 6 feet longer than one fifth of the rope. How long is the rope?



30. We are standing on the top of a 960 feet tall building and launch a small object upward. The object's vertical position, measured in feet, after t seconds is

$$h(t) = -16t^2 + 64t + 960$$

a) How long until the object hits the ground?

b) What is the highest point that the object will reach? When?

31. Find the equation of the parabola if we know that the points $A(-2, -10)$, $B(4, 2)$, and $C(6, -2)$ are on its graph.

32. The budget increased by 15%. If the new budget is 1811 250, how much was the old budget?

33. The first row in a theater has 36 seats in it. The second row has four more seats than the first row. The third row has four more seats than the second row. And so on, each row has four more seats than the row before. If the last row has 168 seats in it, how many seats are there in the entire theater?

34. The sum of five times a number and -10 is 8 less than six times the sum of 7 and the opposite of the number. Find this number.

35. We have invested \$8000 into two bank accounts: one earns 6% interest, the other one earns 9% interest. How much money did we invest into each account if the combined interest was \$660?

36. The hypotenuse of a right triangle is 26 cm. The difference between the other two sides is 14 cm. Find the missing sides.

37. Lisa took 5 exams. The first 4 received scores of 72, 93, 86, and 82. How much did she score on the fifth exam if her average score is 74 points?

38. A number is exactly two greater than its own reciprocal. Find this number.

39. The base of a right triangle is 8 units longer than its height. Find the base if we know that the area of the triangle is 120 unit^2 .

40. A company finds that if they set the price of their new product at \$80, then 3200 items will be sold. For every dollar increase in the price, 10 less tickets would be purchased. What is the greatest revenue possible with this product? What price would guarantee this maximum income?

Answers

1. a) 9 b) 25 c) 1 d) -1 e) $-\frac{1}{8}$ f) $-\frac{1}{9}$ g) 49

2. a) $(-1, 8]$ b) $[3, 5)$ c) $(-\infty, \infty)$ d) $(-3, 14]$ e) $(-\infty, -2) \cup [5, \infty)$ f) \emptyset

3. a) $-\frac{4y^5}{x}$ b) x^9 c) x^8 d) x e) \sqrt{x} f) $\frac{3}{10}$ g) $\frac{xy}{x+y}$ h) 1 i) $\frac{1}{8}$ j) undefined k) $\sqrt[24]{x^{17}}$ l) $-\frac{a}{b^2}$

4. a) $\frac{2-\sqrt{7}}{3}$ and $\frac{2+\sqrt{7}}{3}$ b) If $x = \frac{2-\sqrt{7}}{3}$, then

$$\text{LHS} = 3 \left(\frac{2-\sqrt{7}}{3} \right)^2 = 3 \cdot \frac{(2-\sqrt{7})^2}{3^2} = \frac{3(11-4\sqrt{7})}{9} = \frac{11-4\sqrt{7}}{3}$$

$$\text{RHS} = 4 \left(\frac{2-\sqrt{7}}{3} \right) + 1 = \frac{4(2-\sqrt{7})}{3} + \frac{3}{3} = \frac{8-4\sqrt{7}+3}{3} = \frac{11-4\sqrt{7}}{3}$$

Checking the other solution goes similarly.

5. a) $\frac{x-5}{x+5}$ b) 1 c) $\frac{x+4}{x}$

6. a) $67 - 42\sqrt{2}$ b) -1 c) $\frac{\sqrt{29}-5}{2}$ d) $2\sqrt{x}+3$ e) $7\sqrt{2}$ f) $\frac{3-\sqrt{6}}{5}$ g) -1 h) $\sqrt{3}$ i) 2

7. a) $\sqrt[6]{x}$ b) $\sqrt[6]{x^{13}}$ c) $\sqrt[60]{x}$ d) $\sqrt[4]{x^3}$ e) $\sqrt[30]{x^{13}}$ 8. a) -10 b) $\frac{25}{4}$ c) $15\sqrt{2}-14$

9. a) -3 smallest value b) 18 greatest value c) 10 smallest value d) 0 greatest value

10. a) $x(x+1)$ b) $x(x-1)(x+1)$ c) $x^3(x+1)$ d) $x^3(x^2+1)$ e) $x^4(x-1)(x+1)$ f) $x^3(x^2+1)(x-1)(x+1)$

11. a) $(x-1)(x-5)$ b) $(x-3)^2$ c) cannot be factored d) $(x-3-\sqrt{2})(x-3+\sqrt{2})$ e) $(3x+1)(x-5)$

f) $\left(x - \frac{1+\sqrt{5}}{2}\right) \left(x - \frac{1-\sqrt{5}}{2}\right)$ 12. a) $(x+3)(x-3)$ b) $(x+3)(x-3)$

13. a) cannot be factored b) $(x+\sqrt{5})(x-\sqrt{5})$ 14. a) cannot be factored b) cannot be factored

15. a) (2, 1) b) (7, -4) 16. a) $x=6, y=2, z=-3$ b) $x=-5, y=0, z=1$

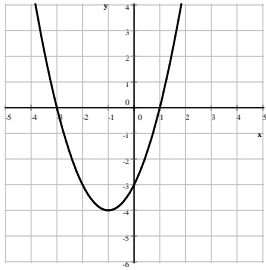
17. a) no solution b) $(-\infty, -4] \cup (27, \infty)$ c) $\left(-\frac{1}{2}, 5\right)$ d) \mathbb{R}

18. a) -4 b) 7, 0, -5 c) 1, 0 d) identity, all numbers are solution e) 0, 16 f) 0, $3-\sqrt{5}$, $3+\sqrt{5}$

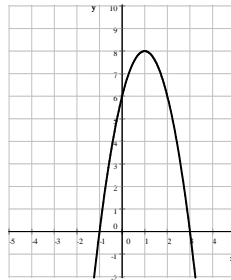
g) no real solution h) 7 (0 is extreme) i) 1 j) no solution (both 3 and 11 are extreme)

k) $\frac{1}{2}, 1$ l) 1 (6 is extreme)

19. a) $y = x^2 + 2x - 3 = (x + 1)^2 - 4$
 $= (x + 3)(x - 1)$ vertex: $(-1, -4)$
 x -intercepts: $(-3, 0)$ and $(1, 0)$
 y -intercept: $(0, -3)$



b) $y = -2x^2 + 4x + 6 = -2(x - 1)^2 + 8$
 $= -2(x + 1)(x - 3)$ vertex: $(1, 8)$
 x -intercepts: $(-1, 0)$ and $(3, 0)$
 y -intercept: $(0, 6)$



20. a) $\frac{587}{900}$ b) $\frac{871}{990}$ c) $\frac{991\,917}{999\,900}$ 21. 21 359 22. 8 ft 23. 69% increase 24. 13 units

25. a) $-\frac{4}{5}(x + 1) = y - 3$ b) $-\frac{1}{4}(x - 3) = y - 8$

26. a) 0 b) undefined c) -1 d) undefined e) $-\frac{3}{2}$ f) $\frac{1}{2}$ g) -1 27. 9 28. $a = 5\sqrt{3}$, $b = 5\sqrt{6}$, $h = 5\sqrt{2}$

29. 45 ft 30. a) 10 seconds b) 1024 ft, after 2 seconds 31. $y = -\frac{1}{2}x^2 + 3x - 2$ 32. 1575 000 33. 3468

34. 4 35. \$2000 at 6% and \$6000 at 9% 36. 10 cm and 24 cm 37. 37 38. $1 + \sqrt{2}$ and $1 - \sqrt{2}$

39. 20 unit 40. \$400 000, with a price of \$200