

1. Label each of the following statements as true or false.

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| a) 3 is a multiple of 3. | h) If a number n is divisible by 2 and 3, then it is also divisible by 6. |
| b) $4 < 4$ | i) If a number n is divisible by 4 and 6, then it is also divisible by 24. |
| c) $5 \geq 5$ | j) Every number divisible by 12 is also divisible by 6. |
| d) $-2 \notin \mathbb{Z}$ | k) Every number divisible by 6 is also divisible by 12. |
| e) $-2 \notin \mathbb{N}$. | l) For all sets A , $\emptyset \subseteq A$. |
| f) 14 is a multiple of 4 or 21 is divisible by 7. | m) $\mathbb{Z} \subseteq \mathbb{N}$ |
| g) 14 is a multiple of 4 and 21 is divisible by 7. | |

2. List all factors of 90.

3. Consider the given numbers. 101010, 1189 188, 35530, 1234 321, 20172017. List all numbers from the list that are divisible by: a) 4 b) 6 c) 9

4. Perform the given division with remainder. $2018 \div 7$

5. Suppose $A = \{2, 3, 4, 5, 7, 8\}$, and $B = \{2, 4, 6, 8, 10\}$, and $C = \{1, 3, 8, 10\}$.

a) Draw a Venn-diagram depicting A and B .

Find each of the following.

b) $(A \cup B) \cap C$ c) $A \cup (B \cap C)$ d) $(A \cup C) \cap B$

6. Recall the following definitions. A **rectangle** is a four-sided polygon with four right angles. A **square** is a rectangle with four equal sides. Let R be the set of all rectangles and S the set of all squares.

a) Label each of the following statements as true or false.

i) Every square is a rectangle. iii) $R \subseteq S$

ii) Every rectangle is a square. iv) $S \subseteq R$

b) Describe x if we know that $x \in R$ and $x \notin S$.

7. Find each of the following sets and if possible, present them by listing their elements.

a) $A = \{a \in \mathbb{N} \mid a < 6\}$ c) $C = \{c \in \mathbb{N} \mid c < 7 \text{ or } c > 3\}$

b) $B = \{b \in \mathbb{N} \mid b < 7 \text{ and } b > 3\}$ d) $D = \{x \in \mathbb{N} \mid x \leq 10 \text{ and } x \text{ is even}\}$

8. Let $A = \{n \in \mathbb{N} : n \text{ is divisible by } 2\}$, $B = \{n \in \mathbb{N} : n \text{ is divisible by } 6\}$. Which (if any) of the following is true?
 $A \subseteq B$ $B \subseteq A$

9. There are thirty students in our Math 99 class. Twenty of them also takes English 101, fifteen of them also takes Speech 101, and ten of them takes both English 101 and Speech 101. How many of the students in Math 99 are taking neither English 101 nor Speech 101? (Hint: draw a Venn Diagram!)

10. The conference had 150 attendees, 65 men and 85 women. When greeting each other, people shook each other's hand. How many handshakes took place if

a) everyone shook hands with everyone else

b) men only shook every other men's hand, women only shook every other women's hand.

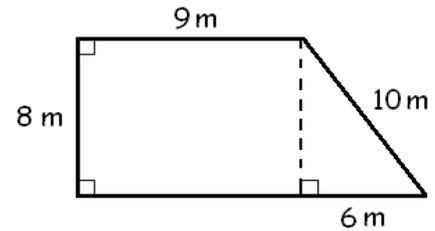
c) every man shook every women's hand.

d*) all 65 men came with their wives, everyone shook hands with everyone else, except for people they came together (i.e. no men shook hands with their own wife.

11. a) List all subsets of $S = \{1, 2, 3, 4\}$
 b) List all two-element subsets of $T = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
12. Evaluate each of the given numerical expressions.
- a) $-2 - 3$ e) $(-2)^2$ i) -2^2 m) $3^2 - 7^2$ q) $10 - 3(-8)$ t) $(10 - 3)(-8)$
 b) $-2(-3)$ f) $(-2)^3$ j) -2^3 n) $(3 - 7)^2$ r) $10 - (3 - 8)$ u) $10(-3 - 8)$
 c) $-(2 - 3)$ g) $(-2)^4$ k) -2^4 o) $3^2 + 7^2$ s) $10(-3 - 8)$
 d) $(-2) - 3$ h) $(-2)^5$ l) -2^5 p) $(3 + 7)^2$
13. Evaluate each of the given numerical expressions.
- a) $(-2)^2$ e) $5(-2)^2$ i) $2^2 + 5^2$ m) $(2 - 5)^2$ q) $(2^2 - 5)^2$ u) $(2 - (-5)^2)$
 b) -2^2 f) $5 - 2^2$ j) $(2 + 5)^2$ n) $(2 - 5^2)$ r) $2^2 - (-5)^2$ v) $2^2(-(-5)^2)$
 c) (-2^2) g) $5(-2^2)$ k) $(2 + 5^2)$ o) $2^2(-5)^2$ s) $2^2 - (-5^2)$ w) $2^2(-(-5)^2)$
 d) $-(2)^2$ h) $5 - (2)^2$ l) $2^2 - 5^2$ p) $2^2(-5^2)$ t) $(2 - (-5))^2$ x) $2^2(-(-5))^2$
14. Simplify each of the following expressions by applying the order of operations agreement. **Show all steps. Perform only one operation in each step.**
- a) $7 \cdot 3^2 - (3 - 2^2 \cdot 5 - 1) \div 2$ f) $\left(\left(\left(8 - 5\right)^2 - 7\right)^2 - 2\right)^2 - 1$ k) $-2^2 - 3(5 - (-2)^2) - (-1)^3$
 b) $\frac{5 - 1 + 2}{-1^2 + (-1)^2}$ g) $\frac{4^2 + 5^2 - 6 \div 2 \cdot 3}{4^2 - 8 \cdot 2}$ l) $-2 - 5(-3^2 - 2(-7))$
 c) $\frac{(-2)^3 - 5(-3) - (-1)^4 + (-3)^2}{-2^2 - (-1)}$ h) $3 + 2(5 + 3(15 - 2^3) - 2^2 - 1)$ m) $|-10 - 7| - |1 - 4|$
 d) $|3 - 8| - (|3| - |8|)$ i) $4(3(2(2^2 - 1) - 1) - 1) + 5$ n) $|-10 - 7 - |1 - 4||$
 e) $2^3 - 2(5 - (-3)^2)^2$ j) $-3^2 - (-24) \div (5 - (-1)^3) \cdot 2$ o) $|-10 - 7||1 - 4||$
 p) $|-10 - |7 - 1 - 4||$
 q) $|-10||-7 - 1 - 4||$
15. Perform the indicated operations. Show all steps.
- a) $8 - 2(7 - 3^2 + 1) + 5$ b) $\left((3^2 - 13)^2 - 11\right)^2 - 1^3$ c) $\sqrt{19 - \sqrt{2\sqrt{25} - 1}}$
16. Let $p = 4$, $q = -3$, and $s = 1$. Evaluate each of the following expressions.
- a) $-q^2 - pq$ b) $\frac{2p - q}{p - (s - q)}$ c) $p^2 - 2s^2$ d) $p^2 - (2s)^2$ e) $\sqrt{|p - q| - 3s}$ f) $p - |q - 3s|$
17. Suppose that $x = 4$ and $y = -3$. Evaluate each of the algebraic expressions.
- a) $2x - y + 1$ b) $-y^2 - 3x^2y$ c) $(-x)^2 - 5y$ d) $5x - 2y + 2x + y$ e) $\left|\frac{x^2 - y^2}{y^2 - x^2}\right|$
18. Consider the equation $x^3 - x^2 + 7 = x^2 + 5x + 1$. Which of the given numbers are solutions of the equation? The given numbers: 0, 1, -1, 2, and -2

19. Consider the inequality $x^2 + 3x \leq x + 24$. Which of the given numbers are solutions of the inequality? The given numbers: 5, 6, 0, -10, 3, and 4

20. Compute the perimeter and area of the figure shown. Include units in your computation and answer.



21. Translate each of the following sentences into algebraic statements.

- a) X is three less than twice Y .
- b) The opposite of x is five greater than the sum of y and half of z .
- c) The product of a and b is fourteen less than three times the sum of a and b .
- d) The sum of m and twice n is one greater than the quotient of m and n .
- e) Three times the difference of x and y is one less than the product of x and the opposite of y .
- f) Suppose that Peter's age is ten less than twice the age of his younger brother, David. If Peter's age is denoted by P and David's age is denoted by D , write an algebraic statement expressing P in terms of D .
- g) Suppose that a cab-fare for one person in Chicago is 2.50 dollars for the first mile and 1.5 dollars for each additional mile. Write an algebraic expression for the price of a cab ride in Chicago of a distance of x miles.
- h) We are thinking of three consecutive numbers. Express them in terms of x if x represents the smallest number.
- i) The longer side of a rectangle is three feet shorter than four times the shorter side. If the shorter side of the rectangle is denoted by x , express the area of the rectangle in terms of x .
- j) Suppose that Ann has A dollars and Beatrix has B dollars. The girls made a bet that Ann lost, so she must pay Beatrix 30 dollars. Express how much money do the girls have after Ann paid.

22. What is the last digit of 7^{2019} ?

23. Compute each of the following.

- a) $-100 + (-99) + (-98) + \dots + 98 + 99 + 100 + 101 + 102$
- b) $-100(-99)(-98)\dots \cdot 98 \cdot 99 \cdot 100 \cdot 101 \cdot 102$

24. (Enrichment) Two mathematicians are having a conversation. Mathematician A asks B about his kids. B answers: "I have three children, the product of their ages is 36." A says: "I still don't know how old your children are." Then B tells A the sum of his three kids' ages. A answers: "I still don't know how old they are. Then B adds: "The youngest one has red hair." Now A knows how old the kids are. Do you?

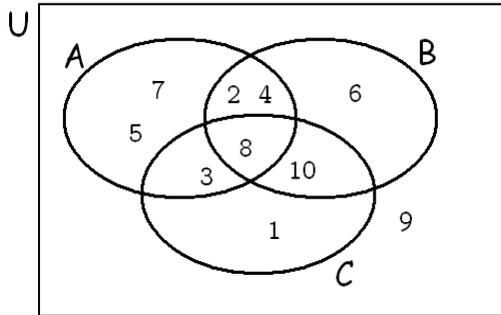
Answers

1. a) true b) false c) true d) false e) true f) true g) false h) true i) false j) true
k) false l) true m) false

2. 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90

3. a) 1189188 b) 101010, 1189188 c) 1189188 4. 288 R 2

5. a) b) {3, 8, 10} c) {2, 3, 4, 5, 7, 8, 10} d) {2, 4, 8, 10}



6. a) i) true ii) false iii) false iv) true
b) x is a rectangle that is NOT a square, i.e. a rectangle that has two sides with different lengths.

7. a) {1, 2, 3, 4, 5} b) {4, 5, 6}
c) \mathbb{N} (all natural numbers) d) {2, 4, 6, 8, 10}

8. $B \subseteq A$ 9. 5

10. a) 11175 b) 5650 c) 5525 d) 11110

11. a) all subsets of $S = \{1, 2, 3, 4\}$

\emptyset ,

{1}, {2}, {3}, {4},

{1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4},

{3, 4},

{1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}

{1, 2, 3, 4}

- b) all two-element subsets of $T = \{1, 2, 3, 4, 5, 6, 7\}$

{1, 2}

{1, 3} {2, 3}

{1, 4} {2, 4} {3, 4}

{1, 5} {2, 5} {3, 5} {4, 5}

{1, 6} {2, 6} {3, 6} {4, 6} {5, 6}

{1, 7} {2, 7} {3, 7} {4, 7} {5, 7} {6, 7}

12. a) -5 b) 6 c) 1 d) -5 e) 4 f) -8 g) 16 h) -32 i) -4 j) -8 k) -16

l) -32 m) -40 n) 16 o) 58 p) 100 q) 34 r) 15 s) -110 t) -56 u) 50

13. a) 4 b) -4 c) -4 d) -4 e) 20 f) 1 g) -20 h) 1 i) 29 j) 49 k) 27 l) -21 m) 9

n) -23 o) 100 p) -100 q) 1 r) -21 s) 29 t) 49 u) -23 v) 100 w) -100 x) 100

14. a) 72 b) undefined c) 5 d) 10 e) -24 f) 3 g) undefined h) 45 i) 61 j) -1 k) -6

l) -27 m) 14 n) 20 o) 31 p) 12 q) 120

15. a) 15 b) 24 c) 4 16. a) 3 b) undefined c) 14 d) 12 e) 2 f) -2

17. a) 12 b) 135 c) 5 d) 31 e) 1 18. -2 and 1 19. 0, 3, and 4 20. $P = 42m$, $A = 96m^2$

21. a) $X = 2Y - 3$ b) $-x = \left(y + \frac{z}{2}\right) + 5$ c) $ab = 3(a + b) - 14$ d) $m + 2n = \frac{m}{n} + 1$

e) $3(x - y) = x(-y) - 1$ f) $P = 2D - 10$ g) $2.5 + (x - 1)1.5$ h) $x, x + 1$, and $x + 2$ i) $x(4x - 3)$

j) Ann has $A - 30$ dollars and Beatrix has $B + 30$. 22. 3 23. a) 203 b) 0

24. Hint: write down all the possible number triples that multiply to 36. Look at the sum of each triple.

Hint for 10 d). Have the 65 couples in a room and let everyone shake hands with everyone else except their spouse. How many handshakes? Have the other 20 women in another room - how many handshakes among themselves? Then let them into the same room - how many handshakes? Then add the three numbers.