

1. Suppose that $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Find each of the following sets.

a) $A = \{x \in U : x > 6 \text{ or } x \text{ is odd}\}$

c) $C = \{x \in U : x \text{ is divisible by 2 or by 3}\}$

b) $B = \{x \in U : x > 6 \text{ and } x \text{ is odd}\}$

d) $D = \{x \in U : x \text{ is divisible by 2 and by 3}\}$

2. Suppose that $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 4, 8, 9\}$, $B = \{1, 4, 5, 7, 9, 10\}$, and $C = \{2, 3, 4, 7, 9\}$. Find each of the following sets.

a) $A \cap B$ c) $B \cap C$ e) $(A \cap B) \cup C$ g) $A \cap \overline{B}$ i) $\overline{A \cup B}$

b) $A \cap C$ d) $A \cap (B \cup C)$ f) $A \cup (B \cap C)$ h) $\overline{A} \cap B$ j) $\overline{A \cap B}$

3. Given a Venn diagram for A , B , and C , shade the region corresponding to each of the given sets.

a) $(A \cap B) \cap C$ b) $A \cap (B \cup C)$ c) $(A \cap B) \cup C$ d) $A \cup (B \cap C)$

4. Label each of the following statements as true or false.

a) If n is an integer such that n^2 is divisible by 12, then n is divisible by 12.

e) If the product xy of two integers is divisible by 6, then x is divisible by 6 or y is divisible by 6.

b) If integer a is divisible by 8 and integer b is divisible by 6, then the product ab is divisible by 48.

f) The sum of two prime numbers is never prime.

c) If integer n is divisible by 8 and by 6, then it is also divisible by 48.

g) The product of two prime numbers is never prime.

d) If the product xy of two integers is divisible by 5, then x is divisible by 5 or y is divisible by 5.

h) The sum of the first five prime numbers is 28.

i) Every square is a rectangle.

j) For every set A and B , $A \subseteq B$ or $B \subseteq A$.

5. Find the prime factorization for each of the following numbers.

a) 4500 b) 1001 c) 7986 d) 120^{120}

6. Suppose that $n = 2^{30} \cdot 3^{50} \cdot 5 \cdot 11^{100}$. What is the prime-factorization of n^3 ?

7. Find the last digit of $2^{41} + 2^{42} + 2^{43} + 2^{44}$.

8. Evaluate each of the following.

a) $|3 - 4| - |6 - 9|$ b) $3 - |4 - 6| - 9$ c) $3|-4 - 6| - 9$ d) $3 - 4|-6 - 9|$ e) $3 - |4 - |6 - 9||$

9. Evaluate each of the following.

a) $(-4^2 - 3(4 - (5 - 8)^2))^3$ b) $\frac{2 - 3(5 - 3^2)}{2^3 - (-1)^4}$ c) $\sqrt{7\sqrt{5\sqrt{2 \cdot 5 - 1} + 1} - 3}$

10. Compute $88 + 95 + 102 + \dots + 1054$

11. Simplify each of the following.

a) $\sqrt[3]{-8}$ b) $(\sqrt[3]{-8})^3$ c) $\sqrt[4]{16}$ d) $-\sqrt[5]{-1}$ e) $(\sqrt[4]{2})^{12}$ f) $(\sqrt[4]{-2})^{12}$ g) $(\sqrt[3]{-2})^{12}$

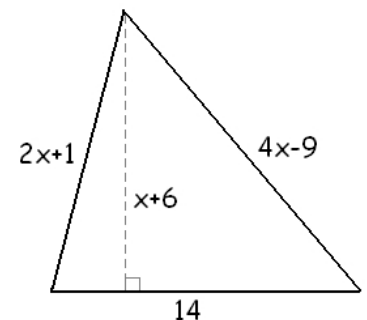
12. Simplify each of the following expressions.

a) $\frac{(-2x^4)^3 x^5}{(-x^3)^2}$ b) $\frac{x(2x)^2 \cdot x^3 \cdot 2x^4}{(x \cdot 2x^2)^3}$ c) $(-(-x)(-x^2)(-x^4))^2$ d) $(-(-x)(-x^2)(-x^4))^3$

13. Suppose that $x = 8 \cdot 10^{18}$ and $y = 2.5 \cdot 10^5$. Compute each of the following. Present your answer using scientific notation.

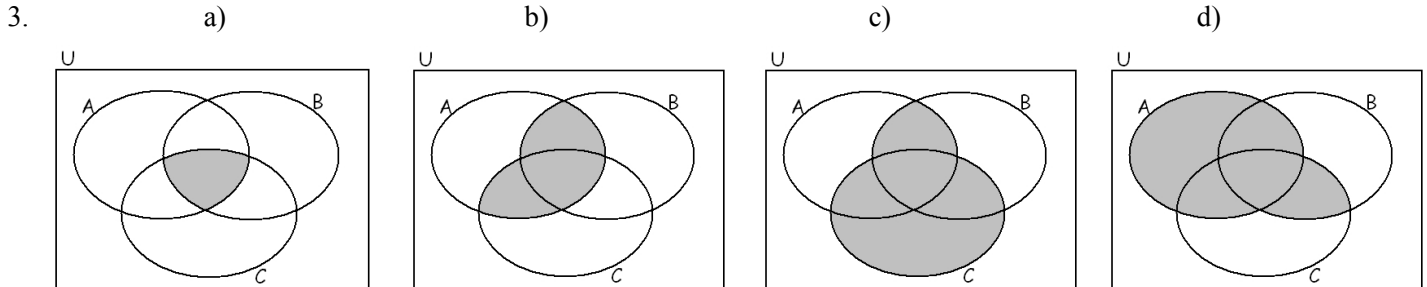
a) xy b) $\frac{x}{y}$ c) $\frac{x}{y^2}$ d) $\frac{x}{y^3}$

14. Consider the expressions $3 \cdot 2^x$ and 6^x . Evaluate both expressions if a) $x = 1$ b) $x = 2$ c) $x = 3$
15. Consider the expression $\frac{2^{n+3} \cdot 5^{n+1}}{10^n}$.
- a) Evaluate the expression for each of the values given for n . i) $n = 1$ ii) $n = 2$ iii) $n = 3$
- b) Can you explain what you noticed in part a)?
16. Simplify each of the following expressions.
- a) $\frac{2^x \cdot 5^{x+2}}{10^x}$ b) $\frac{3^{x+2} \cdot 4^{x+1}}{6^x \cdot 2^{x+1}}$ c) $\frac{8^{x+2}}{2^{3x+1}}$ d) $7 \cdot 3^x - 5 \cdot 3^x + 4 \cdot 3^x$
17. Simplify each of the following expressions.
- a) $(x^6)^2$ b) $\sqrt{x^{12}}$ c) $\sqrt[3]{x^{12}}$ d) $(x^5)^3$ e) $\sqrt[3]{x^{15}}$ f) $\sqrt{x^{100}}$ g) $\sqrt[5]{x^{100}}$
18. Simplify each of the following expressions.
- a) $\sqrt[5]{x^{20}}$ b) $(\sqrt[5]{x})^{20}$ c) $\sqrt[4]{x^{24}}$ d) $(\sqrt[4]{x})^{24}$
19. Suppose that we denote 5^{100} by A . Express each of the following in terms of A .
- a) 5^{101} b) 25^{100} c) 5^{99} d) $5^{102} - 5^{101}$ e) 5^{50}
20. Solve each of the following equations. Make sure to check your solutions.
- a) $5(x - 2) - 3(2x + 1) = -7 - 3(x - 4)$ i) $(-x + 4)^2 - (x + 1)(x - 5) = -4(x + 3)$
- b) $m + 3(4 - m) = 7 - 2(m - 3)$
- c) $3(a - 1) - 3a - 1 = -2(3a - 1)$ j) $3 \left(\frac{\frac{5x + 1}{4} + 3}{-2} + 1 \right) = 15$
- d) $5(x + 1) - 3(x - 2) = 5(2x - 1)$
- e) $2(5x - 1) - 3(-2x + 5) = 1 + 6(x - 3)$
- f) $-4 - 5(1 - 2x) = 1 + 10(x - 1)$ k) $\frac{\frac{x - 1}{3} + 1}{2} - 1 = -2$
- g) $(2x - 3)^2 = (x - 3)(4x + 3)$
- h) $(x + 1)^2 - (x - 3)^2 = -24$
21. Compute the perimeter and area of the right triangle with sides 8 cm, 17 cm, and 15 cm. Include units in your computation and answer.
22. Find the value of x if we know that the triangle shown on the picture has a perimeter A unit and area $2A$ unit².
23. The opposite of a number is five less than twice the difference of the number and eight. Find this number.
24. If we increase all sides of a square by 2 units, the area will increase by 36 unit². How long are the sides before the increase?
25. There were a lot of coins in that jar, all quarters and dimes. The number of dimes was two less than five times the number of quarters. How many of each coins were there if all the coins in the jar were worth 8 dollars and 80 cents? (Hint: think in terms of cents)



Answers

1. a) $\{1, 3, 5, 7, 8, 9, 10\}$ b) $\{7, 9\}$ c) $\{2, 3, 4, 6, 8, 9\}$ d) $\{6\}$
 2. a) $\{1, 4, 9\}$ b) $\{2, 4, 9\}$ c) $\{4, 7, 9\}$ d) $\{1, 2, 4, 9\}$ e) $\{1, 2, 3, 4, 7, 9\}$ f) $\{1, 2, 4, 7, 8, 9\}$
 g) $\{2, 8\}$ h) $\{5, 7, 10\}$ i) $\{3, 6\}$ j) $\{2, 3, 5, 6, 7, 8, 10\}$



4. a) false b) true c) false d) true e) false f) false g) true h) true i) true j) false
 5. a) $2^2 \cdot 3^2 \cdot 5^3$ b) $7 \cdot 11 \cdot 13$ c) $2 \cdot 3 \cdot 11^3$ d) $2^{360} \cdot 3^{120} \cdot 5^{120}$ 6. $2^{90} \cdot 3^{150} \cdot 5^3 \cdot 11^{300}$
 7. 0 8. a) -2 b) -8 c) 21 d) -57 e) 2 9. a) -1 b) 2 c) 5 10. 79369
 11. a) -2 b) -8 c) 2 d) 1 e) 8 f) undefined g) 16
 12. a) $-8x^{11}$ b) x c) x^{14} d) $-x^{21}$
 13. a) $2 \cdot 10^{24}$ b) $3.2 \cdot 10^{13}$ c) $1.28 \cdot 10^8$ d) $5.12 \cdot 10^2$
 14. a) $3 \cdot 2^1 = 6$ and $6^1 = 6$ b) $3 \cdot 2^2 = 12$ and $6^2 = 36$ c) $3 \cdot 2^3 = 24$ and $6^3 = 216$
 15. a) i) $\frac{2^4 \cdot 5^2}{10^1} = 40$ ii) $\frac{2^5 \cdot 5^3}{10^2} = 40$ iii) $\frac{2^6 \cdot 5^4}{10^3} = 40$
 b) If n is part of an algebraic expression and the expression has the same value for every n , then somehow the value of n doesn't matter. This usually indicates some sort of cancellation.

$$\frac{2^{n+3} \cdot 5^{n+1}}{10^n} = \frac{(2^n \cdot 2^3)(5^n \cdot 5^1)}{10^n} = \frac{2^n \cdot 8 \cdot 5^n \cdot 5}{10^n} = \frac{(2^n \cdot 5^n)(8 \cdot 5)}{10^n} = \frac{10^n \cdot 40}{10^n} = 40$$

 16. a) 25 b) 18 c) 32 d) $6 \cdot 3^x$
 17. a) x^{12} b) x^6 c) x^4 d) x^{15} e) x^5 f) x^{50} g) x^{20}
 18. a) x^4 b) x^4 c) x^6 d) x^6 19. a) $5A$ b) A^2 c) $\frac{A}{5}$ d) $20A$ f) \sqrt{A}
 20. a) 9 b) no solution c) 1 d) 2 e) 0 f) all real numbers g) 6 h) -2 i) no solution j) -9 k) -56
 21. $P = 40$ cm $A = 60$ cm² 22. $x = 6$ 23. 6 24. 8 units 25. 12 quarters and 58 dimes