1. Suppose that $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$. Find each of the following sets.

   a) $A = \{x \in U : x \text{ is divisible by 3 or } x \text{ is odd}\}$
   b) $B = \{x \in U : x > 6 \text{ is divisible by 3 and } x \text{ is odd}\}$
   c) $C = \{x \in U : x \text{ is divisible by 4}\}$
   d) $A \cap C$
   e) $B \cup C$

2. Perform each of the given operations.

   a) $(-\infty, 5) \cup (-\infty, 10]$
   b) $(-\infty, 5) \cap (-\infty, 10]$
   c) $(3, 8) \cap [4, 11]$
   d) $(3, 8) \cup [4, 11]$
   e) $(-\infty, 9) \cup [2, \infty)$
   f) $(-\infty, 9) \cap [2, \infty)$
   g) $(-\infty, 1] \cap (3, \infty)$
   h) $(-\infty, 1] \cup (3, \infty)$

3. There are 50 students in the senior class. 35 students are taking English, 28 are taking French, and 15 are taking both English and French. How many students are taking neither English, nor French?

4. Label each of the following statements as true or false.

   a) If n is an integer such that n^2 is divisible by 20, then n^2 is divisible by 400.
   b) If n is an integer greater than 1, then all exponents in the prime-factorization of n^2 are even.
   c) If integer n is divisible by 4 and by 6, then it is also divisible by 24.
   d) The square of an odd number is always odd.
   e) If A and B are sets such that $A \subseteq B$, then $A \cup B = A$.
   f) If A and B are sets such that $A \subseteq B$, then $A \cup B = B$.
   g) There is no even prime number.

5. Find the prime factorization for each of the following numbers.

   a) 10! b) 2016^{100} c) 2009 d) 219 615

   Note: 10! (read as ten factorial is notation for $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$)

6. Find the last digit of $7^{99} + 7^{100} + 7^{101} + 7^{102}$.

7. Suppose that $x = 0.000000000125$ and $y = 45\,000\,000\,000\,000$.

   a) Express x and y using scientific notation.
   b) Compute each of the following. Present your answer using scientific notation.

      i) xy ii) x^2 iii) $x + y$ iv) $\frac{y}{x}$ v) $\sqrt{\frac{y}{x}}$

8. Compute each of the following sums.

   a) $210 + 223 + 236 + ... + 1302$
   b) $100 + 103 + 106 + ... + 847$
   c) $120 + 130 + 140 + ... + 1010$
   d) $\frac{1}{2019} + \frac{2}{2019} + \frac{3}{2019} + ... + \frac{2018}{2019}$

9. Convert each of the following decimals to a fraction of integers. You do NOT have to simplify the fraction.

   a) 2.04 b) $0.\overline{24}$ c) $4.\overline{175}$ d) $0.1\overline{76}$

10. The price of a book was increased from \$80 to \$92. What percentage of an increase does this represent?

11. a) We increased a quantity by 20%. Later, there was an additional increase of 30%. If we express the two subsequent changes as a single change, what percentage of an increase does this represent?

    b) We increased a quantity by 40%. Later, there was decrease of 30%. If we express the two subsequent changes as a single change, what percentage of a change does this represent? Is it an increase or a decrease?

    c) We decreased a quantity by 20%. Later, there was an additional decrease of 10%. If we express the two subsequent changes as a single change, what percentage of a decrease does this represent?

12. Expand the given products.

a) $(2x-1)^3$ b) $(3x+1)(9x^2-3x+1)$ c) $\left(x+\frac{1}{x}\right)^2$

13. Simplify each of the following.

a) $\sqrt[3]{-27}$ b) $\sqrt[4]{-16}$ c) $\sqrt[4]{81}$ d) $-\sqrt[3]{-\frac{1}{8}}$ e) $\sqrt[7]{-1}$ f) $-\left(\sqrt[3]{-2}\right)^3$ g) $\left(\sqrt[4]{-2}\right)^4$ h) $\left(\sqrt[3]{-2}\right)^{12}$

14. Consider the expression $\frac{-2x^2+3x-1}{-2x+1}$. Compute the value of the expression with each of the values of x given.

a) $x=5$ b) $x=-2$ c) $x=\frac{1}{3}$ d) $x=-\frac{1}{2}$ e) $x=\frac{1}{2}$ f) 1

15. Consider the rule $(a^n)^m=a^{nm}$. Simplify $\left(x^{10}\right)^2$. Based on this result, what is $\sqrt{x^{20}}$?

16. Simplify each of the following.

a) $\sqrt{x^{36}}$ b) $\sqrt[3]{x^{36}}$ c) $\sqrt[6]{x^{36}}$ d) $\sqrt[9]{x^{36}}$

17. Recall the definition of $\sqrt[n]{a}$. If n is even, $\sqrt[n]{a}$ is the non-negative number x such that $x^n=a$. If n is odd, $\sqrt[n]{a}$ is the number x such that $x^n=a$. We can re-write $\left(\sqrt[5]{a}\right)^{20}$ as $\left(\sqrt[5]{a}\right)^{5\cdot 4}=\left[\left(\sqrt[5]{a}\right)^5\right]^4=a^4$. Use this technique to simplify each of the following.

a) $\left(\sqrt{2}\right)^6$ b) $\left(\sqrt[3]{2}\right)^6$ c) $\left(\sqrt[3]{x}\right)^{24}$ d) $\left(\sqrt[7]{x}\right)^{21}$

18. Let N denote 6^{2000}. Express each of the following in terms of N.

a) 6^{2001} b) 6^{2002} c) $6^{2002}-6^{2001}$ d) 6^{1998} e) 6^{1000} f) 6^{200}

19. Simplify each of the given expressions.

a) $\frac{6^{2x+1}}{4^{x-1}\cdot 3^{2x+1}}$ b) $\frac{\left(-2x^4y\right)^3\left(-xy^2\right)}{\left(-2xy^2x^3\right)^2}$ c) $-(-x)\left(-x^2\right)(-x)^3$

20. Simplify each of the given expressions.

a) $\frac{x^2-25}{x^2-2x-35}$ b) $\frac{2m-5}{5-2m}$ c) $\frac{x^2+2x}{3x^2-12}$ d) $\frac{x^3-x^2}{x^2-1}$

21. Find the smallest value of each of the given expressions.

a) $x^2-12x+20$ b) $x^2+4x+15$ c*) x^4+18x^2+85

22. Suppose that one number is twenty greater than another. There are many possibilities such as 2 and 22, or -8 and 12, or 0 and 20. Among all these pairs, which one has the smallest product?

23. Solve each of the following equations.

a) $5x-3=-38$

b) $\frac{2}{3}x-\frac{3}{4}=-\frac{5}{12}$

c) $\frac{x-3}{7}=-2$

d) $\frac{x-\frac{5}{6}}{-\frac{3}{8}}=\frac{4}{9}$

e) $3(x+8)=-15$

f) $\frac{3}{8}\left(x-\frac{2}{5}\right)=-\frac{3}{2}$

g) $3(2x-5)-2(5x+3)=3x$

h) $\frac{1}{2}\left(6x-\frac{2}{3}\right)-\frac{5}{6}\left(12x+\frac{1}{2}\right)=-\frac{31}{4}$

i) $\frac{3}{4}x-\frac{1}{2}\left(\frac{2}{3}x-\frac{3}{5}\right)=\frac{1}{20}$

j) $(2x-3)^2-2x(x-5)=3-(x-1)(-2x+3)$

k) $(x+3)^2-(x-1)^2=8(x-1)$

l) $\frac{\frac{2x-5}{3}+1}{-2}-1=5$

m) $x^5=16x^5$

n) $x^5=16x^4$

o) $x^5=16x^3$

p) $x^5=16x$

24. Solve each of the following inequalities. Present the solution set using interval notation.

    a) $(x-3)^2 \geq (x+5)^2$

    b) $\frac{2}{3}\left(x+\frac{1}{2}\right) < \frac{3}{4}x - 1$

    c) $3\left(4-5\left(2x-1\right)\right) - 7 \leq -2\left(-x+22\right)$

    d) $5x - 7 < -x - 3\left(-2x-8\right)$

25. If we increase the side of a square by 2 units, its area will increase by 20 unit^2. How long are the sides of this square?

26. The sum of three consecutive multiples of 5 is 105. Find the value of these numbers.

27. The square of a number is 24 less than ten times the number. Find this number.

28. The tickets for the field trip were purchased yesterday for both students and instructors. Children tickets cost \$6, adult tickets cost \$20. The number of children ticket purchased was three more than twice the number of adults tickets purchased. How many of each were purchased if all of the tickets cost a total of \$274 dollars?

29. One number is 20 less than five times another number. Find these numbers if their product is 1260.

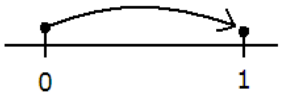
30. One side of a rectangle is five meters shorter than seven times the other side. Find the length of the shorter side if we also know that the perimeter of the rectangle is 278 meters.

31. *Suppose that x is a number such that $x^3 + \frac{1}{x^3} = 5$. What is the value of $x^6 + \frac{1}{x^6}$?

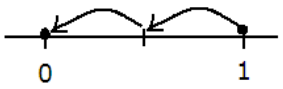
32. *Contributed by Prof. Abdallah Shuaibi.

    Two travelers meet a third one, who is very hungry. He offers the two travelers 8 dollars for a meal. One traveler has three pieces of bread, the other one has five. So the hungry man gives them the 8 dollars, they all sit down and eat all 8 pieces of bread together. Afterwards, the two get into an argument about how to divide up the money. The one who contributed 5 pieces of bread wants to split it to 5 and 3. The other wants to divide the money evenly, 4 and 4. They go to a wise man for advice. They tell him their story and ask him to divide the money between them. The wise man gives the man who had 3 pieces of bread 1 dollar and 7 to the man with 5 pieces of bread. Is this a just or even reasonable decision?

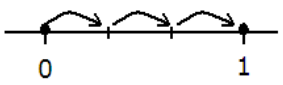
33. *A cricket is sitting on the number line, on zero. It decides to jump to one.



    Then it changes its mind and wants to get back to zero. Since it is a bit tired, it uses two equal jumps to get back to zero.



    Then it changes its mind again, and wants to get back to one. Since it is more tired, it uses three equal jumps to get back to one.



    Then it jumps back to zero, using four equal jumps. Then it jumps back to one, using five equal jumps. And so on, in the nth trip, using n equal jumps, never stopping. Is this cricket going to land on every point between zero and one? Explain your answer.

Answers

1. a) $\{1, 3, 5, 6, 7, 9, 11, 12, 13, 15\}$ b) $\{3, 9, 15\}$ c) $\{4, 8, 12\}$ d) $\{12\}$ e) $\{3, 4, 8, 9, 12, 15\}$

2. a) $(-\infty, 5)$ b) $(-\infty, 10]$ c) $[4, 8)$ d) $(3, 11]$ e) $(-\infty, \infty)$ f) $[2, 9)$ g) $\varnothing$ h) $(-\infty, 1] \cup (3, \infty)$

3. 2 4. a) false b) true c) false d) true e) false f) true g) false

5. a) $2^8 \cdot 3^4 \cdot 5^2 \cdot 7$ b) $2^{500} \cdot 3^{200} \cdot 7^{100}$ c) $7^2 \cdot 41$ d) $3 \cdot 5 \cdot 11^4$ 6. 0

7. a) $x = 1.25 \cdot 10^{-10}$ and $y = 4.5 \cdot 10^{13}$
   b) i) $5.625 \cdot 10^3$ ii) $1.562\,5 \cdot 10^{-20}$ iii) $4.5 \cdot 10^{13}$ iv) $3.6 \cdot 10^{23}$ v) $6 \cdot 10^{11}$

8. a) 64 260 b) 118 375 c) 50 850 d) 1009 9. a) $\dfrac{204}{100}$ b) $\dfrac{24}{99}$ c) $\dfrac{4171}{999}$ d) $\dfrac{175}{990}$ 10. 15%

11. a) 56% increase b) 2% decrease c) 28% decrease

12. a) $8x^3 - 12x^2 + 6x - 1$ b) $27x^3 + 1$ c) $x^2 + \dfrac{1}{x^2} + 2$

13. a) -3 b) undefined c) 3 d) $\dfrac{1}{2}$ e) -1 f) 2 g) undefined h) 16

14. a) 4 b) -3 c) $-\dfrac{2}{3}$ d) $-\dfrac{3}{2}$ e) undefined f) 0 15. x^{10}

16. a) x^{18} b) x^{12} c) x^6 d) x^4 17. a) 8 b) 4 c) x^8 d) x^3

18. a) $6N$ b) $36N$ c) $30N$ d) $\dfrac{N}{36}$ e) $\sqrt{N}$ f) $\sqrt[10]{N}$ 19. a) 48 b) $2x^5y$ c) x^6

20. a) $\dfrac{x-5}{x-7}$ b) -1 c) $\dfrac{x}{3x-6}$ or $\dfrac{x}{3(x-2)}$ d) $\dfrac{x^2}{x+1}$ 21. a) -16 b) 11 c) 4

22. -10 and 10 has the smallest possible product that is -100.

23. a) -7 b) $\dfrac{1}{2}$ c) -11 d) $\dfrac{2}{3}$ e) -13 f) $-\dfrac{18}{5}$ g) -3 h) 1 i) $-\dfrac{3}{5}$ j) -1 k) no solution l) -17 m) 0 n) $0, 16$
   o) $-4, 0, 4$ p) $-2, 0, 2$

24. a) $(-\infty, -1]$ b) $(16, \infty)$ c) $[2, \infty)$ d) $(-\infty, \infty)$ 25. 4 units 26. $30, 35, 40$

27. 19 children tickets and 8 adult tickets 28. -14 with -90 and 18 with 70 29. $4, 6$ 30. 18 m by 121 m 31. 23

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