

- Which number has more divisors, 24 or 36?
- Suppose that $A = 2^{99}$. Express each of the following in terms of A .
 - 2^{100}
 - $2^{99} + 2^{100} - 3 \cdot 2^{101}$
 - 8^{99}
 - $\frac{5 \cdot 2^{99} - 2^{100}}{2^{101}}$
- Find the smallest value of each of the following expressions.
 - $x^2 - 6x + 4$
 - $x^2 + 10x + 7$
 - $x^2 + 6x$
- Find the prime factorization for each of the following.
 - 1200
 - 10^{100}
 - $75 \cdot 80$
- Use the prime factorization to find the greatest common factor and least common multiple of 90 and 135.
- Simplify each of the given expressions.
 - $\frac{2^{x+1} \cdot 3^{x-1}}{6^{x-2}}$
 - $\frac{2^{-1} + 5^{-1}}{2^{-2} - 5^{-2}}$
 - $\frac{2^{-1} \cdot 5^{-1}}{2^{-2} \cdot 5^{-2}}$
 - $\frac{(2x^{-1}y^3)^{-5}}{(2xy^{-2})^{-3}}$
 - $\frac{(-2xy^{-2})^3 (-2^{-2}x^{-3}y)^4}{(-4x^3y)^{-2}}$
- Compute each of the following sums.
 - $42 + 50 + 58 + \dots + 482$
 - $-35 + (-25) + \dots + 755$
 - $31 + 40 + 49 + \dots + 1102$
- Perform the indicated operations and simplify.
 - $(2\sqrt{7} - 1)(\sqrt{7} + 3)$
 - $(3\sqrt{2} - 1)^2$
 - $(\sqrt{2} - 1)^4$
 - $(3\sqrt{7} - 8)(3\sqrt{7} + 8)$
 - $\sqrt{50} - \sqrt{18} + 2\sqrt{72}$
 - $(2\sqrt{5} - 4)^3 (2\sqrt{5} + 4)^3$
 - $\frac{12 - \sqrt{20}}{2}$
 - $\frac{\sqrt{80}}{\sqrt{45}}$
 - $(-\sqrt{2})^8$
 - $(-\sqrt{2})^9$
- Simplify each of the following.
 - $(1, 4) \cup (2, 7)$
 - $(1, 4) \cap (2, 7)$
 - $(-\infty, 3) \cup (-\infty, 5]$
 - $(-\infty, 3) \cap (-\infty, 5]$
 - $(-\infty, 10] \cup (7, \infty)$
 - $(-\infty, 10] \cap (7, \infty)$
 - $(1, 5) \cup [12, \infty)$
 - $(1, 5) \cap [12, \infty)$
- Evaluate each of the given expressions with the values given for the variable(s).
 - $2x^2 - x - 3$ if $x = 2\sqrt{3} - 5$
 - $-a^2 + 5a - 1$ if $a = 4 - \sqrt{3}$
 - $x^2 - 8x + 6$ if $x = 4 - \sqrt{10}$
- Graph each of the following.
 - $y = -\frac{2}{3}x + 1$
 - $x + y = 3$
 - $2x - 3y = -1$
- Graph $y = x^2 - 4x - 5$. State the coordinates of at least five points, including vertex and intercepts.
- Find the perimeter and area of the quadrilateral determined by the points $A(-3, 5)$, $B(-3, -2)$, $C(2, -2)$, and $D(2, 5)$.
- Completely factor each of the following over the integers.
 - $(3a + 1)^2 - 49$
 - $200x + 2x^3$
 - $5x^6 - 80x^2$
 - $x^{10} - 49$
 - $405 - 5x^4$
 - $12x^2 - 110x + 2x^3$
 - $(2x^2 - x + 2)^2 - (2x^2 + x - 4)^2$
- Find the exact value of the distance between the given points.
 - $A(3, -2)$ and $B(-2, 10)$
 - $P(-4, 0)$ and $Q(3, 7)$
 - $M(5, -4)$ and $N(3, 4)$

16. Solve $9x^2 = 12x + 1$ by completing the square. Check one of your solutions.

17. If A is the set of all integers divisible by 5 and B is the set of all integers divisible by 3, what is $A \cap B$?

18. Evaluate $\frac{3xy + 2x^2 - 2y^2}{x + 2y}$ if $x = 2\sqrt{3} - 1$ and $y = 2$

19. Simplify each of the following algebraic expressions.

a) $(x - \sqrt{2})(x + \sqrt{2})$

c) $(x - 2)^3$

f) $(a - 4)(2a + 1) - (a - 3)^2$

b) $(x - 1 + \sqrt{2})(x - 1 - \sqrt{2})$

d) $(a + 1)(a - 2)(a + 2)$

g) $(\sqrt{2} - 1)^4$

e) $(x - 5)(x - 2)(x + 1)$

20. Solve each of the following equations.

a) $-\frac{2}{5}x + \frac{1}{2} = \frac{1}{5}$

g) $(2x - 5)^2 - (2x + 6)^2 = 4x - 11$

b) $\frac{3x + 1}{5} - \frac{2x - 4}{3} = 2x - 15$

h) $60x - 20x^2 = 5x^3$

c) $x(3x - 1) - 2x - 3 = 3(x - 1)^2$

i) $x^4(x^2 + 1)(x^4 + 9) = 0$

d) $\frac{2}{3}\left(x - \frac{1}{2}\right) - \frac{1}{2}\left(x + \frac{2}{3}\right) = \frac{1}{6}(x + 2)$

j) $4(x - 3) - 3(x + 1) = 2x - 10 - (x + 5)$

e) $20x^5 = 5x^4$

k) $(2x - 1)^2 - (x - 2)^2 = 2x^2 + 1$

f) $20x^5 = 5x^3$

l) $x^2 - 6x = -7$

m) $3x^2 - x = 1$

21. Solve each of the following inequalities.

h) $3(x - 2) - 5(6x - 1) \geq 3x - 1$

j) $\frac{2x - 3}{7} - \frac{3x - 1}{2} \leq x - 11$

i) $\frac{3x - 8}{4} < \frac{5x - 17}{3}$

k) $(3x - 1)^2 + (x - 1)^2 \geq 10x^2 + 6$

22. Solve each of the given system of linear equations.

a) $\begin{cases} 2x + 3y = -4 \\ 4x - 5y = -30 \end{cases}$

b) $\begin{cases} -x + 2y = 19 \\ 3x + y = -1 \end{cases}$

c) $\begin{cases} (x - 2)^2 + (y - 1)^2 = x^2 + y^2 - 1 \\ (x - 3)^2 - x^2 = (y + 3)^2 - y^2 \end{cases}$

23. Solve each of the given system of linear equations.

a) $\begin{cases} 2x + 3y - z = 35 \\ 3x - 2y - 4z = 13 \\ x - 5y - 8z = 3 \end{cases}$

b) $\begin{cases} 3x - 4y + 5z = -4 \\ -x + 3y - z = 10 \\ 2x + y - 3z = -7 \end{cases}$

c) $\begin{cases} 5x - 6y + 2z = 37 \\ 2x + 3y - 3z = 23 \\ -2x + 5y + z = -29 \end{cases}$

24. Prove that each of the following numbers is rational by re-writing them as a fraction of two integers.

a) $0.\overline{304} = 0.304040404\dots$ b) $7.\overline{2673} = 7.2673673673673\dots$ c) $0.\overline{9} = 0.99999\dots$

25. The opposite of a number is thirty-five more than the sum of -5 and the number. Find this number.

26. A number is 2 greater than its own reciprocal. Find all real numbers with this property.

27. There is a farm where chickens and cows live. All together, there are 72 heads and 200 legs. How many chickens, how many cows are there?

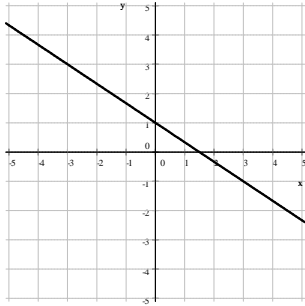
28. A number is 3 less than its own square. Find this number.

29. Last semester Steven took a Spanish class. His course grade was the weighted average of the following: Class Participation: 10%, Homework: 10%, Quizzes: 25%, Exam 1: 25% and Exam 2: 30%.
- a) What was Steven's course average if he earned the following grades (scores):
Class Participation: 95, Homework: 81, Quizzes: 79, Exam 1: 86, Exam 2: 73
- b) Susan is now taking the same class. Exam 2 did not yet take place. Her other scores are:
Class Participation: 82, Homework: 72, Quizzes: 89, Exam 1: 93
What is her current average in the course? 87
- c) Is it still possible for Susan to get an A in the class? If yes, what score does she need to earn on Exam 2?
30. We have some \$5 bills and some \$10 bills. All together, there are 74 bills. How many \$5 bills do we have if the value of all bills is \$430?
31. We invested a total of \$5000 in Stock A and Stock B. After a year, the value of Stock A increased by 8% and the value of Stock B decreased by 2%. If the value of both stocks together increased by a total of \$280, how much was invested into stock A and Stock B?
32. The first row in a theater has 15 seats in it. The second row has three more seats than the first row. The third row has three more seats than the second row. And so on, each row has three more seats than the row before. If the last row has 81 seats in it, how many seats are there in the entire theater?
33. Find all numbers with the following property: if we square the number, the result is the original number.
34. We have a jar full of coins, all dimes and quarters. How many of each coins do we have if all together, we have 154 coins in the total value of \$24.70?
35. Find the equation of a parabola if the following points are on its graph: $(2, 2)$, $(3, 1)$, and $(-1, 17)$.
36. What is the exact value of the height of a square-based pyramid if all of its edges are 16 meters long?
37. There are 120 people in a room. If everyone shakes hands with everyone else, how many handshakes took place?
38. One number is twelve less than twice another. Find these numbers if their product is 320.
39. The hypotenuse of a right triangle is 52 feet long. Find the other sides if we know that the difference between them is 28 ft.
40. One number is 12 less than another. What is the smallest possible value of their product?

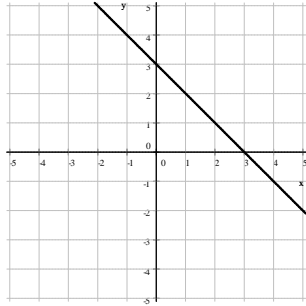
Answers

1. 36 (24 has 8 divisors, 36 has 9) 2. a) $2A$ b) $-9A$ c) A^3 d) $\frac{3}{4}$ 3. a) -5 b) -18 c) -9
4. a) $1200 = 2^4 \cdot 3 \cdot 5^2$ b) $10^{100} = 2^{100} \cdot 5^{100}$ c) $75 \cdot 80 = 2^4 \cdot 3 \cdot 5^3$ 5. lcm = 270 gcd = 45
6. a) 24 b) $\frac{10}{3}$ c) 10 d) $\frac{x^8}{4y^{21}}$ e) $-\frac{1}{2x^3}$ 7. a) 14672 b) 28800 c) 67980
8. a) $11 + 5\sqrt{7}$ b) $19 - 6\sqrt{2}$ c) $17 - 12\sqrt{2}$ d) -1 e) $14\sqrt{2}$ f) 64 g) $6 - \sqrt{5}$ h) $\frac{4}{3}$ i) 16 j) $-16\sqrt{2}$
9. a) $(1, 7)$ b) $(2, 4)$ c) $(-\infty, 5]$ d) $(-\infty, 3)$ e) \mathbb{R} same as $(-\infty, \infty)$ f) $(7, 10]$ g) can not be simplified h) \emptyset
10. a) $76 - 42\sqrt{3}$ b) $3\sqrt{3}$ c) 0

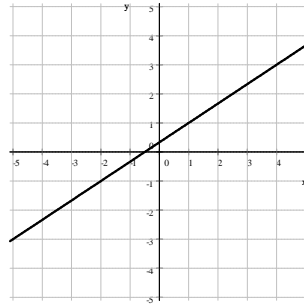
11. a) $y = -\frac{2}{3}x + 1$



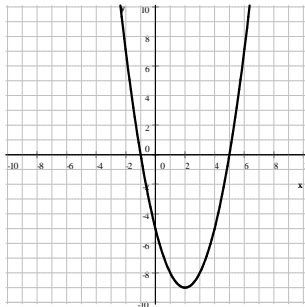
b) $x + y = 3$



c) $2x - 3y = -1$



12. $y = x^2 - 4x - 5 = (x - 2)^2 - 9$
 vertex: $(2, -9)$
 x -intercepts: $(-1, 0)$ and $(5, 0)$
 y -intercept: $(0, -5)$



13. $P = 24$ unit, $A = 35$ unit²

14. a) $3(3a + 8)(a - 2)$ b) $2x(x^2 + 100)$
 c) $5x^2(x^2 + 4)(x + 2)(x - 2)$ d) $(x^5 + 7)(x^5 - 7)$
 e) $-5(x^2 + 9)(x + 3)(x - 3)$ f) $2x(x + 11)(x - 5)$
 g) $-4(2x^2 - 1)(x - 3)$

15. a) 13 units b) $\sqrt{98} = 7\sqrt{2}$ units c) $\sqrt{68} = 2\sqrt{17}$ units

16. $\frac{2 \pm \sqrt{5}}{3}$ Checking with $\frac{2 - \sqrt{5}}{3}$
 $LHS = 9x^2 = 9 \left(\frac{2 - \sqrt{5}}{3} \right)^2 = 9 \cdot \frac{4 + 5 - 4\sqrt{5}}{9} = 9 - 4\sqrt{5}$ and

(16. cont.) $RHS = 12x + 1 = 12 \left(\frac{2 - \sqrt{5}}{3} \right) + 1 = 4(2 - \sqrt{5}) + 1 = 8 - 4\sqrt{5} + 1 = 9 - 4\sqrt{5}$

17. the set of all integers divisible by 15 18. a) 4 b) 0 c) undefined d) $4\sqrt{3} - 4$

19. a) $x^2 - 2$ b) $x^2 - 2x - 1$ c) $x^3 - 6x^2 + 12x - 8$ d) $a^3 + a^2 - 4a - 4$ e) $x^3 - 6x^2 + 3x + 10$ f) $a^2 - a - 13$
 g) $17 - 12\sqrt{2}$ 20. a) $\frac{3}{4}$ b) 8 c) 2 d) There is no solution. e) $0, \frac{1}{4}$ f) $-\frac{1}{2}, 0, \frac{1}{2}$ g) 0 h) 2, 0, -6 i) 0

j) \mathbb{R} (all real numbers are solution) k) 2, -2 l) $3 \pm \sqrt{2}$ m) $\frac{1 \pm \sqrt{13}}{6}$

21. a) $(-\infty, 0]$ b) $(4, \infty)$ c) $[5, \infty)$ d) $\left(-\infty, -\frac{1}{2}\right]$ 22. a) $(-5, 2)$ b) $(-3, 8)$ c) $(3, -3)$

23. a) $x = 3, y = 8, z = -5$ b) $x = -1, y = 4, z = 3$ c) $x = 7, y = -2, z = -5$

24. a) $\frac{301}{990}$ b) $\frac{72601}{9990}$ c) 1 25. -15 26. $1 \pm \sqrt{2}$ 27. 44 chickens and 28 cows 28. $\frac{1 \pm \sqrt{13}}{2}$

29. a) 80.75 b) 87 c) yes, but she must earn 97 (or more on Exam 2) 30. 12 ten-dollar bills and 62 five-dollar bills

31. \$3800 in Stock A and \$1200 in Stock B 32. 1104 33. 0, 1 34. 62 quarters and 92 dimes

35. $y = x^2 - 6x + 10$ 36. $\sqrt{128} \text{ m} = 8\sqrt{2} \text{ m}$ 37. 7140 38. -10 with -32 and 16 with 20

39. 20 ft and 48 ft 40. -36 (6 and -6)