Equations are a fundamental concept and tool in mathematics.

> **Definition**: An **equation** is a statement in which two expressions (algebraic or numeric) are connected with an equal sign.

For example, $3x^2 - x = 4x + 28$ is an equation. So is $x^2 + 5y = -y^2 + x + 2$.

> **Definition**: A **solution** of an equation is a number (or an ordered set of numbers) that, when substituted into the variable(s) in the equation, makes the statement of equality true.

Example 1. a) Verify that -2 is not a solution of the equation $3x^2 - x = 4x + 28$.

b) Verify that 4 is a solution of the equation $3x^2 - x = 4x + 28$.

Solution: a) Consider the equation $3x^2 - x = 4x + 28$ with $x = -2$. We substitute $x = -2$ into both sides of the equation and evaluate the expressions.

If $x = -2$, the left-hand side of the equation is	If $x = -2$, the right-hand side of the equation is
$\text{LHS} = 3x^2 - x$	$\text{RHS} = 4x + 28$
$= 3(-2)^2 - (-2)$	$= 4(-2) + 28$
$= 3 \cdot 4 + 2 = 12 + 2 = 14$	$= -8 + 28 = 20$

Since the two sides are not equal, $14 \neq 20$, the number -2 is not a solution of this equation.

b) Consider the equation $3x^2 - x = 4x + 28$ with $x = 4$. We evaluate both sides of the equation after substituting 4 into x.

If $x = 4$, the left-hand side of the equation is	If $x = 4$, the right-hand side of the equation is
$\text{LHS} = 3x^2 - x$	$\text{RHS} = 4x + 28$
$= 3 \cdot 4^2 - 4$	$= 4 \cdot 4 + 28$
$= 3 \cdot 16 - 4 = 48 - 4 = 44$	$= 16 + 28 = 44$

Since the two sides are equal, $x = 4$ is a solution of this equation.

> **Definition**: To **solve an equation** is to find *all* solutions of it. The set of all solutions is also called the solution set.

Caution! Finding one solution for an equation is not the same as solving it. For example, the number 2 is a solution of the equation $x^3 = 4x$. However, -2 is also a solution of this equation.

If we think about it a little, trial and error is never a legitimate method because there is no way for us to gurantee that there are no other solutions are there. It is impossible for us to try all real numbers because there are infinitely many of them, and we have finite lives.

So we will need to develop systematic methods to solve equations.

We will start with the easiest group of equations, linear equations. There are several types of linear equations, and we will start with the easiest type that is called one-step equations.

To solve a linear equation, we isolate the unknown by applying the same operation(s) to both sides. Consider, for example, Ann and Dewitt who has the same monthly salary. This month they both get a 40 dollar raise. Who is making ore money now? It is clear that if we start with two equal quantities and we add the same amount to them, they will still stay equal. This is the underlying principle of solving equations. We always apply the same operations to both sides in an effort to bring the equation in a simple form such as $x = -2$. The following equations are **one-step equations** because there is only one operation that separates us from the desired form.

Example 2. Solve each of the given equations. Make sure to check your solutions.

a) $x - 8 = 10$ b) $3y = -12$ c) $\dfrac{x}{-3} = 8$ d) $m + 10 = -5$

Equations like these are called **one-step equations** because they can be solved in only one step. We need to isolate the unknown on one side. In order to do that, we perform the inverse operation. (The inverse operation of additon is subtraction and vica versa. The inverse operation of multiplication is division and vica versa.)

Solution: a) In order to isolate the unknown, we add 8 to both sides.

$$\begin{aligned} x - 8 &= 10 \qquad \text{add } 8 \\ x &= 18 \end{aligned}$$

So the only solution of this equation is 18. We can also say that the solution set is $\{18\}$. We should check; if $x = 18$, the left-hand side is

$$\text{LHS} = x - 8 = 18 - 8 = 10 = \text{RHS} \checkmark$$

So our solution, $\boxed{x = 18}$ is correct.

b) In order to isolate the unknown, we divide both sides by 3.

$$\begin{aligned} 3y &= -12 \qquad \text{dividde by } 3 \\ y &= -4 \end{aligned}$$

So the only solution of this equation is -4. We check; if $y = -4$, then

$$\text{LHS} = 3y = 3(-4) = -12 = \text{RHS} \checkmark$$

So our solution, $\boxed{y = -4}$ is correct.

c) In order to isolate the unknown, we multiply both sides by -3.

$$\begin{aligned} \frac{x}{-3} &= 8 \qquad \text{multiply by } -3 \\ x &= -24 \end{aligned}$$

So the only solution of this equation is -24. We check; if $x = -24$, then

$$\text{LHS} = \frac{-24}{-3} = 8 = \text{RHS} \checkmark$$

So our solution, $\boxed{x = -24}$ is correct.

d) In order to isolate the unknown, we we subtract 10 from both sides.

$$\begin{aligned} m+10 &= -5 \qquad \text{subtract } 10 \\ m &= -15 \end{aligned}$$

So the only solution of this equation is -15. We check; if $m=-15$, then

$$\text{LHS} = m+10 = -15+10 = -5 = \text{RHS}$$

So our solution, $\boxed{m=-15}$ is correct.

Discussion: Solve each of the following equations. How are these unusual?

a) $5x=5$ b) $5x=0$ c) $x-4=-4$ d) $\dfrac{x}{3}=0$

Example 3. One side of a rectangle is 12 feet long. Find the length of the other side if the area of the rectangle is 60 square-feet.

Solution: Let us denote the missing side by x. We will write and solve an equation expressing the area of the rectangle.

$$\begin{aligned} 12x &= 60 \qquad \text{divide by } 12 \\ x &= 5 \end{aligned}$$

Thus the other side is $\boxed{5 \text{ feet}}$ long. Note that if we carry the units in the computation, they will work out perfectly.

$$(12\,\text{ft})\,x = 60\,\text{ft}^2 \qquad \text{divide by } 12\,\text{ft} \qquad \text{margin work: } \frac{60\,\text{ft}^2}{12\,\text{ft}} = 5\frac{\text{ft}\cdot\text{ft}}{\text{ft}} = 5\,\text{ft}$$

$$x = 5\,\text{ft}$$

Sometimes we will solve equations in more abstract forms. The following examples are called **formulas** or **literal equations**. While they might be intimidating for students, the ideas and techniques are the same.

Example 4. Solve each of the given equations for the specified variable.

a) Solve $A+B=C$ for A b) $IR=V$ for I

Solution: a) All of A, B, and C are unknown, but the instructions identify A as the unknown in which we are interested. We preted that we know the values of B and C, we just don't care about them. So, first we ask: *what happened to our unknown*? On the left-hand side, we see $A+B$. This means that someone came along and added B to the unknown A. In order to isolate the unknown A, we need to 'undo' this operation, that is, we will subtract B from both sides. Although we do not know the value of B, we still are subtracting the same amount when subtracting B.

$$\begin{aligned} A+B &= C \qquad \text{subtract } B \\ A &= C-B \end{aligned}$$

What is somewhat unsettling is that the expression $C-B$ does not collapse to a number because their values are not known. Either way, the solution is $\boxed{A=C-B}$. We can still check, if $A=C-B$, then the left-hand side is

$$\text{LHS} = A+B = \underbrace{C-B}_{A}+B = C-B+B = C = \text{RHS} \ \checkmark$$

So our solution, $\boxed{A=C-B}$ is correct.

b) Consider now the equation $IR = V$. This equation is from physics, it is called Ohm's law. If we connect a lightbulb to a battery, the electric current created depends on properties of the light bulb and the battery. Ohm's law expresses the connection between resistance of the lightbulb (denoted by R), the potential or voltage of the battery (denoted by V), and the electric current (denoted by I). Our unknown is I. The unknown was multiplied by R. In order to isolate the unknown, we will divide both sides by R.

$$IR = V \qquad \text{divide by } R$$
$$I = \frac{V}{R}$$

So the only solution of this equation is $\boxed{I = \frac{V}{R}}$.

If we can compute with formulas in the abstract, one formula becomes many. We can solve $V = IR$ for I and get $I = \frac{V}{R}$ and also, solve $V = IR$ for R and get $R = \frac{V}{I}$.

There is an application of one-step equations that helps us with a tricky algebraic expression that comes up often. Suppose we want to express that two numbers add up to 10. If label one number by x, how can we label the other number?

Example 5. Suppose that x represents a number. Let y be another number such that the sum of x and y is 10. Express y in terms of x.

Solution: Our numbers are labeled x and y. We state that their sum is 10 and solve the equation for y in terms of x.

$$x + y = 10 \qquad \text{subtract } x$$
$$y = 10 - x$$

So y can be expressed in terms of x as $\boxed{10 - x}$.

Caution! $x - 10$ and $10 - x$ look similar but they are very different. $x - 10$ is a number ten less than x, while $10 - x$ is the number, that, when added to x, results in 10. If we confuse the two, we can quickly check which is which by evaluating the expressions using a few numbers for x. We come up with a few values for x, say -10, -5, 1, 6, 10, and 20. Then we evaluate $x - 10$ and $10 - x$ using these values for x.

x	-10	-5	1	6	10	20
$x - 10$	-20	-15	-9	-4	0	10

and

x	-10	-5	1	6	10	20
$10 - x$	20	15	9	4	0	-10

We can now easily tell which table's columns add up to 10 and which table has columns in which the second number is ten less than the first one. We needed a few values because sometimes we can get unlucky: notice that if x is 10, then both $10 - x$ and $x - 10$ give us the same zero.

Part 2 – Two-Step Equations

Suppose we decide to hide a small object, say a coin. We put the coin on the table, then place an envelope over it, and then, just to be sure, we place a hat on top of the envelope. Let us find the coin! To do that, what do we need to remove, and in what order? We would first remove the hat and then the envelope, right?

This is the basis of solving two-step equations. To isolate the unknown, we will perform the inverse operations, in the reverse order. What happened last can be undone first.

Example 6. Solve each of the given equations. Make sure to check your solutions.

a) $10 = 3x - 11$ b) $3x + 8 = -7$ c) $\frac{t-7}{2} = -8$ d) $\frac{x}{-3} + 4 = 15$

Solution: a) The equation $10 = 3x - 11$ looks unusual in the sense that two-step equations often contain the unknown on the left-hand side. We are always allowed to swap two sides of an equation. If $A = B$, then clearly, also $B = A$. We will do this first. This is an optional step that is always available.

$$\begin{aligned} 10 &= 3x - 11 && \text{we swap the two sides} \\ 3x - 11 &= 10 \end{aligned}$$

We now look at the side that contains x and ask: *What happened to the unknown*? The answer is: *Multiplication by* 3 *and then subtraction of* 11. We need to apply the inverse operations, in reverse order. In this case, this means that we will add 11 to both sides and then divide both sides by 3.

$$\begin{aligned} 3x - 11 &= 10 && \text{add } 11 \\ 3x &= 21 && \text{divide by } 3 \\ x &= 7 \end{aligned}$$

So the only solution of this equation is 7. We check: if $x = 7$, then

$$\text{LHS} = 3x - 11 = 3 \cdot 7 - 11 = 21 - 11 = 10 = \text{RHS} \checkmark$$

So our solution, $\boxed{x = 7}$ is correct.

b) As we look at the equation $3x + 8 = -7$, we ask: *What happened to the unknown*? The answer is: *Multiplication by* 3 *and then addition of* 8. We need to apply the inverse operations, in a reverse order. In this case, this means that we will subtract 8 from both sides and then divide both sides by 3.

$$\begin{aligned} 3x + 8 &= -7 && \text{subtract } 8 \\ 3x &= -15 && \text{divide by } 3 \\ x &= -5 \end{aligned}$$

So the only solution of this equation is -5. We check: if $x = -5$, then

$$\text{LHS} = 3x + 8 = 3(-5) + 8 = -15 + 8 = -7 = \text{RHS} \checkmark$$

So our solution, $\boxed{x = -5}$ is correct.

c) What happened to the unknown? On the left-hand side, there was a subtraction of 7 and then a division by 2. To reverse that, we will multiply both sides by 2 and then add 7 to both sides.

$$\begin{aligned} \frac{t-7}{2} &= -8 && \text{multiply by 2} \\ t-7 &= -16 && \text{add 7} \\ t &= -9 \end{aligned}$$

So the only solution of this equation is -9. We check: if $t=-9$, then

$$\text{LHS} = \frac{t-7}{2} = \frac{-9-7}{2} = \frac{-16}{2} = -8 = \text{RHS} \checkmark$$

So our solution, $\boxed{t=-9}$ is correct.

d) What happened to the unknown? On the left-hand side, there was a division by -3 and then an addition of 4. To reverse that, we will subtract 4 from both sides by and then multiply both sides by -3.

$$\begin{aligned} \frac{x}{-3}+4 &= 15 && \text{subtract 4} \\ \frac{x}{-3} &= 11 && \text{multiply by } -3 \\ x &= -33 \end{aligned}$$

So the only solution of this equation is -33. We check: if $x=-33$, then

$$\text{LHS} = \frac{x}{-3}+4 = \frac{-33}{-3}+4 = 11+4 = 15 = \text{RHS} \checkmark$$

So our solution, $\boxed{x=-33}$ is correct.

Example 7. The longer side of a rectangle is one feet shorter than three times the shorter side. How long is the shorter side if the longer side is 26 feet?

Solution: Let us denote the shorter side by x. Then the longer side can be expressed as $3x-1$.

$$\begin{aligned} 3x-1 &= 26 && \text{add 1} \\ 3x &= 27 && \text{divide by 3} \\ x &= 9 \end{aligned}$$

The shorter side was labeled x. We know now that x is 9. So the shorter side is $\boxed{9 \text{ feet}}$ long.

Example 8. The sum of three times a number and seven is -5. Find this number.

Solution: Let us denote our mystery number by x. The equation will be just the first sentence, translated to algebra. The sum of three times the number and seven is $3x+7$. So our equation is $3x+7=-5$. We will know the number if we solve this equation.

$$\begin{aligned} 3x+7 &= -5 && \text{subtract 7} \\ 3x &= -12 && \text{divide by 3} \\ x &= -4 \end{aligned}$$

Good news! We do not need to check if -4 is indeed the solution of the equation. What if we correctly solved the *wrong* equation? Recall that *we* came up with the equation, it was not given. Instead of checking the number against the equation, we should check if our solution satisfies the conditions stated in the problem. Is it true the sum of three times -4 and seven is -5? Indeed,
$3(-4)+7=-12+7=-5$. Thus our solution, $\boxed{-4}$, is correct.

Example 9. Solve each of the given equations for the unknown indicated.

a) $A=3B-C$ for B b) $A=3(B-C)$ for B

Solution: a) We are to solve the equation $A=3B-C$ for B. Two things happened to the unknown: first a multiplication by 3 and then C was subtracted. To isolate B, we will reverse those operations in the reverse order. This means that we will first add C and then divide by 3.

$$A=3B-C \qquad \text{add } C$$
$$A+C=3B \qquad \text{divide by 3}$$
$$\frac{A+C}{3}=B \qquad \text{and so } \boxed{B=\frac{A+C}{3}}.$$

b) The equation $A=3(B-C)$ is very similar to the previous one, because it involves the same two operations; only the order is different. We first subtract C and then multiply by 3. So we will divide by 3 first and then add C.

$$\begin{aligned} A &= 3(B-C) && \text{divide by 3} \\ \frac{A}{3} &= B-C && \text{add } C \\ \frac{A}{3}+C &= B \end{aligned}$$

Thus our solution is $\boxed{B=\frac{A}{3}+C}$.

Most often a presented method is not the only one possible. We can solve this equation differently, by first distributing 3 and then basically solving an equation very similar to the previous example.

$$\begin{aligned} A &= 3(B-C) && \text{distribute 3} \\ A &= 3B-3C && \text{add } 3C \\ A+3C &= 3B && \text{divide by 3} \\ \frac{A+3C}{3} &= B \end{aligned}$$

Both methods are correct, and the two results are the same, although they might appear different at first. Once we have more algebra skills under our belt, we will be able to verify that the two expressions are really the same.

The next example is not a two-step equation but it can be solved using the same method. We will see many steps. We will perform the inverse operations, in the reverse order. It is sort of like an onion we take apart. We can peel of always the outermost layer.

Example 10. Solve the given equation. $\dfrac{\dfrac{\dfrac{3x-1}{5}+2}{-2}-8}{4}=-2$

Solution: The unknown is on the left-hand side, and lots of operations were done to it. In order, there were: multiplication by 3, subtracting 1, division by 5, adding 2, division by -2, subtraction of 8 and division by 4. We will undo them in the reversed order. This means: first multiply by 4, then add 8, then multiply by -2, then subtract 2, then multiply by 5, then add 1, and finally divide by 3. So, that's the plan.

$$\begin{aligned}
\frac{\dfrac{\dfrac{3x-1}{5}+2}{-2}-8}{4} &= -2 && \text{multiply by 4}\\
\frac{\dfrac{3x-1}{5}+2}{-2}-8 &= -8 && \text{add 8}\\
\frac{\dfrac{3x-1}{5}+2}{-2} &= 0 && \text{multiply by } -2\\
\frac{3x-1}{5}+2 &= 0 && \text{subtract 2}\\
\frac{3x-1}{5} &= -2 && \text{multiply by 5}\\
3x-1 &= -10 && \text{add 1}\\
3x &= -9 && \text{divide by 3}\\
x &= -3
\end{aligned}$$

We check: if $x=-3$, then the left-hand side is

$$\begin{aligned}
\text{LHS} &= \frac{\dfrac{\dfrac{3(-3)-1}{5}+2}{-2}-8}{4} = \frac{\dfrac{\dfrac{-9-1}{5}+2}{-2}-8}{4} = \frac{\dfrac{\dfrac{-10}{5}+2}{-2}-8}{4} = \frac{\dfrac{-2+2}{-2}-8}{4} = \frac{\dfrac{0}{-2}-8}{4}\\
&= \frac{0-8}{4} = \frac{-8}{4} = -2 = \text{RHS} \checkmark
\end{aligned}$$

Thus our solution, $\boxed{x=-3}$ is correct.

Sample Problems

Solve each of the following equations. Make sure to check your solutions.

1. $2x-5=17$
2. $\dfrac{a-10}{5}=-3$
3. $\dfrac{t}{4}-10=-4$
4. $\dfrac{t-5}{12}=4$
5. $2x-7=-3$
6. $\dfrac{x+8}{3}=-2$
7. $\dfrac{x}{3}+8=-2$
8. $-2x+3=3$
9. $3(x+7)=36$
10. $3x-10=-10$
11. $-4x+6=-18$
12. $\dfrac{\dfrac{\frac{5x-1}{7}+3}{5}-10}{3}=-4$
13. Solve each of the given equations for the indicated variables.
    a) $2x+y=18$ for y b) $2x+y=18$ for x c) $3y-5x=15$ for y d) $3y-5x=15$ for x

Solve each of the following application problems.

14. Paul invested his money on the stock market. First he bet on a risky stock and lost half of his money. Then he became a bit more careful and invested money in more conservative stocks that involved less risk but also less profit. His investments made him 80 dollars. If he has 250 dollars in the stock market today, with how much money did he start investing?
15. In a hotel, the first night costs 45 dollars, and all additional nights cost 35 dollars. How long did Mr. Williams stay in the hotel if his bill was 325 dollars?

Practice Problems

Solve each of the following equations. Make sure to check your solutions.

1. $2x-3=-11$
2. $-2x-3=7$
3. $5x-3=17$
4. $\dfrac{x-3}{7}=-2$
5. $\dfrac{x}{7}-3=-1$
6. $-4x-3=13$
7. $\dfrac{a+1}{4}=-9$
8. $5x-6=-6$
9. $\dfrac{x}{7}-1=-3$
10. $-x+5=-7$
11. $\dfrac{2x-1}{7}=-3$
12. $5(x-2)=-20$
13. $\dfrac{x-8}{7}=-2$
14. $3b+13=-5$
15. $\dfrac{x}{3}-7=7$
16. $\dfrac{\dfrac{\frac{3x-1}{2}+4}{-5}-6}{2}+7=2$
17. $\dfrac{\dfrac{5\left(\frac{5x+2}{-7}-7\right)+11}{2}+12}{5}=2$

18. Solve each of the given equations for the indicated variables. You do not have to check your solutions.

    a) $P = 2a + 2b$ for a

    b) $PV = nRT$ for n

    c) $2x + 3y = -12$ for x

    d) $2x + 3y = -12$ for y

    e) $AB + C = D$ for A

Solve each of the following application problems.

19. Three times the difference of x and 7 is -15. Find x.

20. Ann and Bonnie are discussing their financial situation. Ann said: *If you take* 50 *bucks from me and then doubled what is left, I would have* \$300. Bonnie answers: *That's funny. If you doubled my money first and then took* \$50, *then I would have* \$300! How much do they each have?

21. Susan was asked about her age. She answered as follows: My big brother's age is six less than three times my age. How old is Susan if her big brother is 21 years old?

Answers

Discussion

a) 1 b) 0 c) 0 d) 0

One thing that is unusual in this problem is the idea of cancellation. Cancellation results in 0 or 1, depending on the operation.

Sample Problems

22. 11 2. -5 3. 24 4. 53 5. 2 6. -14 7. -30 8. 0 9. 5 10. 0 11. -4 12. -18

13. a) $y = 18 - 2x$ b) $x = \dfrac{18 - y}{2}$ c) $y = \dfrac{15 + 5x}{3}$ d) $x = \dfrac{15 - 3y}{-5}$ or $x = \dfrac{3y - 15}{5}$

14. \$340 15. 9 nights

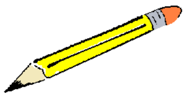
Practice Problems

1. -4 2. -5 3. 4 4. -11 5. 14 6. -4 7. -37 8. 0 9. -14 10. 12 11. -10

12. -2 13. -6 14. -6 15. 42 16. 11 17. -6

18. a) $a = \dfrac{P - 2b}{2}$ b) $n = \dfrac{PV}{RT}$ c) $x = \dfrac{-12 - 3y}{2}$ d) $y = \dfrac{-12 - 2x}{3}$ e) $A = \dfrac{D - C}{B}$

19. 2 20. Ann has \$200 and Bonnie has \$175 21. 9 years



Sample Problems - Solutions

Solve each of the following equations. Make sure to check your solutions.

1. $2x - 5 = 17$

Solution:

$$\begin{aligned} 2x - 5 &= 17 && \text{add 5 to both sides} \\ 2x &= 22 && \text{divide by 2} \\ x &= 11 \end{aligned}$$

We check: if $x = 11$, then

$$\text{RHS} = 2(11) - 5 = 22 - 5 = 17 = \text{LHS}$$

Thus our solution, $\boxed{x = 11}$ is correct.

2. $\dfrac{a - 10}{5} = -3$

Solution:

$$\begin{aligned} \frac{a - 10}{5} &= -3 && \text{multiply both sides by 5} \\ a - 10 &= -15 && \text{add 10 to both sides} \\ a &= -5 \end{aligned}$$

We check: if $a = -5$, then

$$\text{LHS} = \frac{-5 - 10}{5} = \frac{-15}{5} = -3 = \text{RHS}$$

Thus our solution, $\boxed{a = -5}$ is correct.

3. $\dfrac{t}{4} - 10 = -4$

Solution:

$$\begin{aligned} \frac{t}{4} - 10 &= -4 && \text{add 10 to both sides} \\ \frac{t}{4} &= 6 && \text{multiply both sides by 4} \\ t &= 24 \end{aligned}$$

We check: if $t = 24$, then

$$\text{RHS} = \frac{t}{4} - 10 = \frac{24}{4} - 10 = 6 - 10 = -4 = \text{LHS}$$

Thus our solution, $\boxed{t = 24}$ is correct.

4. $\dfrac{t-5}{12}=4$

Solution:

$$\begin{aligned} \frac{t-5}{12} &= 4 && \text{multiply both sides by 12} \\ t-5 &= 48 && \text{add 5 to both sides} \\ t &= 53 \end{aligned}$$

We check: if $t=53$, then $\quad \text{LHS} = \dfrac{53-5}{12} = \dfrac{48}{12} = 4 = \text{RHS}.$

Thus our solution, $\boxed{t=53}$ is correct.

5. $2x-7=-3$

Solution: We apply all operations to both sides.

$$\begin{aligned} 2x-7 &= -3 && \text{add } 7 \\ 2x &= 4 && \text{divide by 2} \\ x &= 2 \end{aligned}$$

We check: if $x=2$, then

$$\text{LHS} = 2(2)-7 = 4-7 = -3 = \text{RHS}$$

Thus our solution, $\boxed{x=2}$ is correct.

6. $\dfrac{x+8}{3}=-2$

Solution: We apply all operations to both sides.

$$\begin{aligned} \frac{x+8}{3} &= -2 && \text{multiply by } 3 \\ x+8 &= -6 && \text{subtract } 8 \\ x &= -14 \end{aligned}$$

We check: $\quad \text{LHS}= \dfrac{-14+8}{3} = \dfrac{-6}{3} = -2 = \text{RHS}$

Thus our solution, $\boxed{x=-14}$ is correct.

7. $\dfrac{x}{3}+8=-2$

Solution: We apply all operations to both sides.

$$\begin{aligned} \frac{x}{3}+8 &= -2 && \text{subtract } 8 \\ \frac{x}{3} &= -10 && \text{multiply by } 3 \\ x &= -30 \end{aligned}$$

We check:

$$\text{LHS} = \frac{-30}{3}+8 = -10+8 = -2 = \text{RHS}$$

Thus our solution, $\boxed{x=-30}$ is correct.

8. $-2x+3=3$

Solution: We apply all operations to both sides.

$$\begin{aligned} -2x+3 &= 3 && \text{subtract } 3 \\ -2x &= 0 && \text{divide by } -2 \\ x &= 0 \end{aligned}$$

We check: if $x=0$, then

$$\text{LHS} = -2\cdot 0+3 = 0+3 = 3 = \text{RHS}$$

Thus our solution, $\boxed{x=0}$ is correct.

9. $3(x+7)=36$

Solution: We apply all operation to both sides,

$$\begin{aligned} 3(x+7) &= 36 && \text{divide by } 3 \\ x+7 &= 12 && \text{subtract } 7 \\ x &= 5 \end{aligned}$$

We check: if $x=5$, then

$$\text{LHS} = 3(5+7) = 3\cdot 12 = 36 = \text{RHS}$$

Thus our solution, $\boxed{x=5}$ is correct.

10. $3x-10=-10$

Solution:

$$\begin{aligned} 3x-10 &= -10 && \text{add } 10 \text{ to both sides} \\ 3x &= 0 && \text{divide by } 3 \\ x &= 0 \end{aligned}$$

We check: if $x=0$, then

$$\text{LHS} = 3\cdot 0-10 = 0-10 = -10 = \text{RHS}$$

Thus our solution, $\boxed{x=0}$ is correct.

11. $-4x+6=-18$

Solution:

$$\begin{aligned} -4x+6 &= -18 && \text{subtract } 6 \\ -4x &= -24 && \text{divide by } -4 \\ x &= 6 \end{aligned}$$

We check:

$$\text{RHS} = -4x+6 = -4\cdot 6+6 = -24+6 = -18 = \text{LHS}$$

Thus our solution, $\boxed{x=6}$ is correct.

12. $\dfrac{\dfrac{\dfrac{5x-1}{7}+3}{5}-10}{3}=-4$

Soluton: This is one of those many-step equations where we have more than two steps, but we simply perform them in the reverse order like we did in the case of two-step equations.

$$\begin{aligned}
\frac{\dfrac{\dfrac{5x-1}{7}+3}{5}-10}{3} &= -4 && \text{multiply by 3}\\
\frac{\dfrac{5x-1}{7}+3}{5}-10 &= -12 && \text{add 10}\\
\frac{\dfrac{5x-1}{7}+3}{5} &= -2 && \text{multiply by 5}\\
\frac{5x-1}{7}+3 &= -10 && \text{subtract 3}\\
\frac{5x-1}{7} &= -13 && \text{multiply by 7}\\
5x-1 &= -91 && \text{add 1}\\
5x &= -90 && \text{divide by 5}\\
x &= -18
\end{aligned}$$

We check: if $x=-18$, then

$$\begin{aligned}
\text{LHS} &= \frac{\dfrac{\dfrac{5(-18)-1}{7}+3}{5}-10}{3} = \frac{\dfrac{\dfrac{-90-1}{7}+3}{5}-10}{3} = \frac{\dfrac{\dfrac{-91}{7}+3}{5}-10}{3} = \frac{\dfrac{-13+3}{5}-10}{3}\\
&= \frac{\dfrac{-10}{5}-10}{3} = \frac{-2-10}{3} = \frac{-12}{3} = -4 = \text{RHS} \checkmark
\end{aligned}$$

Thus our solution, $\boxed{x=-18}$ is correct.

13. Solve each of the given equations for the indicated variables.

a) $2x+y=18$ for y

Solution: We can isolate y by subtracting $2x$ from both sides.

$$2x+y=18 \qquad \text{subtract } 2x$$

$$\boxed{y=18-2x}$$

b) $2x+y=18$ for x

Solution: We can isolate x by subtracting y and then dividing by 2.

$$\begin{aligned}
2x+y &= 18 && \text{subtract } y\\
2x &= 18-y && \text{divide by 2}\\
x &= \frac{18-y}{2}
\end{aligned}$$

So $\boxed{x = \dfrac{18-y}{2}}$.

c) $3y - 5x = 15$ for y

Solution: We can isolate y by first adding $5x$ to both sides and then by dividing by 3.

$$3y - 5x = 15 \qquad \text{add } 5x$$

$$3y = 15 + 5x \qquad \text{divide by } 3$$

$$\boxed{y = \frac{15+5x}{3}}$$

d) $3y - 5x = 15$ for x

We will present two methods to solve this.

Method 1.

$$3y - 5x = 15 \qquad \text{subtract } 3y$$

$$-5x = 15 - 3y \qquad \text{divide by } -5$$

$$\boxed{x = \frac{15-3y}{-5}}$$

The problem with this method is that in the last step we were forced to divide by a negative number. We will later see that division by a negative number has its dangers. This can be avoided with using the following other method.

Method 2.

$$3y - 5x = 15 \qquad \text{add } 5x$$

$$3y = 15 + 5x \qquad \text{subtract } 15$$

$$3y - 15 = 5x \qquad \text{divide by } 5$$

$$\boxed{\frac{3y-15}{5} = x}$$

Naturally, the two solutions are the same, but we do not know enough algebra yet to prove that the two expressions are really the same.

14. Paul invested his money on the stock market. First he bet on a risky stock and lost half of his money. Then he became a bit more careful and invested money in more conservative stocks that involved less risk but also less profit. His investments made him 80 dollars. If he has 250 dollars in the stock market today, with how much money did he start investing?

Solution: Let us denote the amount of money with which Paul started to invest by x. First he lost half of his money, so he had $\frac{x}{2}$. Then he gained 80 dollars end ended up with 250 dollars. So, we can write the equation $\frac{x}{2} + 80 = 250$. We will solve this two-step equation for x. What happened to the unknown was first division by 2 and then addition of 80. To reverse these operations, we will first subtract 80 and then multiply by 2.

$$\begin{aligned} \frac{x}{2} + 80 &= 250 && \text{subtract } 80 \\ \frac{x}{2} &= 170 && \text{multiply by } 2 \\ x &= 340 \end{aligned}$$

So Paul started with 340 dollars. We check: If we lose half of 340,dollars we have 170 dollars left. Then when we add 80,dollars we indeed end up with 250 dollars. So our solution is correct, $\boxed{\text{Paul started with 340 dollars}}$.

15. In a hotel, the first night costs 45 dollars, and all additional nights cost 35 dollars. How long did Mr. Williams stay in the hotel if his bill was 325 dollars?

Solution: Suppose that Mr. Williams stayed for the first night and the an additional x many nights. Then the bill would be $45+x\cdot 35$ or $35x+45$. So we write and then solve the equation $35x+45=325$.

$$\begin{aligned} 35x+45 &= 325 && \text{subtract 45} \\ 35x &= 280 && \text{divide by 35} \\ x &= 8 \end{aligned}$$

Thus Mr. Williams stayed in the hotel for 9 nights. Why not 8 if we got $x=8$? Remember, the first night was counted separately; there was the first night and then $x=8$ additional nights. This is why it is a good idea to read the text of the problem one more time before we state our final answer. So **Mr. Williams stayed 9 nights in the hotel**. We check: the bill for 9 nights would be $45+8(35)=325$, and so our solution is correct.

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