

The words *and* and *or* are used differently in mathematics from every day use. Their meaning in mathematics is more restricted than in the English language. First of all, statements or questions in mathematics that use these words are all yes or no questions.

Imagine a cold Monday morning when Sylvia arrives late to her first class, a mathematics class. To make things worse, it is exam day. The teacher stops her at the door and asks: "Did you bring a pen or a pencil?" What does an answer of yes mean? What does an answer of no mean? An answer of yes means that she either has a pen only, or a pencil only, or both. An answer of no means that she has neither.

Her next class is a drawing class where both pen and pencils are needed. There the teacher might ask her: "Did you bring a pen and pencil?" What does an answer of yes mean? What does an answer of no mean? An answer of yes means that she has both a pen and a pencil. An answer of no means that she has only pen, or only pencil, or neither. This is the only allowed use of the words *and* and *or* in mathematics.

**Definition:** Suppose that A and B are statements. The statement **A or B** is true when A is true, or when B is true, or when both A and B are true. A or B is false if both A and B are false.

The statement **A and B** is true when both A and B is true. A and B is false when either A is false, or B is false, or both A and B are false. We can express this using truth tables.

Truth table for A or B

A	B	A or B
true	true	true
true	false	true
false	true	true
false	false	false

Truth table for A and B

A	B	A and B
true	true	true
true	false	false
false	true	false
false	false	false

When a single statement is formed by connecting two or more statements with *and* or *or*, we call such a statement a **compound statement**.

**Example 1.** Determine whether the given statements are true or false.

- a) *The sky is blue or the Earth is flat.*                      b) *The sky is blue and the Earth is flat.*

**Solution:** These compound statements are made by connecting two statements. These statements are:

*The sky is blue.* - this is true.      and      *The Earth is flat.* - this is false.

- a) *The sky is blue **or** the Earth is flat.*

This is true because one true statement is enough for an 'or' statement to be true.

- b) *The sky is blue **and** the Earth is flat.*

This is false because for an 'and' statement to be true, both statements must be true.

**Example 2.** Determine whether the given statements are true or false.

- a) *The number 8 is greater than 8 and is equal to 8.*                      b) *The number 8 is greater than 8 or is equal to 8.*

**Solution:** *The number 8 is greater than 8* - this is false. *The number 8 is equal to 8* - this is true.

- a) When we connect a true and a false statement with 'and', the compound statement is false.

So, *The number 8 is greater than 8 and is equal to 8.*— is false.

- b) When we connect a true and a false statement with 'or', the compound statement is true.

*The number 8 is greater than 8 or is equal to 8.*—is true, and we write it as  $8 \geq 8$ .

**Example 3.** Consider the numbers 1, 2, 3, 4, 5, 6, and 7.

- a) Find all numbers from the list for which the following is true: The number is even and greater than 3.
- b) Find all numbers from the list for which the following is true: The number is even or greater than 3.

**Solution:** a) We simply take all the numbers on the list, one by one, and check whether the statement is true or false.

*The number 1 is even and greater than 3.* This compound statement is obviously false since both statements making up the compound statement are false. We move on to 2.

*The number 2 is even and greater than 3.* This compound statement is false because the second statement making up the compound statement is false. In case of *and*, both statements need to be true for a true statement. We continue with 3.

*The number 3 is even and greater than 3.* This compound statement is false because both statements making up the compound statement are false. We move on to 4.

*The number 4 is even and greater than 3.* This compound statement is true because both statements making up the compound statement are true. The number 4 is going to be on our list. We move on to 5.

*The number 5 is even and greater than 3.* This compound statement is false because the first statement making up the compound statement is false. In case of *and*, both statements need to be true for a true statement. We continue with 6.

*The number 6 is even and greater than 3.* This compound statement is true because both statements making up the compound statement are true. The number 6 is also going to be on our list. We move on to 7.

*The number 7 is even and greater than 3.* This compound statement is false because the first statement making up the compound statement is false. In case of *and*, both statements need to be true for a true statement.

We found that 4 and 6 are the numbers on the list for which the given compound statement is true.

- b) This problem is very similar to the previous one. The only difference is the word connecting the two statements is *or* and not *and*. For an *or*-statement to be true, just one of the statements (or both) needs to be true. In this light, we test the numbers from the list again.

*The number 1 is even or greater than 3.* This compound statement is false because both statements making up the compound statement are false.

*2 is even or greater than 3.* This compound statement is true because the first statement making up the compound statement is true. In case of *or*, one true statement is enough. 2 is going to be on our list.

*3 is even or greater than 3.* This compound statement is false because both statements making up the compound statement are false.

*4 is even or greater than 3.* This compound statement is true because both statements making up the compound statement are true. 4 is going to be on our list.

*5 is even or greater than 3.* This compound statement is true because the second statement making up the compound statement is true. 5 is going to be on our list.

*6 is even or greater than 3.* This compound statement is true because both statements making up the compound statement are true. 6 is also going to be on our list.

*The number 7 is even or greater than 3.* This compound statement is true because the second statement making up the compound statement is true. 7 is going to be on our list.

We found that 2, 4, 5, 6, and 7 are the numbers on the list for which the given compound statement is true.



## Practice Problems

Label each of the given statements as true or false.

1. A week consists of seven days.
2. Every month consist of 31 days.
3. Water is a liquid at room temperature.
4. Water is frozen solid at a temperature of  $5F^{\circ}$ .
5.  $2 + 2 = 5$
6. 5 is an odd number.
7. 3 is an even number.
8. 2 is less than 10.
9. 2 is greater than 10.
10. 2 is equal to 10.
11. A week consists of seven days, or every month consist of 31 days.
12. A week consists of seven days, and every month consist of 31 days.
13.  $2 + 2 = 5$ , or water is a liquid at room temperature.
14.  $2 + 2 = 5$ , and water is a liquid at room temperature.
15. 2 is less than 10, or 2 is greater than 10.
16. 3 is an even number or  $2 + 2 = 5$ .
17. 2 is equal to 10 or 5 is an odd number.
18. 2 is less than 10 and water is frozen solid at a temperature of  $5F^{\circ}$ .
19. 7 is less than 7 or 7 is equal to 7. (We write this as  $7 \leq 7$ )
20. 7 is less than 7 and 7 is equal to 7.
21. Consider the numbers 1, 2, 3, 4, 5, 6, 7, and 8.
  - a) Find all numbers from the list for which the following is true: The number is odd and less than 6.
  - b) Find all numbers from the list for which the following is true: The number is odd or less than 6.



## Enrichment

1. Interpret A or B or C as  $(A \text{ or } B) \text{ or } C$  and create a truth table for this compound statement. How many different cases are there?
2. Interpret A and B and C as  $(A \text{ and } B) \text{ and } C$  and create a truth table for this compound statement. How many different cases are there?
3. Create a truth table for the compound statement  $(A \text{ and } B) \text{ or } C$ . How many different cases are there?
4. Create a truth table for the compound statement  $A \text{ and } (B \text{ or } C)$ . How many different cases are there?



## Answers

1. true 2. false 3. true 4. true 5. false 6. true 7. false 8. true 9. false 10. false 11. true  
 12. false 13. true 14. false 15. true 16. false 17. true 18. true 19. true 20. false  
 21. a) 1, 3, and 5 b) 1, 2, 3, 4, 5, 7

### Enrichment

2. there are 8 cases

A	B	C	A or B or C
true	true	true	true
true	true	false	true
true	false	true	true
false	true	true	true
true	false	false	true
false	true	false	true
false	false	true	true
false	false	false	false

4. there are 8 cases

A	B	C	(A and B) or C
true	true	true	true
true	true	false	true
true	false	true	true
false	true	true	true
true	false	false	false
false	true	false	false
false	false	true	true
false	false	false	false

3. there are 8 cases

A	B	C	A and B and C
true	true	true	true
true	true	false	false
true	false	true	false
false	true	true	false
true	false	false	false
false	true	false	false
false	false	true	false
false	false	false	false

5. there are 8 cases

A	B	C	A and (B or C)
true	true	true	true
true	true	false	true
true	false	true	true
false	true	true	false
true	false	false	false
false	true	false	false
false	false	true	false
false	false	false	false