

The words *and* and *or* are used differently in mathematics from every day use. Their meaning in mathematics is more restricted than in the English language. First of all, statements or questions in mathematics that use these words are all yes or no questions.

Imagine a cold Monday morning when Sylvia arrives late to her first class, a mathematics class. To make things worse, it is exam day. The teacher stops her at the door and asks: "Did you bring a pen or a pencil?" What does an answer of yes mean? What does an answer of no mean? An answer of yes means that she either has a pen only, or a pencil only, or both. An answer of no means that she has neither.

Her next class is a drawing class where both pen and pencils are needed. There the teacher might ask her: "Did you bring a pen and pencil?" What does an answer of yes mean? What does an answer of no mean? An answer of yes means that she has both a pen and a pencil. An answer of no means that she has only pen, or only pencil, or neither. This is the only allowed use of the words and and or in mathematics.

Suppose that A and B are statements. The statement **A or B** is true when A is true, or when B is true, or when both A and B are true. A or B is false if both A and B are false.

The statement **A and B** is true when both A and B is true. A and B is false when either A is false, or B is false, or both A and B are false. We can express this using truth tables.

Truth table for A or B

A	B	A or B
true	true	true
true	false	true
false	true	true
false	false	false

Truth table for A and B

A	B	A and B
true	true	true
true	false	false
false	true	false
false	false	false

When a single statement is formed by connecting two or more statements with *and* or *or*, we call such a statement a **compound statement**.

Example 1. Determine whether the given statements are true or false.

- a) *The sky is blue or the Earth is flat.* b) *The sky is blue and the Earth is flat.*

Solution: These compound statements are made by connecting two statements. These statements are:

The sky is blue. - this is true. and *The Earth is flat.* - this is false.

- a) *The sky is blue **or** the Earth is flat.*

This is true because one true statement is enough for an 'or' statement to be true.

- b) *The sky is blue **and** the Earth is flat.*

This is false because for an 'and' statement to be true, both statements must be true.

Example 2. Determine whether the given statements are true or false.

- a) *The number 8 is greater than 8 and is equal to 8.* b) *The number 8 is greater than 8 or is equal to 8.*

Solution: *The number 8 is greater than 8* - this is false. *The number 8 is equal to 8* - this is true.

- a) When we connect a true and a false statement with 'and', the compound statement is false.

So, *The number 8 is greater than 8 and is equal to 8.*— is false.

- b) When we connect a true and a false statement with 'or', the compound statement is true.

The number 8 is greater than 8 or is equal to 8.—is true, and we write it as $8 \geq 8$.



Practice Problems

Label each of the given statements as true or false.

1. A week consists of seven days.
2. Every month consist of 31 days.
3. Water is a liquid at room temperature.
4. Water is frozen solid at a temperature of $5F^{\circ}$.
5. $2 + 2 = 5$
6. 5 is an odd number.
7. 3 is an even number.
8. 2 is less than 10.
9. 2 is greater than 10.
10. 2 is equal to 10.
11. A week consists of seven days, or every month consist of 31 days.
12. A week consists of seven days, and every month consist of 31 days.
13. $2 + 2 = 5$, or water is a liquid at room temperature.
14. $2 + 2 = 5$, and water is a liquid at room temperature.
15. 2 is less than 10, or 2 is greater than 10.
16. 3 is an even number or $2 + 2 = 5$.
17. 2 is equal to 10 or 5 is an odd number.
18. 2 is less than 10 and water is frozen solid at a temperature of $5F^{\circ}$.
19. 7 is less than 7 or 7 is equal to 7. (We write this as $7 \leq 7$)
20. 7 is less than 7 and 7 is equal to 7.



Enrichment

1. Interpret A or B or C as $(A \text{ or } B) \text{ or } C$ and create a truth table for this compound statement. How many different cases are there?
2. Interpret A and B and C as $(A \text{ and } B) \text{ and } C$ and create a truth table for this compound statement. How many different cases are there?
3. Create a truth table for the compound statement $(A \text{ and } B) \text{ or } C$. How many different cases are there?
4. Create a truth table for the compound statement $A \text{ and } (B \text{ or } C)$. How many different cases are there?



Answers

Practice Problems

1. true 2. false 3. true 4. true 5. false 6. true 7. false 8. true 9. false
 10. false 11. true 12. false 13. true 14. false 15. true 16. false 17. true
 18. true 19. true 20. false

Enrichment

1. there are 8 cases

A	B	C	A or B or C
true	true	true	true
true	true	false	true
true	false	true	true
false	true	true	true
true	false	false	true
false	true	false	true
false	false	true	true
false	false	false	false

3. there are 8 cases

A	B	C	(A and B) or C
true	true	true	true
true	true	false	true
true	false	true	true
false	true	true	true
true	false	false	false
false	true	false	false
false	false	true	true
false	false	false	false

2. there are 8 cases

A	B	C	A and B and C
true	true	true	true
true	true	false	false
true	false	true	false
false	true	true	false
true	false	false	false
false	true	false	false
false	false	true	false
false	false	false	false

4. there are 8 cases

A	B	C	A and (B or C)
true	true	true	true
true	true	false	true
true	false	true	true
false	true	true	false
true	false	false	false
false	true	false	false
false	false	true	false
false	false	false	false