# Part 1 – Algebraic Expressions

We will now start studying the expressions and statements frequently made in algebra.

> **Definition**: A **numerical expression** is an expression that combines numbers and operations. To **evaluate** a numerical expression is to compute its value.

For example, $3 \cdot 5^2$ is a numerical expression. So are $-\dfrac{12}{3+1}$ and $5^2 - 2^2$ and $-|-5|$. We can evaluate numerical expressions by performing the operations indicated in the expression. Naturally, we must correctly apply the order of operations agreement. For that, we must clearly understand notation. It is a fundamental principle that notation can be objectively understood in only *one* possible way.

**Example 1**. Evaluate each of the given numerical expressions.

a) $3 \cdot 5^2$ b) $-\dfrac{12}{3+1}$ c) $3^2 + 2^2$ d) $(3+2)^2$ e) $-3^2$ f) $(-3)^2$ g) $-|-5|$

**Solution:** a) Between exponentiation and multiplication, we first perform the exponentiation. $3 \cdot 5^2 = 3 \cdot 25 = \boxed{75}$

b) The addition in the denominator must be performed before we divide. (Why?) $-\dfrac{12}{3+1} = -\dfrac{12}{4} = \boxed{-6}$

c) $3^2 + 2^2 = 9 + 4 = \boxed{13}$

d) $(3+2)^2 = 5^2 = \boxed{25}$

Note: The error of confusing $3^2 + 2^2$ with $(3+2)^2$ is called the "Freshman's Dream Error".

e) $-3^2 = \boxed{-9}$

f) $(-3)^2 = \boxed{9}$

Note: In looking at $-3^2$ and $(-3)^2$ , we can interpret the minus sign in front of 3 as 'the opposite of'. That is the same as multiplication by $-1$. Now we can apply order of opertaions, and exponentiation comes before multiplication.

$$-3^2 = -1 \cdot 3^2 = -1 \cdot 9 = -9 \quad \text{but} \quad (-3)^2 = (-3)(-3) = 9$$

In the case of $-3^2$, we take the opposite of the square of three.
In the case of $(-3)^2$, we square the opposite of three.

g) $-|-5| = \boxed{-5}$ This is a perfect example that two minuses don't always make a plus. What happens here?

> **Definition**: An **algebraic expression** is an expression that combines numbers, operations, and variables.

**Variables** are letters that represent numbers. They are subjects to the same rules as numbers. We use variables for different reasons. Often because the variable represents a number we do not know but would like to know. However, there are other reasons. Sometimes we use variables because we would like to discuss general statements. Consider the following statement as an example. "*For every numbers $x$ and $y$,* $x + y = y + x$". This statement is not about the numbers $x$ and $y$; it is rather about the operation addition. No matter what two numbers we add, the order of the numbers in the addition does not matter. We express this property of addition by saying that addition is commutative. In this case, we use variables because we would like to talk about all the numbers at the same time.

In our modern language, we prefer to use $x$ as a variable, especially if it is the type of unknown we want to find. A fundamental principle is that within the context of a problem, a variable has one fixed meaning. Suppose that we are solving a word problem that involves the number of books and the number of pencils. In this case, we can not label both of them $x$, unless we are certain that the number of books is the same as the number of pencils. **To denote quantities that may be different, we must use different letters.** For example, we can denote the number of books by $x$ and the number of pencils by $y$. We might find out later in the problem that the values of $x$ and $y$ might be equal. That is perfectly fine. We must use different variables for quantities that *could* be different.

For example, $3x^2 - 1$ is an algebraic expression. So are $-x + 3$ and $2a - b$ and $5y + 3$. If the values of all variables in an expression are given, we can evaluate it. To evaluate an algebraic expression, we substitute the given values of the variables into the expression, and evaluate the resulting numerical expression.

When subtituting a number into an algebraic expression, it is our responsibility to preserve the indicated operations and their order correctly. Consider the expression $2x$. If $x = 5$, we can not write 25 instead of $2x$, because our notation would indicate a two-digit number with no operation. If $x = -5$, we can not write $2 - 5$ instead of $2x$, because we would incorrectly indicate subtraction instead of the multiplication.

$2x$ when $x = 5 \implies 2 \cdot 5$ or $2(5)$ $\qquad$ $2x$ when $x = -5 \implies 2(-5)$ or $2 \cdot (-5)$

> To evaluate an algebraic expression, we can perform the following steps.
>
> Step 1. Copy the entire expression with one modification: replace each variable by an empty pair of parentheses.
>
> Step 2. Insert the values into the parentheses. Now the problem became an order of operations problem.
>
> Step 3. Drop the unnecessary parentheses and work out the order of operations problem. (It may appear awkward first to create all these parentheses but they are extremely helpful.)

**Example 2**. Evaluate the algebraic expression $3x^2 - x + 5$ given the values of $x$.

a) $x = -2$ $\qquad$ b) $x = 3$

**Solution:** a) We first copy the entire expression, replacing the letter $x$ by little pairs of parentheses.

$$3x^2 - x + 5 = 3(\ )^2 - (\ ) + 5$$

Then we insert the number $-2$ into each pair of parentheses.

$$3x^2 - x + 5 = 3(-2)^2 - (-2) + 5$$

Now the problem became an order of operations problem. We start with the exponent.

$$\begin{aligned} 3(-2)^2 - (-2) + 5 &= 3 \cdot 4 - (-2) + 5 && \text{perform multiplication} \\ &= 12 - (-2) + 5 && \text{subtraction: } 12 - (-2) = 12 + 2 = 14 \\ &= 14 + 5 && \text{addition} \\ &= \boxed{19} \end{aligned}$$

Notice that because we substituted a negative value for $x$, all little parentheses proved to be necessary.

Students' work should look like this:

$$\begin{aligned} 3x^2 - x + 5 &= 3(-2)^2 - (-2) + 5 \\ &= 3 \cdot 4 + 2 + 5 \\ &= 12 + 2 + 5 = \boxed{19} \end{aligned}$$

b) Evaluate $3x^2 - x + 5$ when $x = 3$.

We first copy the entire expression, replacing the letter $x$ by little pairs of paretheses.

$3x^2 - x + 5 = 3(\ )^2 - (\ ) + 5$

Then we insert the number 3 into each pair of parentheses.

$3x^2 - x + 5 = 3(3)^2 - (3) + 5$

Because we substituted a positive number, most parentheses are unnecessary. We will drop them:

$3x^2 - x + 5 = 3(3)^2 - (3) + 5 = 3 \cdot 3^2 - 3 + 5$

Then we solve the resulting order of operations problem. We start with the exponent.

$$\begin{aligned} 3 \cdot 3^2 - 3 + 5 &= 3 \cdot 9 - 3 + 5 && \text{perform multiplication} \\ &= 27 - 3 + 5 && \text{subtraction} \\ &= 24 + 5 && \text{addition} \\ &= \boxed{29} \end{aligned}$$

**Example 3.** We ejected a small object upward from the top of a 720 ft tall building and started measuring time in seconds. We find that $t$ seconds after launching, the vertical position of the object is $-16t^2 + 64t + 720$ feet.

a) Evaluate the given expression with $t = 0$. What does your result mean?

b) Where is the object 5 seconds after launch?

c) Where is the object 9 seconds after launch?

**Solution:** a) We evaluate the expression with $t = 0$.

$$\begin{aligned} -16t^2 + 64t + 720 &= -16 \cdot 0^2 + 64 \cdot 0 + 720 \\ &= -16 \cdot 0 + 64 \cdot 0 + 720 = \boxed{720} \end{aligned}$$

This means that at the time of the launch of the object, it is at a height of 720 feet.

b) To find out where the object is after 5 seconds, we evaluate the expression with $t = 5$.

$$\begin{aligned} -16t^2 + 64t + 720 &= -16 \cdot 5^2 + 64 \cdot 5 + 720 \\ &= -16 \cdot 25 + 64 \cdot 5 + 720 \\ &= -400 + 320 + 720 = -80 + 720 = 640 \end{aligned}$$

So the object is at a height of $\boxed{640 \text{ feet}}$ 5 seconds after launch.

c) To find out where the object is after 9 seconds, we evaluate the expression with $t = 9$.

$$\begin{aligned} -16t^2 + 64t + 720 &= -16 \cdot 9^2 + 64 \cdot 9 + 720 \\ &= -16 \cdot 81 + 64 \cdot 9 + 720 \\ &= -1296 + 576 + 720 = -720 + 720 = 0 \end{aligned}$$

When we started at the top of the building, the location was 720. This means that a height of zero indicates that the object is at the ground. So the object is at a height of $\boxed{0 \text{ feet}}$ 9 seconds after launch.

Every time we enlarge our language, we have to return to old concepts and discuss them in the light of the new concepts. This is the case with the negative sign. When learning about the order of operations, we discussed that if a negative sign can denote subtraction, then it does denote subtraction. If not, then it is a sign describing the thing after it as *negative*, or *opposite of*. This might have seemed silly until now. Why would we say "*the opposite of three*" instead of "*negative three*"? With the introduction of variables, this is the moment when this interpretation becomes useful.

It is a common mistake to assume that the negative sign in $-x$ means that $-x$ is negative. This is incorrect. We are much better off thinking of $-x$ as "*the opposite of* $x$".

**Example 4**. Evaluate $-x$ with the given values of $x$.

a) when $x = 5$ b) when $x = -4$

**Solution:** a) We can carefully use notation: $-x = -(5) = \boxed{-5}$.
We can also use language: $-x$ is the opposite of $x$. If $x$ is 5, its opposite is $-5$.

b) We can carefully use notation: $-x = -(-4) = \boxed{4}$.
We can also use language: $-x$ is the opposite of $x$. If $x$ is $-4$, its opposite is 4.

**Example 5**. Evaluate each of the given expressions with the given values of $x$.

a) $-x^2$ when $x = 5$ c) $(-x)^2$ when $x = 2$ e) $-(-x)^2$ when $x = 6$

b) $-x^2$ when $x = -5$ d) $(-x)^2$ when $x = -2$ f) $-(-x)^2$ when $x = -6$

**Solution:** a) $-x^2$ is the opposite of the square of $x$. We square 5 and then take the opposite.
$-x^2 = -(5)^2 = -1 \cdot 5^2 = \boxed{-25}$

b) We square $-5$ and then take the opposite.
$-x^2 = -(-5)^2 = -1 \cdot (-5)^2 = -1 \cdot 25 = \boxed{-25}$.
If we think about this, it is not so surprising that we got the same answer. The square of a number and its opposite is the same. Then if we take the opposite of both, we still have the same result.

c) $(-x)^2$ means the square of the opposite of $x$. When we carefully substitute $x = 2$, we will have two pairs of parentheses. One came with the expression, the other with the substitution.
$(-x)^2 = (-(2))^2 = (-2)^2 = \boxed{4}$.

d) $(-x)^2$ means the square of the opposite of $x$. When we carefully substitute $x = -2$, we will have two pairs of parentheses. One came with the expression, the other with the substitution.
$(-x)^2 = (-(-2))^2 = 2^2 = \boxed{4}$.

e) $-(-x)^2$ means that we take the opposite of $x$, we square it, and then take the opposite again. At this point, notation might be the safest way.
$-(-x)^2 = -(-(6))^2 = -(-6)^2 = -1 \cdot (-6)^2 = -1 \cdot 36 = \boxed{-36}$.

f) $-(-x)^2$ means that we take the opposite of $x$, we square it, and then take the opposite again.
$-(-x)^2 = -(-(-6))^2 = -(6)^2 = -1 \cdot 6^2 = -1 \cdot 36 = \boxed{-36}$.

**Discussion**: Evaluate each of the following algebraic expressions with $x = 2$ and $x = -2$. How are these results similar to or different from the results in Example 4? Can you explain why?

a) $-x^3$ b) $(-x)^3$ c) $-(-x)^3$

## Part 2 – Algebraic Statements

The most frequantly made statements in algebra are equations and inequalities.

> **Definition:** An **equation** is a pair of numerical or algebraic expressions connected with an equal sign.

For example, $2+3=9$ and $1+2a=5b$ are equations. So are $x+y=y+x$, and $x^3+4=x^2+4x$. Some equations have variables in them, some don't.

> **Definition:** A **solution of an equation** is a value (or ordered pair of values) of the unknown(s) that, when substituted into both sides of the equation, makes the statement of equality true.

**Example 6.** Consider the equation $x^3+4=x^2+4x$. Evaluate both sides of the equation with the given value of $x$ to determine whether it is a solution of the equation or not.

a) $x=3$ b) $x=-2$ c) $x=-1$ d) $x=2$

**Solution:** a) We substitute $x=3$ into both sides of the equation $x^3+4=x^2+4x$ and compare the values. We will denote the left-hand side by LHS and the right-hand side by RHS.

If $x=3$, then

| | |
|---|---|
| LHS $=3^3+4$ <br> $=27+4=31$ | RHS $=3^2+4\cdot 3$ <br> $=9+12=21$ |

The left-hand side is 31, and the right-hand side is 21. Since these are not equal, $x=3$ is not a solution of the given equation. In short, LHS $=31\neq 21=$ RHS, so 3 is not a solution.

b) We substitute $x=-2$ into both sides of the equation $x^3+4=x^2+4x$ and compare the values.

If $x=-2$, then

| | |
|---|---|
| LHS $=(-2)^3+4$ <br> $=-8+4=-4$ | RHS $=(-2)^2+4(-2)$ <br> $=4+(-8)=-4$ |

Since the two sides are equal, $-2$ is a solution of the equation.

c) We substitute $x=-1$ into both sides of the equation $x^3+4=x^2+4x$.

If $x=-1$, then

| | |
|---|---|
| LHS $=(-1)^3+4$ <br> $=-1+4=3$ | RHS $=(-1)^2+4(-1)$ <br> $=1+(-4)=-3$ |

Since the two sides are not equal, $-1$ is not a solution of the equation.

d) We substitute $x=2$ into both sides of the equation $x^3+4=x^2+4x$.

If $x=2$, then

| | |
|---|---|
| LHS $=2^3+4$ <br> $=8+4=12$ | RHS $=2^2+4\cdot 2$ <br> $=4+8=12$ |

Since the two sides are equal, 21 is a solution of the equation.

We have seen that both 2 and $-2$ are solutions of the equation $x^3 + 4 = x^2 + 4x$. We will leave it for the reader to verify that $x = 1$ is also a solution. So, it is not true that equations can have only one solution! To solve an equation means to find *all* solutions of it.

**Example 7.** Consider the equation $1 + 2a = 5b$. Evaluate both sides of the equation with the given ordered pairs of values to determine whether they are a solution of the equation or not.

a) $a = 7$ and $b = 3$ b) $a = 3$ and $b = 7$

**Solution:** a) The pair $a = 7$ and $b = 3$ is often denoted by $(7, 3)$. We substitute these numbers into both sides of the equation $1 + 2a = 5b$.

$$\text{LHS} = 1 + 2 \cdot 7 = 1 + 14 = 15 \quad \text{and} \quad \text{RHS} = 5 \cdot 3 = 15$$

The left-hand side and the right-hand side are both 15. Thus the ordered pair $(7, 3)$ is a solution of the equation.

b) We substitute $a = 3$ and $b = 7$ into both sides of the equation.

$$\text{LHS} = 1 + 2 \cdot 3 = 1 + 6 = 7 \quad \text{and} \quad \text{RHS} = 5 \cdot 7 = 35$$

The values of the right-hand side and the left-hand side are different, and so the ordered pair $(3, 7)$ is not a solution of the equation. Notice that $(7, 3)$ is a solution, but $(3, 7)$ is not. This is why we call such pairs *ordered* pairs.

**Definition:** An **inequality** is a pair of numerical or algebraic expressions connected with an inequality sign.

For example, $3x - 1 \geq 5x - 7$, and $2x < 4y - 5$, and $2x^2 \leq 7x - 4$ are all inequalities.

**Definition:** A **solution of an inequality** is a value (or ordered pair of values) of the unknown(s) that, when substituted into both sides of the inequality, makes the statement of inequality true.

**Example 8.** Consider the equation $3x - 1 \geq 5x - 7$. Evaluate both sides of the equation with the given value of $x$ to determine whether it is a solution of the inequality or not.

a) $x = 5$ b) $x = -5$ c) $x = 3$ d) $x = -3$

**Solution:** a) We evaluate both sides of the inequality $3x - 1 \geq 5x - 7$ with $x = 5$ to see whether the inequality statement is true.

If $x = 5$, then

| | |
|---|---|
| $\text{LHS} = 3 \cdot 5 - 1$ $= 15 - 1 = 14$ | $\text{RHS} = 5 \cdot 5 - 7$ $= 25 - 7 = 18$ |

This means that if $x = 5$, then $3x - 1 \geq 5x - 7$ becomes

$$14 \geq 18$$

This statement is false, therefore 5 is not a solution of the inequality.

b) We evaluate both sides of the inequality with $x = -5$.

If $x = -5$, then

| | |
|---|---|
| $\text{LHS} = 3(-5) - 1$ <br> $= -15 - 1 = -16$ | $\text{RHS} = 5(-5) - 7$ <br> $= -25 - 7 = -32$ |

This means that if $x = -5$, then $3x - 1 \geq 5x - 7$ becomes

$$-16 \geq -32$$

This statement is true, therefore $-5$ is a solution of the inequality.

c) We evaluate both sides of the inequality with $x = 3$.

If $x = 3$, then

| | |
|---|---|
| $\text{LHS} = 3 \cdot 3 - 1$ <br> $= 9 - 1 = 8$ | $\text{RHS} = 5 \cdot 3 - 7$ <br> $= 15 - 7 = 8$ |

This means that if $x = 3$, then $3x - 1 \geq 5x - 7$ becomes $8 \geq 8$. This statement is true, therefore 3 is a solution of the inequality.

d) We evaluate both sides of the inequality with $x = -3$.

If $x = -3$, then

| | |
|---|---|
| $\text{LHS} = 3(-3) - 1$ <br> $= -9 - 1 = -10$ | $\text{RHS} = 5(-3) - 7$ <br> $= -15 - 7 = -22$ |

This means that if $x = -3$, then $3x - 1 \geq 5x - 7$ becomes $-10 \geq -22$. This statement is true, therefore $-3$ is a solution of the inequality.

## Part 3 – Translations

We will often need to translate English sentences into algebraic statements. While this might seem intimidating first, we just need to practice this skill. Here are a few basic statements for a start. Suppose our number is $x$.

| | | | | | |
|---|---|---|---|---|---|
| $y$ is twice as great as $x$. | $\Longrightarrow$ | $y = 2x$ | $y$ is two greater than $x$. | $\Longrightarrow$ | $y = x + 2$ |
| $y$ is three times as great as $x$. | | $y = 3x$ | $y$ is three more than $x$. | | $y = x + 3$ |
| $y$ is half of $x$. | | $y = \frac{x}{2}$ | $y$ is five less than $x$. | | $y = x - 5$ |
| $y$ is one-fifth of $x$. | | $y = \frac{x}{5}$ | | | |

Recall a few additional expressions we might also need.

| | | | |
|---|---|---|---|
| the sum of $a$ and $b$ | $\Longrightarrow$ | $a + b$ | the order of $a$ and $b$ does not matter |
| the product of $a$ and $b$ | | $ab$ | the order of $a$ and $b$ does not matter |
| the differrence of $a$ and $b$ | | $a - b$ | the order of $a$ and $b$ does matter (first mentioned, first written) |
| the quotient of $a$ and $b$ | | $\frac{a}{b}$ | the order of $a$ and $b$ does matter (first mentioned is upstairs) |
| the opposite of $a$ | | $-a$ | |

Notice that the statements "*$y$ is the sum of $x$ and three*" and "*$y$ is three greater than $x$*" will result in the same equation: $y = x + 3$. Similarly, "*$P$ is four less than $Q$*" and "*$P$ is the difference of $Q$ and four*" will both result in $P = Q - 4$. We will see all of these expressions frequently.

Just like in English, we can create many statements by combining these basic expressions and basic statements.

**Example 9.** Translate each of the following statements into an equation.

a) $M$ is the sum of $A$ and three times $B$.

b) $M$ is three times the sum of $A$ and $B$.

c) $y$ is three less than twice $x$.

d) $y$ is twice as much as the sum of three and $x$.

e) $A$ is three more than the product of $B$ and $C$.

f) The opposite of $m$ is five less than one-third of $n$.

**Solution:** a) *$M$ is the sum of $A$ and three times $B$.*

We can first translate "*three times $B$*" to $3B$. Then we have: "*$M$ is the sum of $A$ and $3B$*". Then we can translate "t*he sum of $A$ and $3B$*" to $A + 3B$. Thus the translation is $\boxed{M = A + 3B}$.

b) *$M$ is three times the sum of $A$ and $B$.*

We can first translate the sum of $A$ and $B$ into $A+B$. So now we have: *$M$ is three times* $A+B$. However, the translation $M = 3 \cdot A + B$ would be incorrect. This way we would multiply only $A$ by 3 and not the entire sum. $3 \cdot A + B$ is the sum of three times $A$ and $B$. If we want the sum to be multiplied by 3, we would have to force the multiplication to be performed after the addition. We can easily do that with a pair of parentheses; the correct answer is $\boxed{M = 3(A + B)}$.

c) Given: *$y$ is three less than twice $x$.*

We can translate "*twice $x$*" as $2x$. Then we have: $y$ is three less than $2x$. This is one of the basic statements: $\boxed{y = 2x - 3}$.

d) *$y$ is twice as much as the sum of three and $x$.*

We can first translate "*the sum of three and $x$*" into $3 + x$. Then we have: *$y$ is twice as much as* $3 + x$. This means that we get $y$ if we muttiply $3 + x$ by 2. However, $y = 2 \cdot 3 + x$ would be incorrect. According to order of operations, we would not multiply the sum by 2, only three. So the correct way to express this is $y = 2(x + 3)$. The parentheses overwrites the usual order of operations, so we really multiply the entire sum and not just parts of it. So the translation is $\boxed{y = 2(3 + x)}$.

e) *$A$ is three more than the product of $B$ and $C$.*

The product of $B$ and $C$ is simply $BC$. So now we have that *$A$ is three more than $BC$*. This is again one of the basic ones: $\boxed{A = BC + 3}$.

f) *The opposite of $m$ is five less than one-third of $n$.*

We can translate "*the opposite of $m$*" to $-m$, and "*one-third of $n$*" to $\dfrac{n}{3}$. Then we have: "*$-m$ is five less than $\dfrac{n}{3}$*", which can be translated to $\boxed{-m = \dfrac{n}{3} - 5}$.

Sometimes the same expressions show up "disguised". One such frequently occuring expression is c*onsecutive integers*. How can we translate three consecutive integers?

Consecutive means that they come right after the other, like 3, 4, and 5. If we denote the smallest number by $x$, then the other two would be $x + 1$ and $x + 2$. If we denote the largest number by $x$, then the three numbers would be expressed as $x$, $x - 1$, and $x - 2$. In word problems we often have the freedom of selecting which of the three numbers should we denote by $x$, and the best choice often depends on the particular problem.

How about four consecutive even numbers? First let us look at an example, say 6, 8, 10, and 12. We see that each one is two greater than the one before. So if we denote the smallest even number by $x$, then these numbers can be labeled as $x$, $x+2$, $x+4$, and $x+6$.

## Sample Problems

1. Evaluate each of the following numerical expressions.

   a) $2-5(3-7)$ b) $24-10+2$ c) $-4^2$ d) $(-4)^2$ e) $|3|-|8|$ f) $|3-8|$

2. Evaluate each of the algebraic expressions when $p=-7$ and $q=3$.

   a) $15-p$ b) $pq-|p-2|$ c) $4p-q^3$ d) $\dfrac{q^2-p}{2q+p+1}$ e) $p^2-q^2$ f) $(p-q)^2$ g) $2q^2$ h) $(2q)^2$ i) $15-\dfrac{p+q}{|1-p|}$ j) $(p+q)^2-(5q+2p)^4$ k) $-p^2-p+8$

3. Evaluate the expression $3x^2-x+5$ with the given values of $x$.

   a) $x=0$ b) $x=-1$

4. We ejected a small object upward from the top of a 720 ft tall building and started measuring time in seconds. We find that $t$ seconds after launching, the vertical position of the object is $-16t^2+64t+720$ feet.

   a) Where is the object 2 seconds after launch?

   b) Where is the object 8 seconds after launch?

5. Let $a=-4$, $b=2$, and $x=-3$. Evaluate each of the following expressions.

   a) $a^2-b^2$ b) $(a-b)^2$ c) $a^b-2bx-x^2-2x$ d) $\dfrac{-x^2+(x+2)^2}{(x-1)}$ e) $\dfrac{x-1}{x+3}$

6. Consider the equation $2x^2+x+34=21x-8$. In case of each number given, determine whether it is a solution of the equation or not.

   a) $x=1$ b) $x=3$ c) $x=4$ d) $x=7$

7. Consider the equation $x^2-10x+x^3-4=4(x+5)$. In case of each number given, determine whether it is a solution of the equation or not.

   a) $x=0$ b) $x=-2$ c) $x=-3$ d) $x=2$

8. Consider the equation $3a-2b-1=(a-b)^2+4$. In case of each pair of numbers given, determine whether it is a solution of the equation or not.

   a) $a=8$ and $b=5$ or, as an ordered pair, $(8,5)$

   b) $a=10$ and $b=7$ or, as an ordered pair, $(10,7)$

9. Consider the inequality $3(2y-1)+1\leq 5y-7$. In case of each number given, determine whether it is a solution of the inequality or not.

   a) $y=-10$ b) $y=3$ c) $y=-5$ d) $y=0$

10. Consider the inequality $\frac{2x+1}{3}+5<\frac{3x-1}{2}$. In case of each number given, determine whether it is a solution of the inequality or not.

    a) $x=1$ b) $x=13$ c) $x=7$ d) $x=-5$

11. Translate each of the following statements to an algebraic statement.

    a) $y$ is three less than four times $x$

    b) The opposite of $A$ is one greater than the difference of $B$ and three times $C$.

    c) Twice $M$ is five less than the product of $N$ and the opposite of $M$.

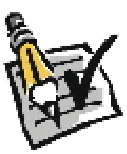
12. The longer side of a rectangle is three units shorter than five times the shorter side. If we label the shorter side by $x$, how can we express the longer side in terms of $x$?

13. Suppose we have three consecutive integers.

    a) Express them in terms of $x$ if $x$ denotes the smallest number.

    b) Express them in terms of $y$ if $y$ denotes the number in the middle.

    c) Express them in terms of $L$ if $L$ denotes the greatest number.



# Practice Problems

1. Evaluate each of the following numerical expressions.

    a) $24-5+1$ c) $-1^2$ e) $-|4|-|7|$ g) $6^2-4^2$

    b) $24\div 3\cdot 2$ d) $(-1)^2$ f) $-|4-7|$ h) $(6-4)^2$

2. Evaluate each of the algebraic expressions when $x=6$ and $y=8$.

    a) $19-y+x$ d) $x^2+y^2$ g) $3(y-x)$ i) $5x-\frac{y}{2}$

    b) $19-(y+x)$ e) $(x+y)^2$ h) $\frac{5x-y}{2}$ j) $\frac{x^2-5x+4}{y-3}$

    c) $2x^2-5y+3$ f) $3y-x$

3. Consider the expression $\frac{6x-3y-xy+2x^2}{2x-y}-3$. Evaluate this expression if

    a) $x=-1$ and $y=2$ or the ordered pair $(-1,2)$ c) $x=3$ and $y=-2$ or the ordered pair $(3,-2)$

    b) $x=-3$ and $y=-6$ or the ordered pair $(-3,-6)$ d) $x=-7$ and $y=4$ or the ordered pair, $(-7,4)$

4. Evaluate $-m^2-m$ if

    a) $m=2$ b) $m=-2$ c) $m=0$ d) $m=5$ e) $m=-5$

5. Evaluate $\frac{8x+x^2-33}{x+11}$ if

    a) $x=0$ b) $x=7$ c) $x=-4$ d) $x=-11$ e) $x=-1$

6. a) It is a common mistake to think that the expressions $2x-3$ and $2x+3$ are opposites. They are not. Evaluate these expressions for the values given below to fill out the table below.

| | $x=2$ | $x=5$ | $x=6$ | $x=10$ | $x=-1$ | $x=-5$ | $x=-8$ |
|---|---|---|---|---|---|---|---|
| $2x-3$ | 1 | | | | | | |
| $2x+3$ | 7 | | | | | | |

b) the opposite of $2x-3$ is actually $-2x+3$. Evaluate these expressions for the values given below to fill out the table below.

| | $x=2$ | $x=5$ | $x=6$ | $x=10$ | $x=-1$ | $x=-5$ | $x=-8$ |
|---|---|---|---|---|---|---|---|
| $2x-3$ | | | | | | | |
| $-2x+3$ | | | | | | | |

7. Evaluate $\dfrac{x-2}{2-x}$ if

a) $x=0$ b) $x=10$ c) $x=2$ d) $x=-13$

8. Evaluate each of the following algebrac expressions with the value(s) given.

a) $3x^2-x+7$ if $x=-1$ b) $-a+5b$ if $a=3$ and $b=-2$ c) $\dfrac{x^x-1}{x-1}$ if $x=2$

9. The absolute value of a number is its distance from zero on the number line. (Recall that distances can never be negative.) If we wanted to know the distance between two numbers on the number line, the absolute value is very helpful. Consider the expression $|a-b|$. Evaluate this expression for each of the pairs of values given, and see whether $|a-b|$ really results in the distance between $a$ and $b$ on the number line.

a) $a=8$ and $b=3$ b) $a=2$ and $b=10$ c) $a=-2$ and $b=-9$ d) $a=2$ and $b=-1$

10. Consider the equation $\dfrac{2x^2-11x-21}{2x+3}=3x-(2x+7)$. In case of each number given, determine whether it is a solution of the equation or not.

a) $x=8$ b) $x=13$ c) $x=10$

11. Consider the equation $-x^2-2x\left(3-x^2\right)=-x+2$. In case of each number given, determine whether it is a solution of the equation or not.

a) $x=0$ b) $x=1$ c) $x=-1$ d) $x=2$ e) $x=-2$

12. Consider the equation $y=\dfrac{5x-3}{2}$. In case of each pair of numbers given, determine whether it is a solution of the equation or not.

a) $x=1$ and $y=1$ or, as an ordered pair, $(1,1)$ c) $x=3$ and $y=6$ or, as an ordered pair, $(3,6)$

b) $x=9$ and $y=4$ or, as an ordered pair, $(9,4)$ d) $x=17$ and $y=41$ or, as an ordered pair, $(17,41)$

13. Consider the equation $(p-q)^2+\dfrac{3p-1}{6-q}=4(p+1)$. In case of each pair of numbers given, determine whether it is a solution of the equation or not.

a) $p=8$ and $q=5$ b) $p=7$ and $q=1$

14. Consider the inequality $-x+2<-x^2+2(x+6)$. In case of each number given, determine whether it is a solution of the inequality or not.

a) $x=-5$ b) $x=-2$ c) $x=0$ d) $x=3$ e) $x=7$

15. Consider the inequality $\dfrac{x}{3}+1\geq\dfrac{x+1}{2}-1$. In case of each number given, determine whether it is a solution of the inequality or not.

a) $x=-9$  b) $x=-3$  c) $x=27$  d) $x=15$  e) $x=-15$

16. Translate each of the given statements to algebraic statements.

a) The difference of $A$ and $B$ is four less than the product of $A$ and the opposite of $B$.

b) If we square $x$, the result is eight less than five times the opposite of $x$.

c) If we subtract ten from $P$, we get a number that is five less than the sum of $Q$ and twice $R$.

d) Four times a number $y$ is one greater than twice the sum of $y$ and seven.

e) The number $x$ is ten greater than its own opposite.

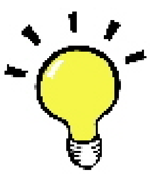
f) The square of the sum of $x$ and $y$ is ten greater than the difference of the square of $x$ and square of $y$.

17. The longer side of a rectangle is seven units longer than three times its shorter side. If we label the shorter side by $x$, express the longer side in terms of $x$.

18. Express four consecutive even numbers if

a) we denote the smallest number by $x$

b) we denote the greatest number by $x$



# Answers

## Sample Problems

1. a) $22$ b) $16$ c) $-16$ d) $16$ e) $-5$ f) $5$

2. a) $22$ b) $-30$ c) $1$ d) undefined e) $40$ f) $100$ g) $18$ h) $36$ i) $17$ j) $15$ k) $-34$

3. a) $5$ b) $9$ 4. a) $784$ ft b) $208$ ft 5. a) $12$ b) $36$ c) $25$ d) $2$ e) undefined

6. a) $37\neq 13$ no b) $55=55$ yes c) $70\neq 76$ no d) $139=139$ yes

7. a) $-4\neq 20$ no b) $12=12$ yes c) $8=8$ yes d) $-12\neq 28$ no 8. a) $13=13$ yes b) $15\neq 13$ no

9. a) $-62\leq -57$ yes b) $16\nleq 8$ no c) $-32\leq -32$ yes d) $-2\nleq -7$ no

10. a) $6\nless 1$ no b) $14<19$ yes c) $10<10$ no d) $2<-8$ no

11. a) $y=4x-3$ b) $-A=(B-3C)+1$ c) $2M=N(-M)-5$ 12. $5x-3$

13. a) $x$, $x+1$, and $x+2$ b) $y-1$, $y$, and $y+1$ c) $L-2$, $L-1$, and $L$

Practice Problems

1. a) $20$ b) $16$ c) $-1$ d) $1$ e) $-11$ f) $-3$ g) $20$ h) $4$ i) $13$ j) $17$

2. a) $17$ b) $5$ c) $35$ d) $100$ e) $196$ f) $18$ g) $6$ h) $11$ i) $26$ j) $2$

3. a) $-1$ b) undefined c) $3$ d) $-7$ 4. a) $-6$ b) $-2$ c) $0$ d) $-30$ e) $-20$

5. a) $-3$ b) $4$ c) $-7$ d) undefined e) $-4$

6. a)

| | $x=2$ | $x=5$ | $x=6$ | $x=10$ | $x=-1$ | $x=-5$ | $x=-8$ |
|---|---|---|---|---|---|---|---|
| $2x-3$ | $1$ | $7$ | $9$ | $17$ | $-5$ | $-13$ | $13$ |
| $2x+3$ | $7$ | $13$ | $15$ | $23$ | $1$ | $-7$ | $-19$ |

b)

| | $x=2$ | $x=5$ | $x=6$ | $x=10$ | $x=-1$ | $x=-5$ | $x=-8$ |
|---|---|---|---|---|---|---|---|
| $2x-3$ | $1$ | $7$ | $9$ | $17$ | $-5$ | $-13$ | $13$ |
| $-2x+3$ | $-1$ | $-7$ | $-9$ | $-17$ | $5$ | $13$ | $-13$ |

7. a) $-1$ b) $-1$ c) undefined d) $-1$ 8. a) $11$ b) $-13$ c) $3$

9. a) $5$ b) $6$ c) $7$ d) $3$ $|a-b|$ always gives us the distance between $a$ and $b$ on the number line

10. a) $1=1$ yes b) $6=6$ yes c) $3=3$ yes

11. a) $0\neq 2$ no b) $-5\neq 1$ no c) $3=3$ yes d) $0=0$ yes e) $-8\neq 4$ no

12. a) $1=1$ yes b) $4\neq 21$ no c) $6=6$ yes d) $41=41$ no

13. a) $32\neq 36$ no b) $40\neq 32$ no

14. a) $7\not< -23$ no b) $4\not< 4$ no c) $2<12$ yes d) $-1<9$ yes e) $-5\not< -23$ no

15. a) $-2\geq -5$ yes b) $0\geq -2$ yes c) $10\not\geq 13$ no d) $6\not\geq 7$ no e) $-4\geq -8$ yes

16. a) $A-B=A(-B)-4$ b) $x^2=5(-x)-8$ c) $P-10=(Q+2R)-5$ d) $4y=2(y+7)+1$
e) $x=-x+10$ f) $(x+y)=x^2-y^2+10$ 17. $3x+7$

18. a) $x$, $x+2$, $x+4$, and $x+6$ b) $x-6$, $x-4$, $x-2$, and $x$

## Solutions - Sample Problems

1. Evaluate each of the following numerical expressions.

a) $2 - 5(3 - 7)$

Solution: We will apply order of operations. First we perform the subtraction in the parentheses.

$$\begin{aligned} 2 - 5(3 - 7) &= && \text{subtraction in parentheses} \\ 2 - 5(-4) &= && \text{multiplication} \\ 2 - (-20) &= && \text{subtraction} \\ 2 + 20 &= \boxed{22} \end{aligned}$$

b) $24 - 10 + 2$

Solution: It is NOT true that addition comes before subtraction. Addition and subtraction are equally strong, so between those two, we perform them left to right. First come, first served.

$$24 - 10 + 2 = 14 + 2 = \boxed{16}$$

c) $-4^2$

Solution: as it was discussed before, $-4^2$ is quite different from $(-4)^2$. This is $-1 \cdot 4^2 = \boxed{-16}$.

d) $(-4)^2$

This is when $-4$ is squared. So $(-4)^2 = -4(-4) = \boxed{16}$

e) $|3| - |8|$

Solution: We subtract the absolute value of 8 from the absolute value of 3. So $|3| - |8| = 3 - 8 = \boxed{-5}$

f) $|3 - 8|$

Solution: This is the absolute value of the difference. Absolute value signs also function of grouping symbols (i.e. parentheses) to overwrite the usual order of operations. So $|3 - 8| = |-5| = \boxed{5}$

2. Evaluate each of the algebraic expressions when $p = -7$ and $q = 3$.

a) $15 - p$

Solution:

Step 1. We re-write the expression with one modification: we replace each variable by an empty pair of parentheses.

Step 2. We insert the values into the parentheses. Now the problem becomes an order of operations problem.

Step 3. We drop the unnecessary parentheses and work out the order of operations problem. (It may appear awkward to create these parentheses but they will later become extremely helpful.)

$$\begin{aligned} &\text{Step 1.} & 15 - p &= 15 - (\ \ ) \\ &\text{Step 2.} & &= 15 - (-7) \\ &\text{Step 3.} & &= 15 + 7 \\ & & &= \boxed{22} \end{aligned}$$

b) $pq - |p-2|$

Solution:

Step 1. $pq - |p-2| = (\ )(\ ) - |(\ ) - 2|$

Step 2. $= (-7)(3) - |-7-2|$

Step 3. $= -7 \cdot 3 - |-9| = -7 \cdot 3 - 9 = -21 - 9 = \boxed{-30}$

c) $4p - q^3$

Solution:

Step 1. $4p - q^3 = 4(\ ) - (\ )^3$

Step 2. $= 4(-7) - (3)^3$

Step 3. $= 4 \cdot 7 - 3^3$ exponentiation

$= 4 \cdot 7 - 27$ multiplication

$= 28 - 27$ subtraction

$= \boxed{1}$

d) $\dfrac{q^2 - p}{2q + p + 1}$

Solution:

Step 1. $\dfrac{q^2 - p}{2q + p + 1} = \dfrac{(\ )^2 - (\ )}{2(\ ) + (\ ) + 1}$

Step 2. $= \dfrac{(3)^2 - (-7)}{2(3) + (-7) + 1}$ drop extra parentheses

$= \dfrac{3^2 - (-7)}{2 \cdot 3 + (-7) + 1}$ exponent upstairs

Step 3. $= \dfrac{9 - (-7)}{2 \cdot 3 + (-7) + 1}$ subtraction upstairs; $9 - (-7) = 9 + 7$

$= \dfrac{16}{2 \cdot 3 + (-7) + 1}$ multiplication

$= \dfrac{16}{6 + (-7) + 1}$ additions, left to right

$= \dfrac{16}{-1 + 1} = \dfrac{16}{0}$ Division by zero is not allowed!

$= \boxed{\text{undefined}}$

e) $p^2 - q^2$

Solution:

$p^2 - q^2 = (\ )^2 - (\ )^2$

$= (-7)^2 - (3)^2 = (-7)^2 - 3^2$ exponents,

$= 49 - 3^2$ left to right

$= 49 - 9$ subtraction

$= \boxed{40}$

f) $(p-q)^2$

Solution:

$$\begin{aligned}(p-q)^2 &= [(\ \ ) - (\ \ )]^2 \\ &= [(-7)-(3)]^2 = [-7-3]^2 && \text{subtraction in parentheses} \\ &= (-10)^2 && \text{exponentiation} \\ &= \boxed{100}\end{aligned}$$

g) $2q^2$

Solution:

$$\begin{aligned}2q^2 &= 2(\ \ )^2 \\ &= 2(3)^2 \\ &= 2\cdot 3^2 && \text{exponentiation} \\ &= 2\cdot 9 && \text{multiplication} \\ &= \boxed{18}\end{aligned}$$

h) $(2q)^2$

Solution:

$$\begin{aligned}(2q)^2 &= [2(\ \ )]^2 \\ &= [2(3)]^2 \\ &= (2\cdot 3)^2 && \text{multiplication in parentheses} \\ &= 6^2 && \text{exponents} \\ &= \boxed{36}\end{aligned}$$

i) $15 - \dfrac{p+q}{|1-p|}$

Solution: From here on, we show computations **in the form they should appear**. Once you wrote down the expression with little parentheses instead of the letters, you can insert the values into it.

$$\begin{aligned}15 - \frac{p+q}{|1-p|} &= 15 - \frac{(\ \ )+(\ \ )}{|1-(\ \ )|} \\ &= 15 - \frac{(-7)+(3)}{|1-3|} \\ &= 15 - \frac{-7+3}{|1-3|} && \text{invisible parentheses! addition on top} \\ &= 15 - \frac{-4}{|1-3|} && \text{subtraction in absolute value sign} \\ &= 15 - \frac{-4}{|-2|} && \text{evaluate the absolute value of 2} \\ &= 15 - \frac{-4}{2} && \text{division} \\ &= 15 - (-2) && \text{subtraction} \\ &= \boxed{17}\end{aligned}$$

j) $(p+q)^2-(5q+2p)^4$

Solution:

$$\begin{aligned}(p+q)^2-(5q+2p)^4 &= [(-7)+(3)]^2-[5(3)+2(-7)]^4 \\ &= (-7+3)^2-(5\cdot 3+2\cdot(-7))^4 && \text{addition in first parentheses} \\ &= (-4)^2-(5\cdot 3+2\cdot(-7))^4 && \text{multiplications in parentheses} \\ &= (-4)^2-(15+2\cdot(-7))^4 && \text{left to right} \\ &= (-4)^2-(15+(-14))^4 && \text{addition in parentheses} \\ &= (-4)^2-1^4 && \text{exponents, left to right} \\ &= 16-1^4 && \text{careful! } 1^4\neq 4 \\ &= 16-1 && \text{subtraction} \\ &= \boxed{15}\end{aligned}$$

k) $-p^2-p+8$

Solution:

$$\begin{aligned}-p^2-p+8 &= -(\ )^2-(\ )+8 \\ &= -(-7)^2-(-7)+8 \\ &= -49+7+8 \\ &= -42+8=\boxed{-34}\end{aligned}$$

3. Evaluate the expression $3x^2-x+5$ with the given values of $x$.

a) $x=0$

Solution: We first copy the entire expression, replacing the letter $x$ by little pairs of paretheses.

$$3x^2-x+5=3(\ )^2-(\ )+5$$

Then we insert the number 0 into each pair of parentheses.

$$3x^2-x+5=3(0)^2-(0)+5$$

Because we substituted zero, most parentheses are unnecessary. We will drop them:

$$3x^2-x+5=3(0)^2-(0)+5=3\cdot 0^2-0+5$$

Then we solve the resulting order of operations problem. We start with the exponent.

$$\begin{aligned}3\cdot 0^2-0+5 &= 3\cdot 0-0+5 && \text{perform multiplication} \\ &= 0-0+5 && \text{subtraction} \\ &= 0+5 && \text{addition} \\ &= \boxed{5}\end{aligned}$$

b) Evaluate $3x^2-x+5$ when $x=-1$.

Solution: We evaluate the expression with $x=-1$.

$$\begin{aligned}3x^2-x+5 &= 3(-1)^2-(-1)+5 \\ &= 3\cdot 1+1+5=3+1+5=\boxed{9}\end{aligned}$$

4. We ejected a small object upward from the top of a 720 ft tall building and started measuring time in seconds. We find that $t$ seconds after launching, the vertical position of the object is $-16t^2 + 64t + 720$ feet.

   a) Where is the object 2 seconds after launch?

   b) Where is the object 8 seconds after launch?

   a) Solution: To find out where the object is after 2 seconds, we evaluate the expression with $t = 2$.

$$\begin{aligned} -16t^2 + 64t + 720 &= -16 \cdot 2^2 + 64 \cdot 2 + 720 \\ &= -16 \cdot 4 + 64 \cdot 2 + 720 \\ &= -64 + 128 + 720 = 64 + 720 = 784 \end{aligned}$$

   So the object is at a height of $\boxed{784 \text{ feet}}$ 2 seconds after launch.

   b) Solution: To find out where the object is after 8 seconds, we evaluate the expression with $t = 8$.

$$\begin{aligned} -16t^2 + 64t + 720 &= -16 \cdot 8^2 + 64 \cdot 8 + 720 \\ &= -16 \cdot 64 + 512 + 720 \\ &= -1024 + 512 + 720 = -512 + 720 = 208 \end{aligned}$$

   So the object is at a height of $\boxed{208 \text{ feet}}$ exactly 8 seconds after launch.

5. Let $a = -4$, $b = 2$, and $x = -3$. Evaluate each of the following expressions.

   a) $a^2 - b^2$
   Solution: First we re-write the expression with one change, we write little pairs of parentheses instead of the letters.

$$a^2 - b^2 = (\ \ )^2 - (\ \ )^2$$

   We now write the values inside the parentheses. From here on this is an order of operations problem.

$$\begin{aligned} a^2 - b^2 &= (-4)^2 - (2)^2 && \text{drop extra parentheses} \\ &= (-4)^2 - 2^2 && \text{exponents} \\ &= 16 - 4 && \text{subtraction} \\ &= \boxed{12} \end{aligned}$$

   b) $(a - b)^2$
   Solution: First we re-write the expression with one modification: we write little pairs of parentheses instead of the letters.

$$(a - b)^2 = ((\ \ ) - (\ \ ))^2$$

   We now write the values inside the parentheses. From here on this is an order of operations problem.

$$\begin{aligned} (a - b)^2 &= ((-4) - (2))^2 && \text{drop extra parentheses} \\ &= (-4 - 2)^2 && \text{subtraction in parentheses} \\ &= (-6)^2 && \text{exponent} \\ &= \boxed{36} \end{aligned}$$

   This and the previous problem are here to remind you that $(a - b)^2$ and $a^2 - b^2$ are two different expressions.

c) $a^b - 2bx - x^2 - 2x$

Solution: First we re-write the expression with one modification only: we write little pairs of parentheses instead of the letters.

$$a^b - 2bx - x^2 - 2x = (\ \ )^{(\ )} - 2(\ )(\ ) - (\ )^2 - 2(\ )$$

We now write the values inside the parentheses. From here on this is an order of operations problem.

$a^b - 2bx - x^2 - 2x =$

$$\begin{aligned}
&= (\ \ )^{(\ )} - 2(\ )(\ ) - (\ )^2 - 2(\ ) \\
&= (-4)^{(2)} - 2(2)(-3) - (-3)^2 - 2(-3) && \text{drop extra parentheses} \\
&= (-4)^2 - 2\cdot 2(-3) - (-3)^2 - 2(-3) && \text{exponents, left to right} \\
&= 16 - 2\cdot 2(-3) - (-3)^2 - 2(-3) \\
&= 16 - 2\cdot 2(-3) - 9 - 2(-3) && \text{multiplications, left to right} \\
&= 16 - 4(-3) - 9 - 2(-3) \\
&= 16 - (-12) - 9 - 2(-3) \\
&= 16 - (-12) - 9 - (-6) && \text{additions, subtractions, left to right} \\
&= 16 + 12 - 9 - (-6) \\
&= 28 - 9 - (-6) \\
&= 19 - (-6) = 19 + 6 = \boxed{25}
\end{aligned}$$

d) $\dfrac{-x^2 + (x+2)^2}{(x-1)}$

Solution: First we re-write the expression with only one modification: we write little pairs of parentheses instead of the letters.

$$\frac{-x^2 + (x+2)^2}{(x-1)} = \frac{-(\ \ )^2 + ((\ \ ) + 2)^2}{((\ \ ) - 1)}$$

We now write the values inside the parentheses. From here on this is an order of operations problem.

$$\begin{aligned}
\frac{-x^2 + (x+2)^2}{(x-1)} &= \frac{-(-3)^2 + ((-3)+2)^2}{((-3)-1)} && \text{drop parentheses} \\
&= \frac{-(-3)^2 + (-3+2)^2}{(-3-1)} && \text{addition in parentheses upstairs} \\
&= \frac{-(-3)^2 + (-1)^2}{(-3-1)} && \text{subtraction downstairs in parentheses} \\
&= \frac{-(-3)^2 + (-1)^2}{(-4)} && \text{drop parentheses} \\
&= \frac{-(-3)^2 + (-1)^2}{-4} && \text{exponents upstairs} \\
&= \frac{-9+1}{-4} && \text{addition} \\
&= \frac{-8}{-4} && \text{division} \\
&= \boxed{2}
\end{aligned}$$

e) $\dfrac{x-1}{x+3}$

Solution: First we re-write the expression with only one modification: we write little pairs of parentheses instead of the letters.

$$\frac{x-1}{x+3}=\frac{(\ )-1}{(\ )+3}$$

We write the values inside the parentheses and evaluate the expression.

$$\frac{x-1}{x+3}=\frac{(-3)-1}{(-3)+3}=\frac{-4}{0}=\boxed{\text{undefined}}$$

6. Consider the equation $2x^2+x+34=21x-8$. In case of each number given, determine whether it is a solution of the equation or not.

a) $x=1$

Solution: We need to substitute 1 for $x$ into both sides of the equation and evaluate those algebraic expressions to see whether the left-hand side equals to the right-hand side.

If $x=1$, then

| | | |
|---|---|---|
| $\text{LHS}=2(1)^2+(1)+34$<br>$=2\cdot 1+1+34$<br>$=2+1+34=37$ | $\text{RHS}=21(1)-8$<br>$=21\cdot 1-8$<br>$=13$ | $2x^2+x+34=21x-8$<br>becomes $37=13$<br>This is false. |

So 1 is not a solution of the equation.

b) $x=3$

Solution: We need to substitute 3 for $x$ into both the left-hand side and right-hand side of the equation and evaluate those algebraic expressions to see whether the left-hand side equals to the right-hand side.

If $x=3$, then

| | | |
|---|---|---|
| $\text{LHS}=2\cdot 3^2+3+34$<br>$=2\cdot 9+3+34$<br>$=18+3+34$<br>$=21+34=55$ | $\text{RHS}=21\cdot 3-8$<br>$=63-8$<br>$=5$ | $2x^2+x+34=21x-8$<br>becomes $55=55$<br>This is true! |

So 3 is a solution of the equation.

c) $x=4$

Solution: We will substitute 4 for $x$ in both sides of the equation and compare the values.

If $x=4$, then

| | | |
|---|---|---|
| $\text{LHS}=2\cdot 4^2+4+34$<br>$=2\cdot 16+4+34$<br>$=32+4+34$<br>$=36+34=70$ | $\text{RHS}=21\cdot 4-8$<br>$=84-8$<br>$=76$ | $2x^2+x+34=21x-8$<br>becomes $70=76$<br>This is false. |

Therefore 4 is not a solution of the equation.

d) $x = 7$

Solution: We will substitute 7 for $x$ into both sides of the equation and evaluate those algebraic expressions to see whether the left-hand side equals to the right-hand side.

| If $x = 7$, then | | | |
|---|---|---|---|
| | $\text{LHS} = 2 \cdot 7^2 + 7 + 34$<br>$= 2 \cdot 49 + 7 + 34$<br>$= 98 + 7 + 34$<br>$= 105 + 34 = 139$ | $\text{RHS} = 21 \cdot 7 - 8$<br>$= 147 - 8$<br>$= 139$ | $2x^2 + x + 34 = 21x - 8$<br>becomes $139 = 139$<br>This is true! |

Therefore 7 is a solution of the equation.

7. Consider the equation $x^2 - 10x + x^3 - 4 = 4(x+5)$. In each case, determine whether the number given is a solution of the equation or not.

a) $x = 0$

Solution: We simply evaluate both sides of the equation when $x = 0$.

| If $x = 0$, then | | | |
|---|---|---|---|
| | $\text{LHS} = 0^2 - 10 \cdot 0 + 0^3 - 4$<br>$= 0 - 0 + 0 - 4$<br>$= -4$ | $\text{RHS} = 4(0+5)$<br>$= 4 \cdot 5$<br>$= 20$ | $x^2 - 10x + x^3 - 4 = 4(x+5)$<br>becomes $-4 = 20$<br>This is false. |

Therefore 0 is not a solution of the equation.

b) $x = -2$

Solution: We simply evaluate both sides of the equation when $x = -2$.

| If $x = -2$, then | | | |
|---|---|---|---|
| | $\text{LHS} = (-2)^2 - 10(-2) + (-2)^3 - 4$<br>$= 4 - 10(-2) + (-8) - 4$<br>$= 4 + 20 - 8 - 4$<br>$= 24 - 8 - 4 = 12$ | $\text{RHS} = 4(-2+5)$<br>$= 4 \cdot 3$<br>$= 12$ | $x^2 - 10x + x^3 - 4 = 4(x+5)$<br>becomes $12 = 12$<br>This is true. |

Therefore $-2$ is a solution of the equation.

c) $x = -3$

Solution: We simply evaluate both sides of the equation when $x = -3$.

| If $x = -3$, then | | | |
|---|---|---|---|
| | $\text{LHS} = (-3)^2 - 10(-3) + (-3)^3 - 4$<br>$= 9 - 10(-3) + (-27) - 4$<br>$= 9 + 30 - 27 - 4$<br>$= 39 - 27 - 4$<br>$= 12 - 4 = 8$ | $\text{RHS} = 4(-3+5)$<br>$= 4 \cdot 2$<br>$= 8$ | $x^2 - 10x + x^3 - 4 = 4(x+5)$<br>becomes $8 = 8$<br>This is true. |

Therefore $-3$ is a solution of the equation.

8. Consider the equation $3a - 2b - 1 = (a-b)^2 + 4$. In case of each pair of numbers given, determine whether it is a solution of the equation or not.

a) $a = 8$ and $b = 5$ or, as an ordered pair, $(8, 5)$

Solution: We will substitute $a = 8$ and $b = 5$ into both sides of the equation and evaluate those algebraic expressions to see whether the left-hand side equals to the right-hand side.

| | | | |
|---|---|---|---|
| If $a = 8$ and $b = 5$, then | $\text{LHS} = 3 \cdot 8 - 2 \cdot 5 - 1$ <br> $= 24 - 2 \cdot 5 - 1$ <br> $= 24 - 10 - 1$ <br> $= 14 - 1 = 13$ | $\text{RHS} = (8-5)^2 + 4$ <br> $= 3^2 + 4$ <br> $= 9 + 4$ <br> $= 13$ | $3a - 2b - 1 = (a-b)^2 + 4$ <br> becomes $13 = 13$ <br> This is true! |

Therefore $(8, 5)$ is a solution of the equation.

b) $a = 10$ and $b = 7$

Solution: We need to substitute $a = 10$ and $b = 7$ into both sides of the equation and evaluate those algebraic expressions to see whether the left-hand side equals to the right-hand side.

| | | | |
|---|---|---|---|
| If $a = 10$ and $b = 7$, then | $\text{LHS} = 3 \cdot 10 - 2 \cdot 7 - 1$ <br> $= 30 - 2 \cdot 7 - 1$ <br> $= 30 - 14 - 1$ <br> $= 16 - 1 = 15$ | $\text{RHS} = (10-7)^2 + 4$ <br> $= 3^2 + 4$ <br> $= 9 + 4$ <br> $= 13$ | $3a - 2b - 1 = (a-b)^2 + 4$ <br> becomes $15 = 13$ <br> This is false. |

Therefore $(10, 7)$ is not a solution of the equation.

9. Consider the inequality $3(2y-1) + 1 \leq 5y - 7$. In case of each number given, determine whether it is a solution of the inequality or not.

a) $y = -10$

Solution: We need to substitute $y = -10$ into both sides of the inequality and evaluate those algebraic expressions to see whether the left-hand side is indeed less than or equal to the right-hand side.

| | | | |
|---|---|---|---|
| If $y = -10$, then | $\text{LHS} = 3(2(-10) - 1) + 1$ <br> $= 3(-20 - 1) + 1$ <br> $= 3(-21) + 1$ <br> $= -63 + 1 = -62$ | $\text{RHS} = 5(-10) - 7$ <br> $= -50 - 7$ <br> $= -57$ | $3(2y-1) + 1 \leq 5y - 7$ <br> becomes $-62 \leq -57$ <br> This is true. |

Thus $-10$ is a solution of the inequality.

b) $y = 3$

Solution: We need to substitute $y = 3$ into both sides of the inequality and evaluate those algebraic expressions to see whether the left-hand side is indeed less than or equal to the right-hand side.

| | | | |
|---|---|---|---|
| If $y = 3$, then | $\text{LHS} = 3(2 \cdot 3 - 1) + 1$ <br> $= 3(6-1)+1$ <br> $= 3 \cdot 5 + 1$ <br> $= 15 + 1 = 16$ | $\text{RHS} = 5 \cdot 3 - 7$ <br> $= 15 - 7$ <br> $= 8$ | $3(2y-1)+1 \le 5y - 7$ <br> becomes $16 \le 8$ <br> This is false. |

Thus 3 is not a solution of the inequality.

c) $y = -5$

Solution: We need to substitute $y = -5$ into both sides of the inequality and evaluate those algebraic expressions to see whether the left-hand side is indeed less than or equal to the right-hand side.

| | | | |
|---|---|---|---|
| If $y = -5$, then | $\text{LHS} = 3(2(-5) - 1) + 1$ <br> $= 3(-10-1)+1$ <br> $= 3(-11) + 1$ <br> $= -33 + 1 = -32$ | $\text{RHS} = 5(-5) - 7$ <br> $= -25 - 7$ <br> $= -32$ | $3(2y-1)+1 \le 5y - 7$ <br> becomes $-32 \le -32$ <br> This is true. |

Thus $-5$ is a solution of the inequality.

d) $y = 0$

Solution: We need to substitute $y = 0$ into both sides of the inequality and evaluate those algebraic expressions to see whether the left-hand side is indeed less than or equal to the right-hand side.

| | | | |
|---|---|---|---|
| If $y = 0$, then | $\text{LHS} = 3(2 \cdot 0 - 1) + 1$ <br> $= 3(0-1)+1$ <br> $= 3(-1) + 1$ <br> $= -3 + 1 = -2$ | $\text{RHS} = 5 \cdot 0 - 7$ <br> $= 0 - 7$ <br> $= -7$ | $3(2y-1)+1 \le 5y - 7$ <br> becomes $-2 \le -7$ <br> This is false. |

Thus 0 is not a solution of the inequality.

10. Consider the inequality $\dfrac{2x+1}{3} + 5 < \dfrac{3x-1}{2}$. In case of each number given, determine whether it is a solution of the inequality or not.

a) $x = 1$

Solution: We need to substitute $x = 1$ into both sides of the inequality and evaluate those algebraic expressions to see whether the left-hand side is indeed less than the right-hand side. The left-hand side:

$$\text{LHS} = \frac{2 \cdot 1 + 1}{3} + 5 = \frac{2+1}{3} + 5 = \frac{3}{3} + 5 = 1 + 5 = 6$$

The right-hand side:

$$\text{RHS} = \frac{3 \cdot 1 - 1}{2} = \frac{3-1}{2} = \frac{2}{2} = 1$$

So the statement $\dfrac{2x+1}{3}+5<\dfrac{3x-1}{2}$ becomes $6<1$. Since this is a false statement, $x=1$ is not a solution of the inequality.

b) $x=13$

Solution: We need to substitute $x=13$ into both sides of the inequality and evaluate those algebraic expressions to see whether the left-hand side is indeed less than the right-hand side. The left-hand side:

$$\text{LHS}=\frac{2(13)+1}{3}+5=\frac{26+1}{3}+5=\frac{27}{3}+5=9+5=14$$

The right-hand side:

$$\text{RHS}=\frac{3(13)-1}{2}=\frac{39-1}{2}=\frac{38}{2}=19$$

So the statement $\dfrac{2x+1}{3}+5<\dfrac{3x-1}{2}$ becomes $14<19$. Since this is a true statement, $x=13$ is a solution of the inequality.

c) $x=7$

Solution: We need to substitute $x=7$ into both sides of the inequality and evaluate those algebraic expressions to see whether the left-hand side is indeed less than the right-hand side. The left-hand side:

$$\text{LHS}=\frac{2(7)+1}{3}+5=\frac{14+1}{3}+5=\frac{15}{3}+5=5+5=10$$

The right-hand side:

$$\text{RHS}=\frac{3(7)-1}{2}=\frac{21-1}{2}=\frac{20}{2}=10$$

So the statement $\dfrac{2x+1}{3}+5<\dfrac{3x-1}{2}$ becomes $10<10$. Since this is a false statement, $x=7$ is not a solution of the inequality.

d) $x=-5$

Solution: We need to substitute $x=-5$ into both sides of the inequality and evaluate those algebraic expressions to see whether the left-hand side is indeed less than the right-hand side. The left-hand side:

$$\text{LHS}=\frac{2(-5)+1}{3}+5=\frac{-10+1}{3}+5=\frac{-9}{3}+5=-3+5=2$$

The right-hand side:

$$\text{RHS}=\frac{3(-5)-1}{2}=\frac{-15-1}{2}=\frac{-16}{2}=-8$$

So the statement $\dfrac{2x+1}{3}+5<\dfrac{3x-1}{2}$ becomes $2<-8$. Since this is a false statement, $x=-5$ is not a solution of the inequality.

11. Translate each of the following statements to an algebraic statement.

a) $y$ is three less than four times $x$

Solution: "*four times* $x$" can be translated as $4x$.

Then we have: "$y$ *is three less than* $4x$", which can be translated as $\boxed{y = 4x - 3}$.

b) The opposite of $A$ is one greater than the difference of $B$ and three times $C$.

Solution: "*The opposite of* $A$" can be translated as $-A$. "*three times* $C$" can be translated as $3C$.

Now we have: "$-A$ *is one greater than the difference of* $B$ *and* $3C$".

"*The difference of* $B$ *and* $3C$" can be translated as $B - 3C$. It is important to write them in the order they are mentioned; first come first served.

Now we have: "$-A$ *is one greater than* $B - 3C$". This can be translated as $\boxed{-A = (B - 3C) + 1}$.

The parentheses turns out to be unnecessary, but it does make sense here. We are adding $1$ to the entire difference $B - 3C$, not just $-3C$.

c) Twice $M$ is five less than the product of $N$ and the opposite of $M$.

Solution: "*Twice* $M$" can be translated to $2M$ and "*the opposite of* $M$" can be translated as $-M$.

Now we have: "$2M$ *is five less than the product of* $N$ *and* $-M$".

"*The product of* $N$ *and* $-M$" can be translated as $N(-M)$ or $N \cdot (-M)$.

So now we have: "$2M$ *is five less than* $N(-M)$". This can be translated as $\boxed{2M = N(-M) - 5}$.

12. The longer side of a rectangle is three units shorter than five times the shorter side. If we label the shorter side by $x$, how can we express the longer side in terms of $x$?

Solution: The longer side is three less than five times the shorter side. (The shorter side is $x$.)

So, the longer side is three less than five times $x$.

Or, the longer side is three less than $5x$.

Or, the longer side is $\boxed{5x - 3}$.

13. Suppose we have three consecutive integers.

a) Express them in terms of $x$ if $x$ denotes the smallest number.

Solution: To get from one consecutive integer to the next one, we simply need to add $1$. So the middle number is $x + 1$ and the largest number we obtain by adding $1$ again, so we get $x + 2$. The answer is $\boxed{x, x + 1, x + 2}$.

b) Express them in terms of $y$ if $y$ denotes the number in the middle.

Solution: then the smallest number is one less than $y$, which is $y - 1$, and the largest number is one greater than $y$, which is $y + 1$. So the answer is $\boxed{y - 1, y, y + 1}$.

c) Express them in terms of $L$ if $L$ denotes the greatest number.

Solution: Then we have to subtract one to get to the middle number, and two to get to the smallest number. So the answer is $\boxed{L - 2, L - 1, L}$.

