Sample Problems

1. (Natural numbers) Evaluate each of the algebraic expressions when $p = 7$ and $q = 3$.

   a) $15 - p$
   b) pq
   c) $4p - q$
   d) $p - 2q$
   e) $p^2 - q^2$
   f) $(p - q)^2$
   g) $2q^2$
   h) $(2q)^2$
   i) $15 - \dfrac{p+q}{5}$
   j) $(p+q)^2 - (5q - 2p)^4$

2. (Integers) Let $a = -4$, $b = 2$, and $x = -3$. Evaluate each of the following expressions.

   a) $a^2 - b^2$
   b) $(a - b)^2$
   c) $a^b - 2bx - x^2 - 2x$
   d) $\dfrac{-x^2 + (x+2)^2}{(x-1)}$
   e) $\dfrac{x-1}{x+3}$

Practice Problems

1. (Natural numbers) Evaluate each of the algebraic expressions when $x = 6$ and $y = 8$.

   a) $19 - y + x$
   b) $19 - (y + x)$
   c) $2x^2 - 5y + 3$
   d) $x^2 + y^2$
   e) $(x + y)^2$
   f) $3y - x$
   g) $3(y - x)$
   h) $\dfrac{5x - y}{2}$
   i) $5x - \dfrac{y}{2}$
   j) $\dfrac{x^2 - 5x + 4}{y - 3}$

2. (Natural numbers) Consider the expression $\dfrac{6x - 3y - xy + 2x^2}{2x - y} - 3$. Evaluate this expression if

   a) $x = 5$ and $y = 1$ b) $x = 5$ and $y = 2$ c) $x = 5$ and $y = 3$ d) $x = 4$ and $y = 1$

3. (Integers) Consider the expression $\dfrac{6x - 3y - xy + 2x^2}{2x - y} - 3$. Evaluate this expression if

   a) $x = -1$ and $y = 2$ b) $x = -3$ and $y = -6$ c) $x = 3$ and $y = -2$ d) $x = -7$ and $y = 4$

4. (Integers) Evaluate $-m^2 - m$ if

   a) $m = 2$ b) $m = -2$ c) $m = 0$ d) $m = 5$ e) $m = -5$

5. (Integers) Evaluate $\dfrac{8x + x^2 - 33}{x + 11}$ if

   a) $x = 0$ b) $x = 7$ c) $x = -4$ d) $x = -11$ e) $x = -1$

6. (Integers) a) It is a common mistake to think that the expressions $2x-3$ and $2x+3$ are opposites. They are not. Evaluate these expressions for the values given below to fill out the table below.

	$x=2$	$x=5$	$x=6$	$x=10$	$x=-1$	$x=-5$	$x=-8$
$2x-3$	1						
$2x+3$	7						

b) the opposite of $2x-3$ is actually $-2x+3$. Evaluate these expressions for the values given below to fill out the table below.

	$x=2$	$x=5$	$x=6$	$x=10$	$x=-1$	$x=-5$	$x=-8$
$2x-3$							
$-2x+3$							

7. (Integers) Evaluate $\dfrac{x-2}{2-x}$ if

a) $x=0$ b) $x=10$ c) $x=2$ d) $x=-13$

Answers - Sample Problems

1. a) 8 b) 21 c) 25 d) 1 e) 40 f) 16 g) 18 h) 36 i) 13 j) 99
2. a) 12 b) 36 c) 25 d) 2 e) undefined

Answers - Practice Problems

1. a) 17 b) 5 c) 35 d) 100 e) 196 f) 18 g) 6 h) 11 i) 26 j) 2
2. a) 5 b) 5 c) 5 d) 4
3. a) -1 b) undefined c) 3 d) -7
4. a) -6 b) -2 c) 0 d) -30 e) -20
5. a) -3 b) 4 c) -7 d) undefined e) -4
6. a)

	$x=2$	$x=5$	$x=6$	$x=10$	$x=-1$	$x=-5$	$x=-8$
$2x-3$	1	7	9	17	-5	-13	13
$2x+3$	7	13	15	23	1	-7	-19

b)

	$x=2$	$x=5$	$x=6$	$x=10$	$x=-1$	$x=-5$	$x=-8$
$2x-3$	1	7	9	17	-5	-13	13
$-2x+3$	-1	-7	-9	-17	5	13	-13

7. a) -1 b) -1 c) undefined d) -1

Sample Problems - Solutions

Evaluate each of the algebraic expressions when $p = 7$ and $q = 3$.

1. a) $15 - p$

Solution: Step 1. We re-write the expression with one modification: we replace each variable by an empty pair of parenthses.
Step 2. We insert the values into the parentheses. Now the problem becomes an order of operations problem.
Step 3. We drop the unnecessary parentheses and work out the order of operations problem. (It may appear awkward to create these parentheses but they will later become extremely helpful.)

$$\begin{array}{lrcl} \text{Step 1.} & 15 - p & = & 15 - (\ ) \\ \text{Step 2.} & & = & 15 - (7) \\ \text{Step 3.} & & = & 15 - 7 \\ & & = & 8 \end{array}$$

b) pq

Solution:

$$\begin{array}{lrcl} \text{Step 1.} & pq & = & (\ )(\ ) \\ \text{Step 2.} & & = & (7)\,(3) \\ \text{Step 3.} & & = & 21 \end{array}$$

c) $4p - q$

Solution:

$$\begin{array}{lrcll} \text{Step 1.} & 4p - q & = & 4(\ ) - (\ ) & \\ \text{Step 2.} & & = & 4\,(7) - (3) & \\ \text{Step 3.} & & = & 4 \cdot 7 - 3 & \text{multiplication} \\ & & = & 28 - 3 & \text{subtraction} \\ & & = & 25 & \end{array}$$

d) $p - 2q$

Solution:

$$\begin{array}{lrcll} \text{Step 1.} & p - 2q & = & (\ ) - 2(\ ) & \\ \text{Step 2.} & & = & (7) - 2\,(3) & \\ \text{Step 3.} & & = & 7 - 2 \cdot 3 & \text{multiplication} \\ & & = & 7 - 6 & \text{subtraction} \\ & & = & 1 & \end{array}$$

e) $p^2 - q^2$

Solution:

$$\begin{array}{lrcll} & p^2 - q^2 & = & (\ )^2 - (\ )^2 & \\ & & = & (7)^2 - (3)^2 & \\ & & = & 7^2 - 3^2 & \text{exponents,} \\ & & = & 49 - 3^2 & \text{left to right} \\ & & = & 49 - 9 & \text{subtraction} \\ & & = & 40 & \end{array}$$

f) $(p-q)^2$
Solution:

$$\begin{aligned}(p-q)^2 &= [(\ )-(\ )]^2 \\ &= [(7)-(3)]^2 \\ &= (7-3)^2 && \text{subtraction in parentheses} \\ &= 4^2 && \text{exponentiation} \\ &= 16\end{aligned}$$

g) $2q^2$
Solution:

$$\begin{aligned}2q^2 &= 2(\ )^2 \\ &= 2(3)^2 \\ &= 2\cdot 3^2 && \text{exponentiation} \\ &= 2\cdot 9 && \text{multiplication} \\ &= 18\end{aligned}$$

h) $(2q)^2$
Solution:

$$\begin{aligned}(2q)^2 &= [2(\ )]^2 \\ &= [2(3)]^2 \\ &= (2\cdot 3)^2 && \text{multiplication in parentheses} \\ &= 6^2 && \text{exponents} \\ &= 36\end{aligned}$$

i) $15-\dfrac{p+q}{5}$
Solution: From here on, we show computations **in the form they should appear**. Once you wrote down the expression with little parentheses instead of the letters, you can insert the values into it.

$$\begin{aligned}15-\frac{p+q}{5} &= 15-\frac{(\ )+(\ )}{5} \\ &= 15-\frac{(7)+(3)}{5} \\ &= 15-\frac{7+3}{5} && \text{invisible parentheses!} \\ &= 15-\frac{10}{5} && \text{division} \\ &= 15-2 && \text{subtraction} \\ &= 13\end{aligned}$$

j) $(p+q)^2-(5q-2p)^4$
Solution:

$$\begin{aligned}(p+q)^2-(5q-2p)^4 &= [(7)+(3)]^2-[5(3)-2(7)]^4 \\ &= (7+3)^2-(5\cdot 3-2\cdot 7)^4 && \text{addition in parentheses} \\ &= 10^2-(5\cdot 3-2\cdot 7)^4 && \text{multiplications in parentheses} \\ &= 10^2-(15-2\cdot 7)^4 && \text{left to right}\end{aligned}$$

$$\begin{aligned} &= 10^2 - (15-14)^4 && \text{subtraction in parentheses} \\ &= 10^2 - 1^4 && \text{exponents, left to right} \\ &= 100 - 1^4 && \text{careful! } 1^4 \neq 4 \\ &= 100 - 1 \\ &= 99 \end{aligned}$$

2. Let $a = -4$, $b = 2$, and $x = -3$. Evaluate each of the following expressions.

a) $a^2 - b^2$

Solution: First we re-write the expression with one change, we write little pairs of parentheses instead of the letters.

$$a^2 - b^2 = (\ \ )^2 - (\ \ )^2$$

We now write the values inside the parentheses. From here on this is an order of operations problem.

$$\begin{aligned} a^2 - b^2 &= (-4)^2 - (2)^2 && \text{drop extra parentheses} \\ &= (-4)^2 - 2^2 && \text{exponents} \\ &= 16 - 4 && \text{subtraction} \\ &= 12 \end{aligned}$$

b) $(a-b)^2$

Solution: First we re-write the expression with one change, we write little pairs of parentheses instead of the letters.

$$(a-b)^2 = ((\ \ ) - (\ \ ))^2$$

We now write the values inside the parentheses. From here on this is an order of operations problem.

$$\begin{aligned} (a-b)^2 &= ((-4) - (2))^2 && \text{drop extra parentheses} \\ &= (-4-2)^2 && \text{subtraction in parentheses} \\ &= (-6)^2 && \text{exponent} \\ &= 36 \end{aligned}$$

This and the previous problem is here to remind you that $(a-b)^2$ and $a^2 - b^2$ are two different expressions.

c) $a^b - 2bx - x^2 - 2x$

Solution: First we re-write the expression with one modification only: we write little pairs of parentheses instead of the letters.

$$a^b - 2bx - x^2 - 2x = (\ \ )^{(\ \ )} - 2\,(\ \ )\,(\ \ ) - (\ \ )^2 - 2\,(\ \ )$$

We now write the values inside the parentheses. From here on this is an order of operations problem.

$a^b - 2bx - x^2 - 2x =$

$$\begin{aligned} &= (\ \ )^{(\ \ )} - 2\,(\ \ )\,(\ \ ) - (\ \ )^2 - 2\,(\ \ ) \\ &= (-4)^{(2)} - 2\,(2)\,(-3) - (-3)^2 - 2\,(-3) && \text{drop extra parentheses} \\ &= (-4)^2 - 2\cdot 2\,(-3) - (-3)^2 - 2\,(-3) && \text{exponents, left to right} \\ &= 16 - 2\cdot 2\,(-3) - (-3)^2 - 2\,(-3) \\ &= 16 - 2\cdot 2\,(-3) - 9 - 2\,(-3) && \text{multiplications, left to right} \\ &= 16 - 4\,(-3) - 9 - 2\,(-3) \\ &= 16 - (-12) - 9 - 2\,(-3) \end{aligned}$$

$$\begin{aligned} &= 16-(-12)-9-(-6) && \text{additions, subtractions, left to right}\\ &= 16+12-9-(-6)\\ &= 28-9-(-6)\\ &= 19-(-6)\\ &= 19+6\\ &= 25 \end{aligned}$$

d) $\dfrac{-x^2+(x+2)^2}{(x-1)}$

Solution: First we re-write the expression with only one modification: we write little pairs of parentheses instead of the letters.

$$\frac{-x^2+(x+2)^2}{(x-1)} = \frac{-(\ \ )^2+((\ \ )+2)^2}{((\ \ )-1)}$$

We now write the values inside the parentheses. From here on this is an order of operations problem.

$$\begin{aligned} \frac{-x^2+(x+2)^2}{(x-1)} &= \frac{-(-3)^2+((-3)+2)^2}{((-3)-1)} && \text{drop parentheses}\\ &= \frac{-(-3)^2+(-3+2)^2}{(-3-1)} && \text{addition in parentheses upstairs}\\ &= \frac{-(-3)^2+(-1)^2}{(-3-1)} && \text{subtraction downstairs in parentheses}\\ &= \frac{-(-3)^2+(-1)^2}{(-4)} && \text{drop parentheses}\\ &= \frac{-(-3)^2+(-1)^2}{-4} && \text{exponents upstairs}\\ &= \frac{-9+1}{-4} && \text{addition}\\ &= \frac{-8}{-4} && \text{division}\\ &= 2 \end{aligned}$$

e) $\dfrac{x-1}{x+3}$

Solution: First we re-write the expression with only one modification: we write little pairs of parentheses instead of the letters.

$$\frac{x-1}{x+3} = \frac{(\ \ )-1}{(\ \ )+3}$$

We write the values inside the parentheses and evaluate the expression.

$$\frac{x-1}{x+3} = \frac{(-3)-1}{(-3)+3} = \frac{-4}{0} = \text{undefined}$$

