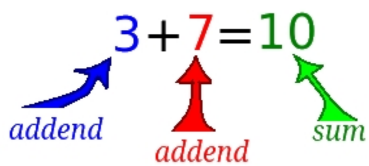
The **set of all natural numbers**, (sometimes also called the set of all counting numbers), denoted by $\mathbb{N}$, is the set

$$\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$$

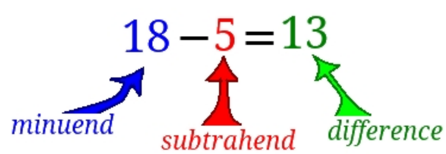
It is easy to imagine that the set of all natural numbers was the first set of numbers at which human beings looked. The four basic operations can be defined and performed on natural numbers as follows.

Addition, denoted by $+$, is defined as we usually think of addition: the addition of two natural numbers is obtaining the total amount of those quantities combined. In the statement $3 + 7 = 10$, we say that 3 and 7 are **addend**s and 10 is called the **sum** of 3 and 7.



If we add two natural numbers, the sum is always a natural number. This is called **closure**. The set of all natural numbers is closed under addition: If n and m are natural numbers, then $n + m$ is also a natural number.

Subtraction, denoted by $-$, is a mathematical operation that represents the operation of removing objects from a collection. In the statement $18 - 5 = 13$, we say that 18 is the **minuend** and 5 is the **subtrahend**, and 13 is called the **difference** of 18 and 5.

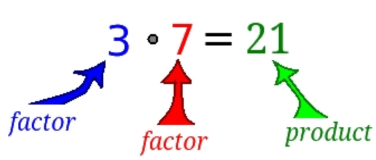


If we subtract a natural number from another natural number, the difference may or may not exits within $\mathbb{N}$. For example, the subtraction $8 - 5$ results in a natural number but the subtraction $3 - 11$ does not. In other words, the set of all natural numbers is NOT closed under subtraction.

Multiplication, denoted by $\cdot$, or by $\times$, or by nothing at all between two objects, is defined as we usually think of multiplication

$$3 \cdot 7 = 21 \quad \text{or} \quad 3 \times 7 = 21 \quad \text{or} \quad (3)\,7 = 21 \quad \text{or} \quad 3\,(7) = 21$$

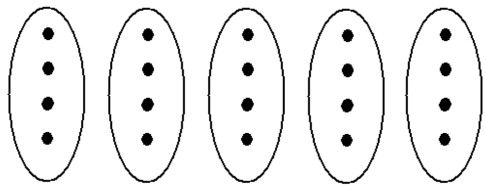
Please note that in the last two equations, the **parentheses do not indicate multiplication**. The parentheses helps us interpret 3 and 7 as two separate numbers and not the number 37. Once we see two numbers with no operation sign between them, that NOTHING indicates multiplication. In the statement $3 \cdot 7 = 21$, we say that 21 is the **product** of 3 and 7. We also say that 3 and 7 are **divisors** or factors of 21.



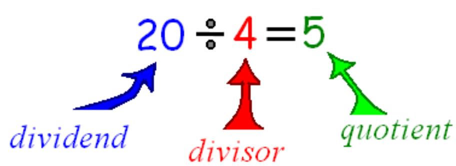
If we multiply two natural numbers, the product is always a natural number. In other words, the set of all natural numbers is closed under multiplication: if n and m are natural numbers, then nm is also a natural number.

Division, denoted by $\div$ or by / is defined as we usually think of division. One example is: if we have 20 dots and we circle together every four dots, how many packages of four do we obtain? The answer is clearly five, because five packages of four will account for 20 dots.

$$20 \div 4 = 5 \quad \text{or} \quad \frac{20}{4} = 5$$



In the statement $20 \div 4 = 5$, we say that 20 is the **dividend**, 4 is the **divisor**, and 5 is called the **quotient** of 20 and 4.



If we divide a natural number by another natural number, the quotient may or may not exist within $\mathbb{N}$. For example, the division $12 \div 3$ results in a natural number, but the division $20 \div 7$ does not. In other words, the set of all natural numbers is NOT closed under division.

Discussion:

1. Consider the multiplication $4 \cdot 9 = 36$ and the division $30 \div 6 = 5$. While in the division all of 30, 6, and 5 have different names, both 4 and 9 are simply called factors in the multiplication. Can you explain why this will not cause any problems?
2. Under which of the four basic operations (addition, subtraction, multiplication, division) is $\mathbb{N}$ closed?

Enrichment

If you haven't already done so, read our lecture notes on axioms, because you need to understand the concepts of axioms and theorems for this exercise.

1. The set of all natural numbers, $\mathbb{N}$ was not defined rigorously. Instead, we gave an intuitive description of $\mathbb{N}$. However, mathematicians insisted on axiomatizing $\mathbb{N}$. This means that they established a set of axioms from which many theorems can be derived. The resulting collection of true statements is the same as what we get starting with the intuitive definition.

   Research the Peano axioms. Feel free to start at Wikipedia. What are the Peano axioms?

   Note: when you look at the Peano axioms, you may notice that according to Wikipedia, 0 is a natural number. In our class, 0 was not defined as a natural number. Do not let yourself be annoyed or confused by the difference. Both conventions are very common.

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