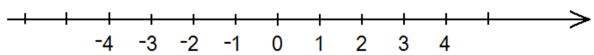
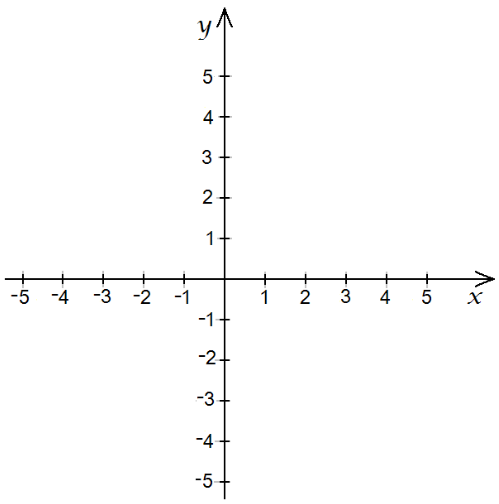
In mathematics, we often deal with algebraic objects such as numbers and equations. We also deal with geometric objects such as lines, points, or triangles. The relationship between things of algebraic and geometric nature is an important recurring theme within mathematics. We will discuss how algebraic properties manifest geometrically and vica versa. The beautiful and fascinating relationship between geometry and algebra starts with the concept of the number line.

There is a one-to-one correspondance between the set of all real numbers and the set of points on a straight line. In other words, for every real number, there exists exactly one point on the line; and for every point on the line, there exists exactly one real number. We display this correspondance using the **number line**, representing the set of all real numbers.

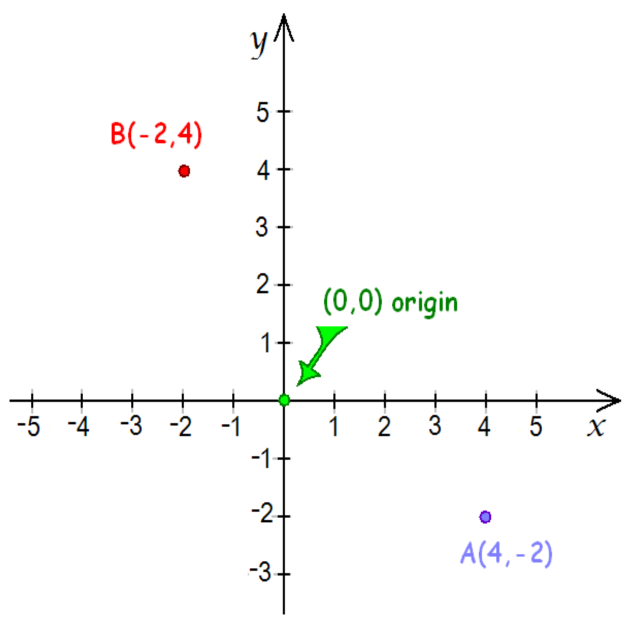




When we use two identical copies of the real number line, we create a **rectangular coordinate system**. This type of a coordinate system is also called a **Cartesian coordinate system**, after French mathematician and philosopher René Descartes who published this idea first in 1637. The horizontal line is called the $x-$**axis**, and the vertical line the $y-$**axis**. The arrows on the axes indicate the positive direction.

Every point in the plane can be uniquely described by an **ordered pair of real numbers** such as $(4, -2)$. The first number in the pair describes the $x-$coordinate (or horizontal address) of the point. The second number in the pair describes the $y-$coordinate (or vertical address) of the point. The word *ordered* is important since $A(4, -2)$ and $B(-2, 4)$ are different points. The point $(0, 0)$, where the two axes intersect each other, is called the **origin**.

Suprising facts: the $x-$axis is the set of all points whose $y-$coordinate is zero. Similarly, the $y-$axis is the set of all points whose $x-$coordinate is zero.



Theorem: Suppose that $A(x_A, y_A)$ and $B(x_B, y_B)$ are two points given. Let M denote the midpoint of line segment AB. The coordinates of the $M(x_M, y_M)$ are the average of the corresponding coordinates of A and B.

$$x_M = \frac{x_A + x_B}{2} \text{ and } y_M = \frac{y_A + y_B}{2}$$

Example 1. Find the midpoint of the line segment AB where points A and B are given as $A(-5, 4)$ and $B(3, -4)$.

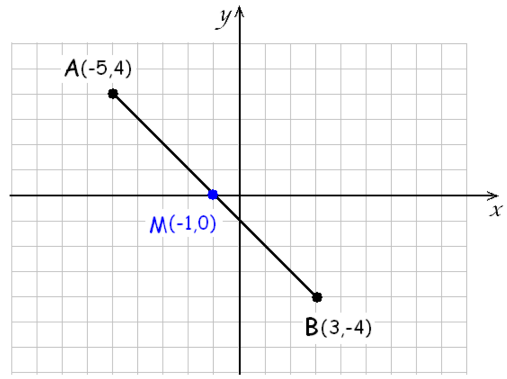
Solution: The $x-$coordinate of the midpoint is the average of the $x-$coordinates of A and B.

$$x_M = \frac{x_A + x_B}{2} = \frac{-5+3}{2} = \frac{-2}{2} = -1$$

Similarly, the $y-$coordinate of the midpoint is the average of the $y-$coordinates of A and B.

$$y_M = \frac{y_A + y_B}{2} = \frac{4+(-4)}{2} = \frac{0}{2} = 0$$

Thus the midpoint is $M(-1, 0)$.



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