Sample Problems

1. Solve each of the following system of linear equations.

   a) $\begin{cases} 3x - 2y = 26 \\ 2x + 5y = -27 \end{cases}$ b) $\begin{cases} x + 5y = 8 \\ 15y = -3x + 18 \end{cases}$ c) $\begin{cases} 2y - x = -50 \\ \frac{1}{2}x = y + 25 \end{cases}$

2. There is an animal farm where chickens and cows live. All together, there are 53 heads and 174 legs. How many chickens and how many cows are there on the farm?

3. We invested \$10000 into two bank accounts. One account earns 14% per year, the other account earns 8% per year. How much did we invest into each account if after the first year, the combined interest from the two accounts is \$1238?

4. We have a jar of coins, all pennies and dimes. All together, we have 372 coins, and the total value of all coins in the jar is \$20.91. How many pennies are there in the jar?

5. How many gallons of each of a 4% and an 11% salt solutions should be mixed to obtain 35 gallons of a 7% solution?

Practice Problems

1. Solve each of the following system of linear equations.

   a) $\begin{cases} 2x - y = -8 \\ x + 2y = -9 \end{cases}$ d) $\begin{cases} \frac{1}{2}x + \frac{1}{4}y = -1 \\ \frac{1}{2}y - \frac{1}{3}x = 6 \end{cases}$ g) $\begin{cases} 3x - 2y = 2 \\ 2x + 3y = 5 \end{cases}$

   b) $\begin{cases} 2(p-1) - 3(q-1) = 24 \\ p + q = -6 \end{cases}$ e) $\begin{cases} 3x - y = 5 \\ 2(y-1) = 6(x-2) \end{cases}$ h) $\begin{cases} x - 2y = 5 \\ 2x + 3y = 10 \end{cases}$

   c) $\begin{cases} 3x - y = 10 \\ \frac{1}{3}y - 2 = x \end{cases}$ f) $\begin{cases} 2a + b = 17 \\ a + b = 5 \end{cases}$ i) $\begin{cases} 0.5x - 1.2y = -1.21 \\ x + 3.2y = 2.06 \end{cases}$

2. Given the equations of two straight lines, find both coordinates of all intersection points.

   a) $2x - 5y = -41$ and $x + y = 4$

   b) $x + y = -5$ and $2y = -2x - 10$

   c) $y = \frac{3}{4}x - 2$ and $3x - 4y = -24$

   d) $5x - y = -35$ and $y = -\frac{3}{4}x + \frac{1}{2}$

   e) $y = -\frac{2}{3}x + 2$ and $2x + 3y = 6$

3. There is an animal farm where chickens and cows live. All together, there are 60 heads and 164 legs. How many chickens and how many cows are there on the farm?

4. We invested \$6000 into two bank accounts. One account earns 7% per year, the other account earns 11% per year. How much did we invest into each account if after the first year, the combined interest from the two accounts is \$520?

5. We have 51 coins, all dimes and quarters, in the total value of \$7.05. How many quarters and how many dimes are there?

6. We invested \$7600 into two bank accounts. One account earns 9% per year, the other account earns 13% per year. How much did we invest into each account if after the first year we have a total of \$8508 in the accounts?

7. How many gallons of each of a 67% and a 40% salt solutions should be mixed to obtain 27 gallons of a 49% solution?

8. How many gallons of each of a 45% and a 30% sugar solutions should be mixed to obtain 135 gallons of a 37% solution?

Sample Problems – Answers

1. a) $x = 4, y = -7$ b) there is no solution
   c) x can be any number, and then $y = \frac{1}{2}x - 25$
2. 19 chickens and 34 cows
3. \$7300 at 14% and \$2700 at 8%
4. \$4200 at 14% and \$2800 at 9%
5. 20 gallons of 4% solution with 15 gallons of 11% solution

Practice Problems – Answers

1. a) $x = -5, y = -2$ b) $p = 1, q = -7$ c) there is no solution d) $x = -6, y = 8$
   e) infintely many solutions; x can take any value, and then $y = 3x - 5$ f) $a = 12, b = -7$
   g) $x = \frac{16}{13}, y = \frac{11}{13}$ h) $x = 5, y = 0$ i) $x = -0.5, y = 0.8$
2. a) $(-3, 7)$ b) all points on $y = -x - 5$ are common; the two lines given are identical
   c) no common points; the two lines given are parallel d) $(-6, 5)$
   e) all points on $y = -\frac{2}{3}x + 2$ are common; the two lines given are identical.
3. 38 chickens, 22 cows
4. \$3500 at 7% and \$2500 at 11%
5. 38 dimes and 13 quarters
6. \$2000 at 9% and \$5600 at 13%
7. 9 gallons of the 67% solution and 18 gallons of the 40% solution
8. 63 gallons of the 45% solution and 72 gallons of the 30% solution

Sample Problems – Solutions

1. a) $\begin{cases} 3x - 2y = 26 \\ 2x + 5y = -27 \end{cases}$

Solution: We will eliminate x by multiplying the first equation by -2 and the second equation by 3, and then by adding the two equations.

$$\begin{array}{rcrcrcl} 3x - 2y & = & 26 & / \cdot (-2) & \Longrightarrow & -6x + 4y = -52 \\ 2x + 5y & = & -27 & / \cdot 3 & \Longrightarrow & 6x + 15y = -81 \end{array}$$

We now add the equations

$$\begin{array}{rcl} -6x + 4y & = & -52 \\ 6x + 15y & = & -81 \end{array}$$

$$\begin{array}{rcll} 19y & = & -133 & \text{divide by 19} \\ y & = & -7 & \end{array}$$

Now that we know the value of y, we use the first equation to find the value of x.

$$\begin{array}{rcll} 3x - 2(-7) & = & 26 & \\ 3x + 14 & = & 26 & \text{subtract 14} \\ 3x & = & 12 & \text{divide by 3} \\ x & = & 4 & \end{array}$$

Thus the solution is $x = 4,\ y = -7$. We check: the pair should be a solution for both equations.

$$\begin{array}{rcl} 3(4) - 2(-7) & = & 12 + 14 = 26 \ \checkmark \\ 2(4) + 5(-7) & = & 8 - 35 = -27 \ \checkmark \end{array}$$

Thus our solution is correct.

b) $\begin{cases} x + 5y = 8 \\ 15y = -3x + 18 \end{cases}$

Solution: We first transform the seond equation to have a similar structure to that of the first equation.

$$\begin{array}{rcll} 15y & = & -3x + 18 & \text{add } 3x \\ 3x + 15y & = & 18 & \text{divide by 3} \\ x + 5y & = & 6 & \end{array}$$

So now we have to solve the system

$$\begin{cases} x + 5y = 8 \\ x + 5y = 6 \end{cases}$$

will eliminate x by multiplying the second equation by -1 and then by adding the two equations.

$$\begin{array}{c} x + 5y = 8 \\ -x - 5y = -6 \\ 0 = 2 \end{array}$$

Since there is no value for x or y that could make the statement $0 = 2$ true, there is no solution for this system. A linear system like this is called an **inconsistent system**.

c) $\begin{cases} 2y - x = -50 \\ \frac{1}{2}x = y + 25 \end{cases}$

Solution: We will transform both equations to bring them to a common, convenient form. We re-arrange the left-hand side in the first equation

$$-x + 2y = -50$$

and

$$\begin{aligned} \frac{1}{2}x &= y + 25 && \text{multiply by 2} \\ x &= 2y + 50 && \text{subtract } 2y \\ x - 2y &= 50 \end{aligned}$$

So now we have to solve the following system

$$\begin{cases} -x + 2y = -50 \\ x - 2y = 50 \end{cases}$$

we can eliminate x by adding the two equations and obtain the equation $0 = 0$. This is in fact true for all values of x. What happens here, the first equation establishes a connection between x and y, namely, $y = \frac{1}{2}x - 25$. The second equation does not contain any new information, it is just a disguised re-statement of the first equation. A system like this is called a **dependent system**. This system has infinitely many solutions. x can take any value, and then y must be $y = \frac{1}{2}x - 25$. Thus, there are infinitely many solutions.

2. There is an animal farm where chickens and cows live. All together, there are 53 heads and 174 legs. How many chickens and how many cows are there on the farm?

Solution: We will denote the number of chickens by x and the number of cows by y. The first equation will express the number of heads, the second equation will express the number of legs.

$$\begin{aligned} x + y &= 53 \\ 2x + 4y &= 174 \end{aligned}$$

To eliminate x, we multiply the first equation by -1 and divide the second equation by 2.

$$\begin{aligned} -x - y &= -53 \\ x + 2y &= 87 \end{aligned}$$

Now we add the two equations.

$$y = 34$$

Now that we know the value of y, we use the first equation to find x.

$$\begin{aligned} x + 34 &= 53 && \text{subtract } 34 \\ x &= 19 && \Longrightarrow x = 19, \quad y = 34 \end{aligned}$$

Thus we have 19 chickens and 34 cows. We check: the number of heads is $19 + 34 = 53$, and the number of legs is $2\,(19) + 4\,(34) = 38 + 136 = 174$. So our solution is correct.

3. We invested \$10 000 into two bank accounts. One account earns 14% per year, the other account earns 8% per year. How much did we invest into each account if after the first year, the combined interest from the two accounts is \$1238?
Solution: Let us denote the amount invested at 14% by x and the amount invested at 8% by y. The two equations express that

$$\begin{aligned} x+y &= 10\,000 \quad \text{the amounts add up to \$10 000} \\ 0.14x+0.08y &= 1238 \quad \text{the interests earned add up to \$1238} \end{aligned}$$

We solve the system of equation by elimination. But let us first make the second equation simpler:

$$\begin{aligned} 0.14x+0.08y &= 1238 \quad \text{multiply by 100} \\ 14x+8y &= 123800 \quad \text{divide by 2} \\ 7x+4y &= 61900 \end{aligned}$$

We now have

$$\begin{aligned} x+y &= 10000 \\ 7x+4y &= 61900 \end{aligned}$$

We will multiply the first equation by -4 to eliminate y.

$$\begin{aligned} -4x-4y &= -40000 \\ 7x+4y &= 61900 \end{aligned}$$

We add the equations and solve for x.

$$\begin{aligned} 3x &= 21900 \quad \text{divide by 3} \\ x &= 7300 \end{aligned}$$

Thus we invested \$7300 at 14%. The other amount is then from the first equation:

$$\begin{aligned} 7300+y &= 10000 \\ y &= 2700 \end{aligned}$$

We invested \$7300 at 14% and \$2700 at 8%. We check: the amounts add up to \$7300 + \$2700 = \$10 000. The interest from the accounts are

$$\begin{aligned} 14\% \text{ of } 7300 \text{ is } 0.14\,(7300) &= 1022 \text{ and} \\ 8\% \text{ of } 2700 \text{ is } 0.08\,(2700) &= 216 \end{aligned}$$

Since $1022 + 216 = 1238$, our solution is correct.

4. We invested \$7000 into two bank accounts. One account earns 14% per year, the other account earns 9% per year. How much did we invest into each account if after the first year, the combined interest from the two accounts is \$840?
Solution: Let us denote the amount invested at 14% by x and the amount invested at 9% by y. The two equations express that

$$\begin{aligned} x+y &= 7000 \qquad \text{the amounts add up to \$7000} \\ 0.14x+0.09y &= 840 \qquad \text{the interests earned add up to \$840} \end{aligned}$$

We solve the system of equation by elimination. But let us first make the second equation simpler:

$$\begin{aligned} 0.14x+0.09y &= 840 \qquad \text{multiply by 100} \\ 14x+9y &= 84\,000 \end{aligned}$$

We now have

$$\begin{aligned} x+y &= 7000 \\ 14x+9y &= 84\,000 \end{aligned}$$

We will multiply the first equation by -9 and leave alone the second equation. Then we add the two equations:

$$\begin{aligned} -9x-9y &= -63\,000 \\ 14x+9y &= 84\,000 \end{aligned}$$

We add the two equations - y is cancelled out

$$\begin{aligned} 5x &= 21\,000 \qquad \text{divide by 5} \\ x &= 4200 \end{aligned}$$

Then $y = 7000 - x = 7000 - 4200 = 2800$. Thus we invested \$4200 at 14% and \$2800 at 9%. We check: the amounts add up to \$4200 + \$2800 = \$7000. The interest from the accounts are:

$$14\% \text{ of } 4200 \text{ is } 0.14\,(4200) = 588 \text{ and } 9\% \text{ of } 2800 \text{ is } 0.09\,(2800) = 252$$

Since $588 + 252 = 840$, our solution is correct.

5. How many gallons of each of a 4% and an 11% salt solutions should be mixed to obtain 35 gallons of a 7% solution?

   Solution: Solution: Let us denote by x the amount of 4% solution and by y the amount of 11% solution. Clearly, the two amounts should add up to 35 gallons, giving us the equation $x + y = 35$.

	Amount of Solution (gallons)	Percentage	Amount of Solvant (gallons)
Component 1	x	0.04	$0.04x$
Component 2	y	0.11	$0.11y$
Mixture	35	0.07	

   Since we have two unknown variables, we will need two equations. The first one is easy, the volume of the mixture should be 35.

$$x + y = 35$$

   We obtain the second equation by stating that the amount of solvant in the components must add up to the amount of solvant. (In other words, the last entry in the third row can be written in two different ways: the product of 35 and 7%; and the sum of $0.04x$ and 0.11)

$$0.07\,(35) = 0.04x + 0.11y$$

   And this equation can be immediately made much nicer by simply multiplying both sides by 100. Then we have:

$$\begin{aligned} 7\,(35) &= 4x + 11y \\ 4x + 11y &= 245 \end{aligned}$$

   So our system is now

$$\begin{cases} x + y = 35 \\ 4x + 11y = 245 \end{cases}$$

   We solve this system using elimination. We will multiply the first equation by -4 and then add the two equations, thereby cancelling out x.

$$\begin{aligned} -4x - 4y &= -140 \\ 4x + 11y &= 245 \end{aligned}$$

   Add the two equation:

$$\begin{aligned} 7y &= 105 \qquad \text{divide by 7} \\ y &= 15 \end{aligned}$$

   If $y = 15$, then the other amount, denoted by x can be found using the equation $x + y = 35$.

$$x + 15 = 35 \quad \Longrightarrow \quad x = 20$$

   Thus we need to mix 20 gallons of 4% solution with 15 gallons of 11% solution.

   We check our solution: suppose we mix the two solutions specified above. We need to find how much solution and how much solvant we have, hoping that the amount of solvant indeed will be 8% of the amount of mixture.

	Amount of Solution			Amount of Solvant
Component 1	20 gallons	of 4% solution	$\Longrightarrow$	$0.04\,(20) = 0.8$ gallons
Component 2	15 gallons	of 11% solution	$\Longrightarrow$	$0.11\,(15) = 1.65$ gallons
	$\Downarrow$			$\Downarrow$
	35 gallons			$0.8 + 1.65 = 2.45$ gallons

   7% of 35 is $0.07\,(35) = 2.45$ Thus our solution is correct.

