Sample Problems

1. In each case, find the slope of the line determined by the two points given.
   a) $(5,-1)$ and $(1,3)$ b) $(2,1)$ and $(6,3)$ c) $(8,-1)$ and $(-1,-1)$ d) $(7,3)$ and $(7,-4)$
2. For each straight line given, find its slope.
   a) $y=-2x+5$. b) $2x-5y=8$ c) $y=3$ d) $7(x-3)+x-2y=2y+1$
   e) $x=-\frac{1}{2}$ f) a line parallel to $y=3x-1$ g) a line perpendicular to $y=3x-1$
3. Find an equation for the straight line that has slope 4 and $y-$intercept $(0,-5)$.
4. Find an equation for the straight line that has slope $-\frac{1}{2}$ and passes through the point $(4,1)$.
5. Find an equation of the straight line that is parallel to $3x-2y=12$ and passes through the point $(8,-1)$.
6. Find an equation of the straight line that is perpendicular to $x+5y=-3$ and passes through the point $(-1,4)$.
7. Find an equation of the straight line that passes through the points $(3,-1)$ and $(1,5)$.

Practice Problems

1. In each case, find the slope of the line determined by the two points given.
   a) $(-2,-3)$ and $(3,-1)$ b) $(12,-7)$ and $(6,-3)$ c) $(2,5)$ and $(2,10)$ d) $(12,9)$ and $(-6,9)$
2. For each straight line given, find its slope.
   a) $y=\frac{3}{4}x$ b) $3x+10y=1$ c) $y=0$ d) $3(y-2)-2(5-x)=3x-4$
   e) $x=11$ f) a line parallel to $y=\frac{1}{2}x+7$ g) a line perpendicular to $y=\frac{1}{2}x+7$
3. Find an equation for the straight line that has slope -2 and $y-$intercept $(0,7)$.
4. Find an equation for the straight line that has slope $\frac{2}{3}$ and passes through the point $(-6,7)$.
5. Find an equation of the straight line that is parallel to $3x-2y=12$ and passes through the point $(8,-1)$.
6. Find an equation of the straight line that is perpendicular to $x+5y=-3$ and passes through the point $(-1,4)$.
7. Find an equation of the straight line that passes through the points $(2,10)$ and $(5,-2)$.
8. Find an equation of the straight line that passes through the points $(-4,5)$ and $(6,0)$.

Sample Problems - Answers

1. a) -1 b) $\frac{1}{2}$ c) 0 d) This line has no slope
2. a) -2 b) $\frac{2}{5}$ c) 0 d) 2 e) This line has no slope f) 3 g) $-\frac{1}{3}$
3. $y = 4x - 5$
4. $y = -\frac{1}{2}x + 3$
5. $y = \frac{3}{2}x - 13$
6. $y = 5x + 9$
7. $y = -3x + 8$

Practice Problems - Answers

1. a) $\frac{2}{5}$ b) $-\frac{2}{3}$ c) This line has no slope d) 0
2. a) $\frac{3}{4}$ b) $-\frac{3}{10}$ c) 0 d) $\frac{1}{3}$ e) This line has no slope f) $\frac{1}{2}$ g) -2
3. $y = -2x + 7$
4. $y = \frac{2}{3}x + 11$
5. $y = \frac{3}{2}x - 13$
6. $y = 5x + 9$
7. $y = -4x + 18$
8. $y = -\frac{1}{2}x + 3$

Sample Problems - Solutions

1. In each case, find the slope of the line determined by the two points given.

   (a) $(5,-1)$ and $(1,3)$ -1
   Solution: We find the slope determined by the points. $(5,-1)=(x_1,y_1)$ and $(1,3)=(x_2,y_2)$ using the slope formula.
   $$m=\frac{\text{rise}}{\text{run}}=\frac{y_2-y_1}{x_2-x_1}=\frac{3-(-1)}{1-5}=\frac{4}{-4}=-1$$

   (b) $(2,1)$ and $(6,3)$ $\frac{1}{2}$
   Solution: We find the slope determined by the points. $(2,1)=(x_1,y_1)$ and $(6,3)=(x_2,y_2)$ using the slope formula.
   $$m=\frac{\text{rise}}{\text{run}}=\frac{y_2-y_1}{x_2-x_1}=\frac{3-1}{6-2}=\frac{2}{4}=\frac{1}{2}$$

   (c) $(8,-1)$ and $(-1,-1)$ 0
   Solution: We find the slope determined by the points. $(8,-1)=(x_1,y_1)$ and $(-1,-1)=(x_2,y_2)$ using the slope formula.
   $$m=\frac{\text{rise}}{\text{run}}=\frac{y_2-y_1}{x_2-x_1}=\frac{-1-(-1)}{-1-8}=\frac{0}{-9}=0$$

   (d) $(7,3)$ and $(7,-4)$. This line has no slope
   Solution: We find the slope determined by the points. $(7,3)=(x_1,y_1)$ and $(7,-4)=(x_2,y_2)$ using the slope formula.
   $$m=\frac{\text{rise}}{\text{run}}=\frac{y_2-y_1}{x_2-x_1}=\frac{-4-3}{7-7}=\frac{-7}{0}=\text{undefined}$$

2. For each straight line given, find its slope.

   (a) the line $y=-2x+5$. -2
   Solution: The equation of the line is given in slope-intercept form. We simply read the coefficient of x.

   (b) the line $2x-5y=8$ $\frac{2}{5}$
   Solution: We first bring the equation of the line to its slope-intercept form by solving for y.. Then we simply read the coefficient of x.
   $$\begin{aligned}2x-5y&=8 && \text{add } 5y\\ 2x&=5y+8 && \text{subtract } 8\\ 2x-8&=5y && \text{divide by } 5\\ \frac{2x-8}{5}&=y\\ y&=\frac{2}{5}x-\frac{8}{5}\end{aligned}$$

   (c) the line $y=3$ 0
   Solution: The equation of the line is given in slope-intercept form. We simply read the coefficient of x. In this case, we don't see x because its coefficient is zero. We can re-write it as $y=0x+3$

(d) the line $7(x-3)+x-2y=2y+1$ 2
Solution: We first bring the equation of the line to its slope-intercept form by solving for y.. Then we simply read the coefficient of x.

$$\begin{aligned}
7(x-3)+x-2y &= 2y+1 && \text{distribute}\\
7x-21+x-2y &= 2y+1 && \text{combine like terms}\\
8x-2y-21 &= 2y+1 && \text{add } 2y\\
8x-21 &= 4y+1 && \text{subtract 1}\\
8x-22 &= 4y && \text{divide by 4}\\
\frac{8x-22}{4} &= y\\
y &= \frac{8}{4}x-\frac{22}{4}\\
y &= 2x-\frac{11}{2}
\end{aligned}$$

(e) the line $x=-\frac{1}{2}$. This line has no slope
Solution: We can not bring this equation to its slope-intercept form by solving for y because y does not appear in it. This means that there is no slope. If this is confusing, select any two points from the line, say $\left(-\frac{1}{2},1\right)$ and $\left(-\frac{1}{2},5\right)$ and apply the slope formula as in problem #1d). It will be clear that m is undefined since we will end up dividing by zero.

(f) a line parallel to $y=3x-1$. 3
Solution: The line $y=3x-1$ has slope 3. Parallel lines have the same slope.

(g) a line perpendicular to $y=3x-1$. $-\frac{1}{3}$
Solution: The line $y=3x-1$ has slope 3. Perpendicular lines have slopes that are negative reciprocals of each other. The negative reciprocal of 3 is $-\frac{1}{3}$.

3. Find an equation for the straight line that has slope 4 and $y-$intercept $(0,-5)$. $y=4x-5$
Solution: The equation is very easy to find with the data given. The slope-intercept form of the line is

$$y=mx+b$$

and we know that $m=4$ and $b=-5$. thus the answer is: $y=4x-5$

4. Find an equation for the straight line that has slope $-\frac{1}{2}$ and passes through the point $(4,1)$. $y=-\frac{1}{2}x+3$
Solution: We will present two methods.
Method 1 . Slope-intercept form. We will be looking for the slope-intercept form of the straight line, $y=mx+b$. We will know the answer once we have found the values of m and b. We already know that $m=-\frac{1}{2}$. Thus we have so far

$$y=-\frac{1}{2}x+b$$

We will find the value of b by using the fact that the line passes through the point $(4,1)$. Since the point $(4,1)$ is on the straight line $y=-\frac{1}{2}x+b$, the coordinates of the point is a solution of the equation. This will give us an equation for b that we can solve.

$$\begin{aligned} y &= -\frac{1}{2}x+b & &\text{the point } (4,1) \text{ is on the line} \\ 1 &= -\frac{1}{2}(4)+b \\ 1 &= -2+b \\ 3 &= b \end{aligned}$$

Thus the equation of the line is $y=-\frac{1}{2}x+3$.

Method 2. Point-slope form. We know that the slope is $m=-\frac{1}{2}$ and a point on the line is $(4,1)$. We can write the line's equation in one easy step:

$$y-1=-\frac{1}{2}(x-4)$$

Why would this be the correct answer? First, the equation above is clearly a line with slope $-\frac{1}{2}$. Also, the point $(4,1)$ is on this line, since the coordinates $x=4$ and $y=1$ are clearly a solution of the equation. Thus, this is the right line. We just simplify the equation

$$\begin{aligned} y-1 &= -\frac{1}{2}(x-4) \\ y-1 &= -\frac{1}{2}x+2 \\ y &= -\frac{1}{2}x+3 \end{aligned}$$

5. Find an equation of the straight line that is parallel to $3x-2y=12$ and passes through the point $(8,-1)$. $y=\frac{3}{2}x-13$

Solution: We start with the slope of $3x-2y=12$. We bring the equation to its slope-intercept form by solving for y.

$$\begin{aligned} 3x-2y &= 12 & &\text{add } 2y \\ 3x &= 2y+12 & &\text{subtract } 12 \\ 3x-12 &= 2y & &\text{divide by } 2 \\ y &= \frac{3}{2}x-6 \end{aligned}$$

Thus the line $3x-2y=12$ has slope $\frac{3}{2}$. Since parallel, our line must have the same slope. Now the problem is like the previous one: the slope and a point is given. Using the point-slope form, we write

$$\begin{aligned} y-(-1) &= \frac{3}{2}(x-8) & &\text{and simplify.} \\ y+1 &= \frac{3}{2}x-\frac{3}{2}(8) \\ y+1 &= \frac{3}{2}x-12 \\ y &= \frac{3}{2}x-13 \end{aligned}$$

6. Find an equation of the straight line that is perpendicular to $x+5y=-3$ and passes through the point $(-1,4)$. $y=5x+9$
Solution: We start with the slope of $x+5y=-3$. We bring the equation $x+5y=-3$ to its slope-intercept form by solving for y.

$$\begin{aligned} x+5y &= -3 && \text{subtract } x \\ 5y &= -x-3 && \text{divide by 5} \\ y &= -\frac{1}{5}x-\frac{3}{5} \end{aligned}$$

Thus the line $x+5y=-3$ has slope $-\frac{1}{5}$. Since they are perpendicular, the slope of our line is the negative reciprocal of $-\frac{1}{5}$, which is 5. Now the problem is like the previous one: the slope and a point is given. Using the point-slope form, we write

$$\begin{aligned} y-4 &= 5(x-(-1)) && \text{and simplify.} \\ y-4 &= 5(x+1) \\ y-4 &= 5x+5 \\ y &= 5x+9 \end{aligned}$$

7. Find an equation of the straight line that passes through the points $(3,-1)$ and $(1,5)$. $y=-3x+8$
Solution: We will presesnt three different methods.
Method 1. Slope-intercept form. We will be looking for the slope-intercept form of the straight line, $y=mx+b$. We will know the answer once we have found the values of m and b.
We first find the slope determined by the points. If the point $(3,-1)=(x_1,y_1)$ and $(1,5)=(x_2,y_2)$, then the slope formula gives us

$$m=\frac{\text{rise}}{\text{run}}=\frac{y_2-y_1}{x_2-x_1}=\frac{5-(-1)}{1-3}=\frac{6}{-2}=-3$$

Now we know that the slope is -3, and so the equation so far is

$$y=-3x+b$$

We will find the value of b by using the fact that the line passes through the point $(1,5)$. (Any of the two given points can be used.) Since the point $(1,5)$ is on the straight line $y=-3x+b$, the coordinates of the point is a solution of the equation. This will give us an equation for b that we can solve.

$$\begin{aligned} y &= -3x+b && \text{the point } (1,5) \text{ is on the line} \\ 5 &= -3(1)+b \\ 5 &= -3+b \\ 8 &= b \end{aligned}$$

Thus the equation of the line is $y=-3x+8$.

Method 2. Point-slope form.
We first find the slope determined by the points. If the point $(3,-1)=(x_1,y_1)$ and $(1,5)=(x_2,y_2)$, then the slope formula gives us

$$m=\frac{\text{rise}}{\text{run}}=\frac{y_2-y_1}{x_2-x_1}=\frac{5-(-1)}{1-3}=\frac{6}{-2}=-3$$

We know that the slope is $m=-3$ and a point on the line is $(1,5)$. We can write the line's equation in one easy step:

$$y-5=-3(x-1)$$

Why would this be the correct answer? First, the equation above is clearly a line with slope -3. Also, the point $(1,5)$ is on this line, since the coordinates $x=1$ and $y=5$ are clearly a solution of the equation. Thus, this is the right line. We just simplify the equation

$$\begin{aligned} y-5 &= -3(x-1) \\ y-5 &= -3x+3 \\ y &= -3x+8 \end{aligned}$$

Method 3. This is an application of systems of equations.
We are looking for the slope-intercept form of the straight line, $y=mx+b$. We will know the answer once we have found the value of m and b. Both points $(1,5)$ and $(3,-1)$ are on the line, and so their coordinates are solution of the equation.

$$y = mx+b$$

$$\begin{aligned} 5 &= m(1)+b \quad \text{since } (1,5) \text{ is on the line, and} \\ -1 &= m(3)+b \quad \text{since } (3,-1) \text{ is on the line} \end{aligned}$$

This gives us a system of linear equations in m and b:

$$\begin{cases} m+b=5 \\ 3m+b=-1 \end{cases}$$

This system can easily be solved by elimination. We multiply the first equation by -1.

$$\begin{aligned} -m-b &= -5 \\ 3m+b &= -1 \end{aligned}$$

We add the two equations to eliminate b, and solve the equation for b.

$$\begin{aligned} 2m &= -6 \\ m &= -3 \end{aligned}$$

Now that we know the value of m, we find b by sustituting $m=-3$ into the first equation (any of the two can be used) and solve for b.

$$\begin{aligned} -3+b &= 5 \\ b &= 8 \end{aligned}$$

Thus $m=-3$ and $b=8$ and so the equation is $y=-3x+8$.

