Sample Problems

1. $\sqrt{3x-2}=x$
2. $\sqrt[3]{3x-6}=3$
3. $10+\sqrt{4x-7}=7$
4. $5+\sqrt{x+15}=x$
5. $2\sqrt{x-1}=x-4$
6. $2\sqrt{x+4}=1+\sqrt{2x+9}$
7. $5\sqrt{x}+1=3\sqrt{x}+17$
8. $\sqrt{2x+5}+5=x$
9. $\sqrt{2x+5}-\sqrt{x-1}=\sqrt{x+2}$
10. $\sqrt[3]{x^3+26}=x+2$
11. $\sqrt{3x+1}-\sqrt{x-4}=3$
12. $\sqrt{x+10}+10=x$
13. $\sqrt[3]{x^3+208}=x+4$
14. $\sqrt{x-1}+\sqrt{x-4}=\sqrt{4x-11}$
15. $\sqrt{4x+6}=\sqrt{x+1}-\sqrt{x+5}$

Practice Problems

1. $\sqrt{3x-5}=4$
2. $2\sqrt{a-1}+7=1$
3. $3\sqrt{7x+1}+2=20$
4. $\sqrt{x+3}=x-9$
5. $2\sqrt{x+5}=x-3$
6. $\sqrt{2p+4}-\sqrt{p+3}=1$
7. $\sqrt[3]{x^3+16}=x+4$
8. $\sqrt{18+x}=x-2$
9. $\sqrt{w}+\sqrt{w+3}=3$
10. $\sqrt{2x+1}+\sqrt{5-x}=4$
11. $2\sqrt{x}-\sqrt{x-3}=\sqrt{x+7}$
12. $2\sqrt{y+4}+\sqrt{y-5}=\sqrt{9y+7}$
13. $\sqrt{b-2}+b=8$
14. $\sqrt{3x+1}-\sqrt{x+4}=1$
15. $\sqrt[3]{x-8}-\sqrt[3]{4x+1}=0$
16. $\sqrt{5m-9}=\sqrt{5m}-3$
17. $\sqrt{3k-5}-\sqrt{3k}=-1$
18. $\sqrt{1-x}+1=x+12$
19. $2=\sqrt{x^2+1}-x$
20. $\sqrt{3x+1}-4=8-2\sqrt{3x+1}$
21. $\sqrt{x}=\sqrt{10+3\sqrt{x}}$
22. $\sqrt[3]{x^3+7}=x+1$
23. $\sqrt{x-6}=\sqrt{x+2}-4$
24. $\sqrt{6x+7}-\sqrt{3x+3}=1$

Sample Problems – Answers

1.) $1, 2$ 2.) 11 3.) no real solution 4.) 10 5.) 10 6.) 0 7.) 64 8.) 10

9.) 2 10.) $-3, 1$ 11.) $5, 8$ 12.) 15 13.) $-6, 2$ 14.) 5 15.) no real solution

Practice Problems – Answers

1.) 7 2.) no real solution 3.) 5 4.) 13 5.) 11 6.) 6 7.) -2 8.) 7 9.) 1

10.) $4, \frac{20}{9}$ 11.) $\frac{25}{8}$ 12.) 21 13.) 6 14.) 5 15.) -3 16.) $\frac{9}{5}$ 17.) 3

18.) -8 19.) $-\frac{3}{4}$ 20.) 5 21.) 25 22.) $-2, 1$ 23.) no real solution 24.) $\frac{1}{3}, -1$

Sample Problems – Solutions

1. $\sqrt{3x-2}=x$ $1,2$
Solution:

$$\begin{aligned} \sqrt{3x-2} &= x && \text{square} \\ 3x-2 &= x^2 && \text{reduce one side to zero} \\ 0 &= x^2-3x+2 && \text{factor} \\ 0 &= (x-2)(x-1) && \Longrightarrow \quad x_1=2 \quad \text{and} \quad x_2=1 \end{aligned}$$

We check: if $x=2$, then

$$\begin{aligned} \text{LHS} &= \sqrt{3(2)-2}=\sqrt{4}=2 \\ \text{RHS} &= 2 \end{aligned}$$

and if $x=1$, then

$$\begin{aligned} \text{LHS} &= \sqrt{3(1)-2}=\sqrt{1}=1 \\ \text{RHS} &= 1 \end{aligned}$$

Thus the solution set is: $\{1,2\}$

2. $\sqrt[3]{3x-6}=3$ 11
Solution:

$$\begin{aligned} \sqrt[3]{3x-6} &= 3 && \text{raise both sides to the third power} \\ 3x-6 &= 27 && \text{add 6} \\ 3x &= 33 && \text{divide by 3} \\ x &= 11 \end{aligned}$$

We check: if $x=11$, then

$$\text{LHS}=\sqrt[3]{3(11)-6}=\sqrt[3]{27}=3=\text{RHS}$$

3. $10+\sqrt{4x-7}=7$ no real solution
Solution:

$$\begin{aligned} 10+\sqrt{4x-7} &= 7 && \text{subtract 10} \\ \sqrt{4x-7} &= -3 \end{aligned}$$

Since the square root of no real number is negative, there is no real solution.

4. $5+\sqrt{x+15}=x$ 10
Solution:

$$\begin{aligned} 5+\sqrt{x+15} &= x && \text{subtract 5} \\ \sqrt{x+15} &= x-5 && \text{square both sides} \\ x+15 &= (x-5)^2 && \text{FOIL right hand side} \\ x+15 &= x^2-10x+25 && \text{reduce one side to zero} \\ 0 &= x^2-11x+10 && \text{factor} \\ 0 &= (x-1)(x-10) && \Longrightarrow \quad x_1=1 \quad \text{and } x_2=10 \end{aligned}$$

We check: If $x = 1$, then

$$\begin{aligned} \text{LHS} &= 5 + \sqrt{1+15} = 5 + \sqrt{16} = 5 + 4 = 9 \\ \text{RHS} &= 1 \\ \text{RHS} &\neq \text{LHS} \end{aligned}$$

Thus $x = 1$ is NOT a solution.

If $x = 10$, then

$$\begin{aligned} \text{LHS} &= 5 + \sqrt{10+15} = 5 + \sqrt{25} = 5 + 5 = 10 \\ \text{RHS} &= 10 \\ \text{RHS} &= \text{LHS} \end{aligned}$$

Thus $x = 10$ is the only solution.

5. $2\sqrt{x-1} = x - 4$ 10
Solution:

$$\begin{aligned} 2\sqrt{x-1} &= x - 4 && \text{square both sides} \\ 4(x-1) &= (x-4)^2 && \text{FOIL, distribute} \\ 4x - 4 &= x^2 - 8x + 16 && \text{reduce one side to zero} \\ 0 &= x^2 - 12x + 20 && \text{factor} \\ 0 &= (x-2)(x-10) && \Longrightarrow \quad x_1 = 2 \quad \text{and} \quad x_2 = 10 \end{aligned}$$

We check: If $x = 2$, then

$$\begin{aligned} \text{LHS} &= 2\sqrt{2-1} = 2\sqrt{1} = 2(1) = 2 \\ \text{RHS} &= 2 - 4 = -2 \\ \text{RHS} &\neq \text{LHS} \end{aligned}$$

Thus $x = 2$ is NOT a solution.
If $x = 10$, then

$$\begin{aligned} \text{LHS} &= 2\sqrt{10-1} = 2\sqrt{9} = 2(3) = 6 \quad \text{and} \quad \text{RHS} = 10 - 4 = 6 \\ \text{RHS} &= \text{LHS} \end{aligned}$$

Thus $x = 10$ is the only solution.

6. $2\sqrt{x+4} = 1 + \sqrt{2x+9}$ 0
Solution:

$$\begin{aligned} 2\sqrt{x+4} &= 1 + \sqrt{2x+9} && \text{square both sides} \\ \left(2\sqrt{x+4}\right)^2 &= \left(1 + \sqrt{2x+9}\right)^2 && \\ 2^2\left(\sqrt{x+4}\right)^2 &= \left(1 + \sqrt{2x+9}\right)\left(1 + \sqrt{2x+9}\right) && \\ 4(x+4) &= 1 + \sqrt{2x+9} + \sqrt{2x+9} + 2x + 9 && \text{combine like terms} \\ 4x + 16 &= 2x + 10 + 2\sqrt{2x+9} && \text{subtract } 2x \\ 2x + 16 &= 10 + 2\sqrt{2x+9} && \text{subtract } 10 \\ 2x + 6 &= 2\sqrt{2x+9} && \text{factor out } 2 \end{aligned}$$

$$
\begin{aligned}
2(x+3) &= 2\sqrt{2x+9} && \text{divide by 2}\\
x+3 &= \sqrt{2x+9} && \text{square both sides}\\
(x+3)^2 &= 2x+9 &&\\
x^2+6x+9 &= 2x+9 && \text{reduce one side to zero}\\
x^2+4x &= 0 && \text{factor}\\
x(x+4) &= 0 \quad \Longrightarrow \quad x_1 = 0 \quad \text{and} \quad x_2 = -4 &&
\end{aligned}
$$

We check: If $x = 0$, then

$$\text{LHS} = 2\sqrt{0+4} = 2\sqrt{4} = 2 \cdot 2 = 4 \quad \text{and} \quad \text{RHS} = 1 + \sqrt{2(0)+9} = 1+3 = 4$$

Thus $x = 0$ is indeed a solution.
If $x = -4$, then

$$
\begin{aligned}
\text{LHS} &= 2\sqrt{(-4)+4} = 2\sqrt{0} = 2(0) = 0\\
\text{RHS} &= 1 + \sqrt{2(-4)+9} = 1 + \sqrt{-8+9} = 1 + \sqrt{1} = 1+1 = 2\\
\text{RHS} &\neq \text{LHS}
\end{aligned}
$$

Thus $x = -4$ is NOT a solution. The only solution is $x = 0$.

7. $5\sqrt{x} + 1 = 3\sqrt{x} + 17$ 64
Solution:

$$
\begin{aligned}
5\sqrt{x}+1 &= 3\sqrt{x}+17 && \text{subtract } 3\sqrt{x}\\
2\sqrt{x}+1 &= 17 && \text{subtract 1}\\
2\sqrt{x} &= 16 && \text{divide by 2}\\
\sqrt{x} &= 8 && \text{square both sides}\\
x &= 64 &&
\end{aligned}
$$

We check: If $x = 64$, then

$$\text{LHS} = 5\sqrt{64} + 1 = 5(8) + 1 = 41 \quad \text{and} \quad \text{RHS} = 3\sqrt{64} + 17 = 3(8) + 17 = 24 + 17 = 41$$

Thus $x = 64$ is indeed a solution.

8. $\sqrt{2x+5} + 5 = x$ 10
Solution:

$$
\begin{aligned}
\sqrt{2x+5}+5 &= x && \text{subtract 5}\\
\sqrt{2x+5} &= x-5 && \text{square}\\
2x+5 &= x^2-10x+25 && \text{reduce one side to zero}\\
0 &= x^2-12x+20 && \text{factor}\\
0 &= (x-2)(x-10) \quad \Longrightarrow \quad x_1 = 2 \quad \text{and} \quad x_2 = 10 &&
\end{aligned}
$$

We check: If $x = 2$, then

$$
\begin{aligned}
\text{LHS} &= \sqrt{2(2)+5} + 5 = \sqrt{4+5} + 5 = \sqrt{9} + 5 = 3+5 = 8 \quad \text{and} \quad \text{RHS} = 2\\
\text{RHS} &\neq \text{LHS}
\end{aligned}
$$

Thus $x = 2$ is NOT a solution.
If $x = 10$, then

$$
\begin{aligned}
\text{LHS} &= \sqrt{2(10)+5} + 5 = \sqrt{20+5} + 5 = \sqrt{25} + 5 = 5+5 = 10\\
\text{RHS} &= 10\\
\text{RHS} &= \text{LHS}
\end{aligned}
$$

Thus $x = 10$ is the only solution.

9. $\sqrt{2x+5} - \sqrt{x-1} = \sqrt{x+2}$ 2

Solution:

$$\begin{aligned}
\sqrt{2x+5} - \sqrt{x-1} &= \sqrt{x+2} && \text{add } \sqrt{x-1} \\
\sqrt{2x+5} &= \sqrt{x+2} + \sqrt{x-1} && \text{square} \\
\left(\sqrt{2x+5}\right)^2 &= \left(\sqrt{x+2} + \sqrt{x-1}\right)^2 \\
2x+5 &= \left(\sqrt{x+2} + \sqrt{x-1}\right)\left(\sqrt{x+2} + \sqrt{x-1}\right) \\
2x+5 &= \underbrace{\sqrt{x+2}\sqrt{x+2}}_{\text{F}} + \underbrace{\sqrt{x+2}\sqrt{x-1}}_{\text{O}} + \underbrace{\sqrt{x-1}\sqrt{x+2}}_{\text{I}} + \underbrace{\sqrt{x-1}\sqrt{x-1}}_{\text{L}} \\
2x+5 &= x+2+2\sqrt{x-1}\sqrt{x+2}+x-1 \\
2x+5 &= 2x+1+2\sqrt{x-1}\sqrt{x+2} && \text{subtract } 2x \\
5 &= 1+2\sqrt{(x-1)(x+2)} && \text{subtract 1} \\
4 &= 2\sqrt{(x-1)(x+2)} && \text{divide by 2} \\
2 &= \sqrt{(x-1)(x+2)} && \text{square} \\
4 &= (x-1)(x+2) && \text{FOIL} \\
4 &= x^2+x-2 && \text{reduce one seide to zero} \\
0 &= x^2+x-6 && \text{factor} \\
0 &= (x+3)(x-2) \quad \Longrightarrow \quad x_1 = -3 \quad \text{and} \quad x_2 = 2
\end{aligned}$$

We check: If $x = -3$, then

$$\text{LHS} = \sqrt{2(-3)+5} - \sqrt{(-3)-1} = \sqrt{-1} - \sqrt{-4} = \text{undefined}$$

Since the left hand side is undefined, $x = -3$ is NOT a solution.
If $x = 2$, then

$$\begin{aligned}
\text{LHS} &= \sqrt{2(2)+5} - \sqrt{2-1} = \sqrt{9} - \sqrt{1} = 3-1 = 2 \\
\text{RHS} &= \sqrt{2+2} = \sqrt{4} = 2 \\
\text{RHS} &= \text{LHS}
\end{aligned}$$

Thus $x = 2$ is the only solution.

10. $\sqrt[3]{x^3+26} = x+2$ $-3, 1$

Solution:

$$\begin{aligned}
\left(\sqrt[3]{x^3+26}\right)^3 &= (x+2)^3 \\
x^3+26 &= (x+2)^3 \\
x^3+26 &= x^3+6x^2+12x+8 \\
0 &= x^3+6x^2+12x+8-x^3-26 \\
0 &= 6x^2+12x-18 \\
0 &= 6\left(x^2+2x-3\right) \\
0 &= 6(x+3)(x-1) \quad \Longrightarrow \quad x_1 = -3 \quad \text{and} \quad x_2 = 1
\end{aligned}$$

We check both answers: if $x = -3$, then

$$\begin{aligned}
\text{LHS} &= \sqrt[3]{(-3)^3+26} = \sqrt[3]{-27+26} = \sqrt[3]{-1} = -1 \\
\text{RHS} &= (-3)+2 = -1
\end{aligned}$$

thus -3 does work. If $x=1$, then

$$\begin{aligned} \text{LHS} &= \sqrt[3]{1^3+26} = \sqrt[3]{1+26} = \sqrt[3]{27} = 3 \\ \text{RHS} &= 1+2 = 3 \end{aligned}$$

The solution set is $\{-3, 1\}$.

11. $\sqrt{3x+1} - \sqrt{x-4} = 3$ $5, 8$
Solution:

$$\begin{aligned} \sqrt{3x+1} - \sqrt{x-4} &= 3 && \text{add } \sqrt{x-4} \text{ to both sides} \\ \sqrt{3x+1} &= 3 + \sqrt{x-4} && \text{square both sides} \\ 3x+1 &= \left(3+\sqrt{x-4}\right)^2 \\ 3x+1 &= \left(3+\sqrt{x-4}\right)\left(3+\sqrt{x-4}\right) \\ 3x+1 &= 9 + 3\sqrt{x-4} + 3\sqrt{x-4} + x - 4 \\ 3x+1 &= x + 5 + 6\sqrt{x-4} && \text{subtract } x \\ 2x+1 &= 5 + 6\sqrt{x-4} && \text{subtract } 5 \\ 2x-4 &= 6\sqrt{x-4} \\ 2(x-2) &= 6\sqrt{x-4} && \text{divide by } 2 \\ x-2 &= 3\sqrt{x-4} && \text{square both sides} \\ (x-2)^2 &= 9(x-4) && \text{FOIL, distribute} \\ x^2-4x+4 &= 9x-36 && \text{reduce one side to zero} \\ x^2-13x+40 &= 0 && \text{factor} \\ (x-5)(x-8) &= 0 \implies x_1 = 5 \quad \text{and} \quad x_2 = 8 \end{aligned}$$

We check: if $x=5$, then

$$\text{LHS} = \sqrt{3(5)+1} - \sqrt{5-4} = \sqrt{16} - \sqrt{1} = 4 - 1 = 3 = \text{RHS}$$

If $x=8$, then

$$\text{LHS} = \sqrt{3(8)+1} - \sqrt{8-4} = \sqrt{25} - \sqrt{4} = 5 - 2 = 3 = \text{RHS}$$

The solution set is $\{5, 8\}$.

12. $\sqrt{x+10} + 10 = x$ 15
Solution:

$$\begin{aligned} \sqrt{x+10} + 10 &= x && \text{subtract } 10 \\ \sqrt{x+10} &= x - 10 && \text{square} \\ x+10 &= (x-10)^2 \\ x+10 &= x^2 - 20x + 100 && \text{reduce one side to zero} \\ 0 &= x^2 - 21x + 90 && \text{factor} \\ 0 &= (x-6)(x-15) \implies x_1 = 6 \quad \text{and} \quad x_2 = 15 \end{aligned}$$

We check: if $x=6$, then

$$\begin{aligned} \text{LHS} &= \sqrt{6+10} + 10 = \sqrt{16} + 10 = 4 + 10 = 14 \\ \text{RHS} &= 6 \\ \text{LHS} &\neq \text{RHS} \end{aligned}$$

If $x = 15$, then

$$\text{LHS} = \sqrt{15+10} + 10 = \sqrt{25} + 10 = 5 + 10 = 15 = \text{RHS}$$

since $x = 6$ doesn't work, the only solution is 15. In set notation: the solution set is $\{15\}$.

13. $\sqrt[3]{x^3+208} = x+4$ $-6, 2$

Solution:

$$\begin{aligned}
\sqrt[3]{x^3+208} &= x+4 && \text{raise both sides to the third power}\\
\left(\sqrt[3]{x^3+208}\right)^3 &= (x+4)^3\\
x^3+208 &= x^3+12x^2+48x+64 && \text{subtract } x^3\\
208 &= 12x^2+48x+64 && \text{reduce one side to zero}\\
0 &= 12x^2+48x-144 && \text{factor out 12}\\
0 &= 12\left(x^2+4x-12\right) && \text{factor}\\
0 &= 12(x+6)(x-2) \implies x_1 = -6 \text{ and } x = 2
\end{aligned}$$

Solution: -6 and 2. We check. They both work.

14. $\sqrt{x-1} + \sqrt{x-4} = \sqrt{4x-11}$ 5

Solution:

$$\begin{aligned}
\sqrt{x-1} + \sqrt{x-4} &= \sqrt{4x-11} && \text{square}\\
\left(\sqrt{x-1} + \sqrt{x-4}\right)^2 &= \left(\sqrt{4x-11}\right)^2\\
\left(\sqrt{x-1} + \sqrt{x-4}\right)\left(\sqrt{x-1} + \sqrt{x-4}\right) &= 4x-11 && \text{FOIL}\\
\underbrace{\sqrt{x-1}\sqrt{x-1}}_{\text{F}} + \underbrace{\sqrt{x-1}\sqrt{x-4}}_{\text{O}} + \underbrace{\sqrt{x-4}\sqrt{x-1}}_{\text{I}} + \underbrace{\sqrt{x-4}\sqrt{x-4}}_{\text{L}} &= 4x-11\\
x-1+2\sqrt{x-1}\sqrt{x-4}+x-4 &= 4x-11 && \text{combine like terms}\\
2x-5+2\sqrt{x-1}\sqrt{x-4} &= 4x-11 && \text{subtract } 2x\\
-5+2\sqrt{x-1}\sqrt{x-4} &= 2x-11 && \text{add 5}\\
2\sqrt{x-1}\sqrt{x-4} &= 2x-6\\
2\sqrt{x-1}\sqrt{x-4} &= 2(x-3) && \text{divide by 2}\\
\sqrt{(x-1)(x-4)} &= x-3 && \text{square}\\
(x-1)(x-4) &= (x-3)^2 && \text{FOIL}\\
x^2-5x+4 &= x^2-6x+9 && \text{subtract } x^2\\
-5x+4 &= -6x+9 && \text{add } 6x\\
x+4 &= 9 && \text{subtract 4}\\
x &= 5
\end{aligned}$$

We check:

$$\begin{aligned}
\text{LHS} &= \sqrt{5-1} + \sqrt{5-4} = \sqrt{4} + \sqrt{1} = 2+1 = 3\\
\text{RHS} &= \sqrt{4x-11} = \sqrt{4(5)-11} = \sqrt{9} = 3
\end{aligned}$$

And so the solution set is $\{5\}$.

15. $\sqrt{4x+6} = \sqrt{x+1} - \sqrt{x+5}$ no real solution
Solution:

$$\begin{aligned}
\sqrt{4x+6} &= \sqrt{x+1} - \sqrt{x+5} && \text{square} \\
\left(\sqrt{4x+6}\right)^2 &= \left(\sqrt{x+1} - \sqrt{x+5}\right)^2 \\
4x+6 &= \underbrace{\sqrt{x+1}\sqrt{x+1}}_{\text{F}} - \underbrace{\sqrt{x+1}\sqrt{x+5}}_{\text{O}} - \underbrace{\sqrt{x+5}\sqrt{x+1}}_{\text{I}} + \underbrace{\sqrt{x+5}\sqrt{x+5}}_{\text{L}} \\
4x+6 &= x+1-2\sqrt{(x+1)(x+5)}+x+5 \\
4x+6 &= 2x+6-2\sqrt{(x+1)(x+5)} && \text{subtract } 2x \\
2x+6 &= 6-2\sqrt{(x+1)(x+5)} && \text{subtract } 6 \\
2x &= -2\sqrt{(x+1)(x+5)} && \text{divide by } 2 \\
x &= -\sqrt{(x+1)(x+5)} && \text{square} \\
x^2 &= \left(-\sqrt{(x+1)(x+5)}\right)^2 \\
x^2 &= (x+1)(x+5) && \text{FOIL right hand side} \\
x^2 &= x^2+6x+5 && \text{subtract } x^2 \\
0 &= 6x+5 && \text{subtract } 5 \\
-5 &= 6x && \text{divide by } 6 \\
-\frac{5}{6} &= x
\end{aligned}$$

We check: if $x = -\frac{5}{6}$, then

$$\begin{aligned}
\text{LHS} &= \sqrt{4\left(-\frac{5}{6}\right)+6} = \sqrt{-\frac{10}{3}+6} = \sqrt{-\frac{10}{3}+\frac{18}{3}} = \sqrt{\frac{8}{3}} \\
\text{RHS} &= \sqrt{-\frac{5}{6}+1} - \sqrt{-\frac{5}{6}+5} = \sqrt{\frac{1}{6}} - \sqrt{\left(-\frac{5}{6}\right)+\frac{30}{6}} = \sqrt{\frac{1}{6}} - \sqrt{\frac{25}{6}} = \frac{\sqrt{1}}{\sqrt{6}} - \frac{\sqrt{25}}{\sqrt{6}} \\
&= \frac{1}{\sqrt{6}} - \frac{5}{\sqrt{6}} = \frac{1-5}{\sqrt{6}} = -\frac{4}{\sqrt{6}}
\end{aligned}$$

Since the left hand side is positive, and the right hand side is negative, these two numbers can not be equal. This equation has no real solution.

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