

Sample Problems

Simplify each of the following expressions. Assume that a represents a positive number.

1. $\sqrt{32}$

2. $\sqrt{45}$

3. $\sqrt{48x^5y^3}$

4. $\sqrt{125} - 3\sqrt{80} + \sqrt{45}$

5. $\frac{\sqrt{24}}{\sqrt{54}}$

6. $\sqrt{80a^{11}} - 2\sqrt{180a^{11}} + 3\sqrt{245a^{11}}$

7. $(\sqrt{7} + 2)(\sqrt{7} - 2)$

8. $(\sqrt{7} - 2)^2$

9. $(\sqrt{3} - 1)^3$

10. $(\sqrt{5x} - 2)(\sqrt{5x} + 3)$

11. $(2 - \sqrt{x})(3 + 2\sqrt{x})$

12. $(\sqrt{x} - \sqrt{2})^2$

Practice Problems

Simplify each of the following expressions. Assume that a represents a positive number.

1. $\sqrt{20}$

2. $\sqrt{300} - 2\sqrt{75} + \sqrt{12}$

3. $3\sqrt{20} - 4\sqrt{45} + 2\sqrt{245}$

4. $5\sqrt{18a^5} - 7\sqrt{32a^5} + 4\sqrt{50a^5}$

5. $(\sqrt{5} + 2)(\sqrt{5} - 2)$

6. $(\sqrt{5} - 2)^2$

7. $(\sqrt{5} - 2)^3$

8. $(3\sqrt{5} - 1)(7\sqrt{5} + 2)$

Sample Problems – Answers

1. $4\sqrt{2}$ 2. $3\sqrt{5}$ 3. $4x^2y\sqrt{3xy}$ 4. $-4\sqrt{5}$ 5. $\frac{2}{3}$ 6. $13a^5\sqrt{5a}$ 7. 3
 8. $11 - 4\sqrt{7}$ 9. $-10 + 6\sqrt{3}$ 10. $5x + \sqrt{5x} - 6$ 11. $6 + \sqrt{x} - 2x$ 12. $x - 2\sqrt{2x} + 2$

Practice Problems – Answers

1. $2\sqrt{5}$ 2. $2\sqrt{3}$ 3. $8\sqrt{5}$ 4. $7a^2\sqrt{2a}$ 5. 1 6. $9 - 4\sqrt{5}$ 7. $17\sqrt{5} - 38$ 8. $103 - \sqrt{5}$

Sample Problems – Solutions

Simplify each of the following expressions. Assume that a represents a positive number.

1. $\sqrt{32}$

Solution: We first factor the number under the square root sign into two factors, where the first factor is the largest square we can find in the number. Then this first part can come out from under the square root sign and become a coefficient.

$$\sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{16}\sqrt{2} = \boxed{4\sqrt{2}}$$

2. $\sqrt{45}$

Solution: We first factor the number under the square root sign into two factors, where the first factor is the largest square we can find in the number. Then this first part can come out from under the square root sign and become a coefficient.

$$\sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9}\sqrt{5} = \boxed{3\sqrt{5}}$$

3. $\sqrt{48x^5y^3}$

Solution: We first factor the expressions under the square root sign into two factors, where the first factor is the largest square we can find in the expression. Then this first part can come out from under the square root sign.

$$\sqrt{48x^5y^3} = \sqrt{16x^4y^2 \cdot 3xy} = \sqrt{16x^4y^2}\sqrt{3xy} = \boxed{4x^2y\sqrt{3xy}}$$

4. $\sqrt{125} - 3\sqrt{80} + \sqrt{45}$

Solution: We simplify each expression as in the previous problem. Then we combine like radicals.

$$\begin{aligned} \sqrt{125} - 3\sqrt{80} + \sqrt{45} &= \sqrt{25 \cdot 5} - 3\sqrt{16 \cdot 5} + \sqrt{9 \cdot 5} \\ &= \sqrt{25}\sqrt{5} - 3\sqrt{16}\sqrt{5} + \sqrt{9}\sqrt{5} \\ &= 5\sqrt{5} - 3 \cdot 4 \cdot \sqrt{5} + 3\sqrt{5} \\ &= 5\sqrt{5} - 12\sqrt{5} + 3\sqrt{5} \\ &= (5 - 12 + 3)\sqrt{5} = \boxed{-4\sqrt{5}} \end{aligned}$$

5. $\frac{\sqrt{24}}{\sqrt{54}}$

Solution: We simplify both radical expressions and simplify.

$$\frac{\sqrt{24}}{\sqrt{54}} = \frac{\sqrt{4 \cdot 6}}{\sqrt{9 \cdot 6}} = \frac{\sqrt{4}\sqrt{6}}{\sqrt{9}\sqrt{6}} = \frac{2\sqrt{6}}{3\sqrt{6}} = \boxed{\frac{2}{3}}$$

6. $\sqrt{80a^{11}} - 2\sqrt{180a^{11}} + 3\sqrt{245a^{11}}$

Solution:

$$\begin{aligned} \sqrt{80a^{11}} - 2\sqrt{180a^{11}} + 3\sqrt{245a^{11}} &= \\ \sqrt{16a^{10} \cdot 5a} - 2\sqrt{36a^{10} \cdot 5a} + 3\sqrt{49a^{10} \cdot 5a} &= \\ \sqrt{16a^{10}}\sqrt{5a} - 2\sqrt{36a^{10}}\sqrt{5a} + 3\sqrt{49a^{10}}\sqrt{5a} &= \\ 4a^5\sqrt{5a} - 2(6a^5)\sqrt{5a} + 3(7a^5)\sqrt{5a} &= \\ 4a^5\sqrt{5a} - 12a^5\sqrt{5a} + 21a^5\sqrt{5a} &= \\ (4 - 12 + 21)a^5\sqrt{5a} &= \boxed{13a^5\sqrt{5a}} \end{aligned}$$

Note: We can rearrange the final answer as $13a^5\sqrt{5a} = 13\sqrt{5a^5}\sqrt{a}$. This other form is just as correct and might even be preferable in some cases.

7. $(\sqrt{7} + 2)(\sqrt{7} - 2)$

Solution:

$$(\sqrt{7} + 2)(\sqrt{7} - 2) = \sqrt{7}\sqrt{7} - 2\sqrt{7} + 2\sqrt{7} - 4 = 7 - 4 = \boxed{3}$$

8. $(\sqrt{7} - 2)^2$

Solution:

$$\begin{aligned} (\sqrt{7} - 2)^2 &= (\sqrt{7} - 2)(\sqrt{7} - 2) \\ &= \sqrt{7}\sqrt{7} - 2\sqrt{7} - 2\sqrt{7} + 4 \\ &= 7 - 4\sqrt{7} + 4 = \boxed{11 - 4\sqrt{7}} \end{aligned}$$

9. $(\sqrt{3} - 1)^3$

Solution: We will first work out $(\sqrt{3} - 1)^2$ and then multiply the result by $(\sqrt{3} - 1)$.

$$\begin{aligned} (\sqrt{3} - 1)^3 &= (\sqrt{3} - 1)(\sqrt{3} - 1)(\sqrt{3} - 1) \\ &= (\sqrt{3}\sqrt{3} - 1\sqrt{3} - 1\sqrt{3} + 1)(\sqrt{3} - 1) \\ &= (3 - 2\sqrt{3} + 1)(\sqrt{3} - 1) \\ &= (4 - 2\sqrt{3})(\sqrt{3} - 1) \\ &= 4\sqrt{3} - 4 - 2\sqrt{3}\sqrt{3} + 2\sqrt{3} \\ &= 4\sqrt{3} - 4 - 2 \cdot 3 + 2\sqrt{3} \\ &= 4\sqrt{3} - 4 - 6 + 2\sqrt{3} \\ &= \boxed{-10 + 6\sqrt{3}} \end{aligned}$$

10. $(\sqrt{5x} - 2)(\sqrt{5x} + 3)$

Solution:

$$\begin{aligned} (\sqrt{5x} - 2)(\sqrt{5x} + 3) &= \sqrt{5x}\sqrt{5x} + 3\sqrt{5x} - 2\sqrt{5x} - 2 \cdot 3 = 5x + 3\sqrt{5x} - 2\sqrt{5x} - 6 \\ &= \boxed{5x + \sqrt{5x} - 6} \end{aligned}$$

11. $(2 - \sqrt{x})(3 + 2\sqrt{x})$

Solution:

$$(2 - \sqrt{x})(3 + 2\sqrt{x}) = 6 + 4\sqrt{x} - 3\sqrt{x} - 2\sqrt{x}\sqrt{x} = \boxed{6 + \sqrt{x} - 2x}$$

12. $(\sqrt{x} - \sqrt{2})^2$

Solution:

$$\begin{aligned} (\sqrt{x} - \sqrt{2})^2 &= (\sqrt{x} - \sqrt{2})(\sqrt{x} - \sqrt{2}) && \text{we FOIL} \\ &= \sqrt{x}\sqrt{x} - \sqrt{x}\sqrt{2} - \sqrt{2}\sqrt{x} + \sqrt{2}\sqrt{2} \\ &= x - \sqrt{2x} - \sqrt{2x} + 2 = \boxed{x - 2\sqrt{2x} + 2} \end{aligned}$$