Sample Problems

Perform the indicated operations and simplify.

1. $\dfrac{3x}{x-2}-\dfrac{x+4}{x-2}$
2. $\dfrac{10}{x-y}-\dfrac{5}{y-x}$
3. $\dfrac{1}{x-y}-\dfrac{1}{x+y}$
4. $\dfrac{2}{p-5}-\dfrac{p+11}{p^2-2p-15}$
5. $\dfrac{x^2-5x+78}{18x+x^2-208}-\dfrac{x}{x+26}$
6. $\left[\left(\dfrac{1}{b}-b\right)\div\left(1-\dfrac{1}{b}\right)\right](1+b)$

Practice Problems

Perform the indicated operations and simplify.

1. $\dfrac{x+y}{2}-\dfrac{x-y}{2}$
2. $\dfrac{3a}{a-1}-\dfrac{a+2}{a-1}$
3. $\dfrac{a+1}{a-b}-\dfrac{b-1}{b-a}$
4. $\dfrac{5y+1}{y+5}+\dfrac{y^2-1}{y+5}$
5. $\dfrac{1}{p-5}-\dfrac{1}{p+5}$
6. $\dfrac{x-5}{x+2}-\dfrac{3}{2-x}-\dfrac{14-x}{x^2-4}$
7. $\dfrac{3}{x+1}-\dfrac{2}{x-1}+\dfrac{x+3}{x^2-1}$
8. $\dfrac{2(m+2)}{4m+m^2-12}-\dfrac{1}{m-2}$
9. $\dfrac{1}{x-2}-\dfrac{3}{x+6}$

Sample Problems – Answers

1. 2
2. $\dfrac{15}{x-y}$ or $-\dfrac{15}{y-x}$
3. $\dfrac{2y}{(x-y)(x+y)}$ or $\dfrac{2y}{x^2-y^2}$
4. $\dfrac{1}{p+3}$
5. $\dfrac{3}{x-8}$
6. $-(b+1)^2$

Practice Problems – Answers

1. y
2. 2
3. $\dfrac{a+b}{a-b}$
4. y
5. $\dfrac{10}{p^2-25}$
6. $\dfrac{x-1}{x+2}$
7. $\dfrac{2}{x+1}$
8. $\dfrac{1}{m+6}$
9. $\dfrac{-2x+12}{x^2+4x-12}$

Sample Problems – Solutions

Perform the indicated operations and simplify.

1. a) $\dfrac{3x}{x-2}-\dfrac{x+4}{x-2}=\quad 2$

Solution: This is a subtraction of fractions. The denominators are the same, the only difficulty is that we are subtracting expressions instead of numbers. The second pair of parentheses is essential. $\dfrac{3x}{x-2}-\dfrac{x+4}{x-2}=\dfrac{(3x)-(x+4)}{x-2}=\dfrac{3x-x-4}{x-2}=\dfrac{2x-4}{x-2}=\dfrac{2(x-2)}{x-2}=2$

2. $\dfrac{10}{x-y}-\dfrac{5}{y-x}=\quad\dfrac{15}{x-y}$ or $-\dfrac{15}{y-x}$

Solution: This problem is much easier when we realize that the denominators are opposites of each other, since the opposite of $y-x$ is $-1(y-x)=-y+x=x-y$

$$\frac{10}{x-y}-\frac{5}{y-x}=\frac{10}{x-y}-\frac{(-1)\,5}{(-1)(y-x)}=\frac{10}{x-y}-\frac{-5}{x-y}=\frac{10-(-5)}{x-y}=\frac{15}{x-y}$$

3. $\dfrac{1}{x-y}-\dfrac{1}{x+y}=\quad\dfrac{2y}{(x-y)(x+y)}$ or $\dfrac{2y}{x^2-y^2}$

Solution: We first bring the two fractions to the common denominator, which is $(x+y)(x-y)=x^2-y^2$

$$\begin{aligned}\frac{1}{x-y}-\frac{1}{x+y}&=\frac{1\cdot(x+y)}{(x-y)(x+y)}-\frac{1\cdot(x-y)}{(x+y)(x-y)}\\&=\frac{x+y}{(x-y)(x+y)}-\frac{x-y}{(x-y)(x+y)}\end{aligned}$$

We are now ready to subtract. We need to be extremely careful to subtract the ENTIRE expression, and not just its first term. To do that, we need to insert parentheses around these expressions.

$$\frac{(x+y)-(x-y)}{(x-y)(x+y)}=\frac{x+y-x+y}{(x-y)(x+y)}=\frac{2y}{(x-y)(x+y)}\quad\text{or}\quad\frac{2y}{x^2-y^2}$$

The final answer can be presented as $\dfrac{2y}{(x-y)(x+y)}$ or $\dfrac{2y}{x^2-y^2}$. They are both equally correct.

4. $\dfrac{2}{p-5}-\dfrac{p+11}{p^2-2p-15}=\quad\dfrac{1}{p+3}$

Solution: The denominator of the second fraction factors. $p^2-2p-15=(p+3)(p-5)$. We will now bring these fractions to the common denominator.

$$\frac{2}{p-5}-\frac{p+11}{p^2-2p-15}=\frac{2(p+3)}{(p-5)(p+3)}-\frac{p+11}{(p-5)(p+3)}$$

We can now perform the subtraction of polynomials in the numerator

$$\frac{2(p+3)-(p+11)}{(p-5)(p+3)}=\frac{2p+6-p-11}{(p-5)(p+3)}=\frac{p-5}{(p-5)(p+3)}=\frac{1}{p+3}$$

5. $\dfrac{x^2-5x+78}{18x+x^2-208}-\dfrac{x}{x+26}= \quad \dfrac{3}{x-8}$

Solution: The denominator of the first fraction factors. $x^2+18x-208=(x+26)(x-8)$. We will now bring these fractions to the common denominator.

$$\frac{x^2-5x+78}{x^2+18x-208}-\frac{x}{x+26}=\frac{x^2-5x+78}{(x+26)(x-8)}-\frac{x(x-8)}{(x+26)(x-8)}$$

We can now perform the subtraction of polynomials in the numerator

$$\frac{\left(x^2-5x+78\right)-x(x-8)}{(x+26)(x-8)}=\frac{x^2-5x+78-x^2+8x}{(x+26)(x-8)}=\frac{3x+78}{(x+26)(x-8)}$$

We are not done yet: after we factor out the greatest common factor from the numerator, the result simplifies.

$$\frac{3x+78}{(x+26)(x-8)}=\frac{3(x+26)}{(x+26)(x-8)}=\frac{3}{x-8}$$

6. $\left[\left(\dfrac{1}{b}-b\right)\div\left(1-\dfrac{1}{b}\right)\right](1+b)= \quad -(b+1)^2$

Solution: Let us perform the subtractions first.

$$\frac{1}{b}-b=\frac{1}{b}-\frac{b}{1}=\frac{1}{b}-\frac{b(b)}{1(b)}=\frac{1}{b}-\frac{b^2}{b}=\frac{1-b^2}{b} \quad \text{and}$$

$$1-\frac{1}{b}=\frac{1}{1}-\frac{1}{b}=\frac{1(b)}{1(b)}-\frac{1}{b}=\frac{b}{b}-\frac{1}{b}=\frac{b-1}{b}$$

Thus

$$\left[\left(\frac{1}{b}-b\right)\div\left(1-\frac{1}{b}\right)\right](1+b)=\left[\frac{1-b^2}{b}\div\frac{b-1}{b}\right](1+b)$$

We re-write the division as multiplication by the reciprocal, and then cancel.

$$\left[\frac{1-b^2}{\not b}\cdot\frac{\not b}{b-1}\right](1+b)=\frac{1-b^2}{b-1}(1+b)$$

This expression can be further simplified, since $1-b^2=(1+b)(1-b)$ via the difference of squares theorem, and $1-b$ and $b-1$ are opposites.

$$\frac{1-b^2}{b-1}(1+b)=\frac{(1-b)(1+b)}{b-1}(1+b)=\frac{-1(b-1)(1+b)}{b-1}(1+b)=-(b+1)^2$$

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