Caution! Currently, square roots are defined differently in different textbooks. In conversations about mathematics, approach the concept with caution and first make sure that everyone in the conversation uses the same definition of square roots.

Definition: Let A be a non-negative number. Then the **square root of** A (notation: $\sqrt{A}$) is the non-negative number that, if squared, the result is A. If A is negative, then $\sqrt{A}$ is undefined.

For example, consider $\sqrt{25}$. The square-root of 25 is a number, that, when we square, the result is 25. There are two such numbers, 5 and -5. The square root is defined to be the non-negative such number, and so $\sqrt{25}$ is 5. On the other hand, $\sqrt{-25}$ is undefined, because there is no real number that, when we square, we get a negative number.

Example 1: Evaluate each of the given numerical expressions.

a) $\sqrt{49}$ b) $-\sqrt{49}$ c) $\sqrt{-49}$ d) $-\sqrt{-49}$

Solution: a) $\sqrt{49} = \boxed{7}$

b) $-\sqrt{49} = -1 \cdot \sqrt{49} = -1 \cdot 7 = \boxed{-7}$

c) $\sqrt{-49} = \boxed{\text{undefined}}$

d) $-\sqrt{-49} = -1 \cdot \sqrt{-49} = \boxed{\text{undefined}}$

Square roots, when stretched over entire expressions, also function as grouping symbols.

Example 2: Evaluate each of the following expressions.

a) $\sqrt{25} - \sqrt{16}$ b) $\sqrt{25-16}$ c) $\sqrt{144+25}$ d) $\sqrt{144} + \sqrt{25}$

Solution: a) $\sqrt{25} - \sqrt{16} = 5 - 4 = \boxed{1}$

b) $\sqrt{25-16} = \sqrt{9} = \boxed{3}$

c) $\sqrt{144+25} = \sqrt{169} = \boxed{13}$

d) $\sqrt{144} + \sqrt{25} = 12 + 5 = \boxed{17}$

Definition: Let A be any real number. Then the **third root of** A (notation: $\sqrt[3]{A}$) is the number that, if raised to the third power, the result is A. (We also refer to $\sqrt[3]{A}$ as cube root of A).

Notice that this definition is much simpler than the previous one. If we square 3 and -3, we get 9 in both cases. For this reason, there are two candidates for $\sqrt{9}$ and no candidate for $\sqrt{-9}$. Third roots behave more pleasantly. If we cube (same as raise to the third power) 3 and -3, we get 27 and -27. Thus, cube roots exist of both positive and negative numbers, and there is no ambiguity on the choice of the cube root. Simply $\sqrt[3]{8}$ is 2 and $\sqrt[3]{-8}$ is -2.

Example 3: Evaluate each of the given numerical expressions.

a) $\sqrt[3]{125}$ b) $-\sqrt[3]{8}$ c) $\sqrt[3]{-27}$ d) $-\sqrt[3]{-64}$

Solution: a) $\sqrt[3]{125} = \boxed{5}$

b) $-\sqrt[3]{8} = -1 \cdot \sqrt[3]{8} = -1 \cdot 2 = \boxed{-2}$

c) $\sqrt[3]{-27} = \boxed{-3}$

d) $-\sqrt[3]{-64} = -1 \cdot \sqrt[3]{-64} = -1\,(-4) = \boxed{4}$

Just as with square roots, third roots, can also serve as grouping symbols.

Example 4: Evaluate each of the following expressions

a) $\sqrt[3]{10^2+5^2}$ b) $\sqrt[3]{36-100}$

Solution: a) $\sqrt[3]{10^2+5^2} = \sqrt[3]{125} = \boxed{5}$

b) $\sqrt[3]{36-100} = \sqrt[3]{-64} = \boxed{-4}$

Definition: Let n be any positive integer, greater than 1, and let A be any real number.

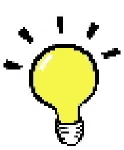
Suppose that n is an even number. If A is negative, then the nth root of A, denoted by $\sqrt[n]{A}$, is undefined. If A is non-negative, then $\sqrt[n]{A}$ is the non-negative number that, when raised to the nth power, the result is A.

Suppose that n is an odd number. Then the nth root of A, denoted by $\sqrt[n]{A}$, is the number that, when raised to the nth power, the result is A.

Practice Problems

Simplify each of the given expressions.

1. $\sqrt[3]{-8}$
2. $\sqrt[5]{32}$
3. $\sqrt[4]{1}$
4. $\sqrt{0}$
5. $\sqrt{-64}$
6. $-\sqrt[3]{-27}$
7. $\sqrt[3]{-64}$
8. $\sqrt[5]{-0}$
9. $\sqrt{16}$
10. $\sqrt[4]{16}$
11. $\sqrt[7]{-1}$
12. $\sqrt[5]{-243}$
13. $\sqrt{-100}$
14. $\sqrt[6]{-64}$
15. $\sqrt{0}$
16. $\sqrt[4]{-1}$
17. $\sqrt[5]{-1}$
18. $\sqrt[99]{-1}$
19. $\sqrt[100]{-1}$
20. $-\sqrt[5]{-32}$
21. $-\sqrt[4]{-16}$
22. $\sqrt[3]{2\cdot 5-37}$
23. $\sqrt[4]{1}-\sqrt[5]{-36+4}$
24. $\sqrt[6]{3^2-2^3}$
25. $\sqrt[7]{3\cdot 5-(-4)^2}$
26. $\sqrt[4]{(-2)^4-4^2}$
27. $\sqrt[3]{\sqrt{3(-4)^2+1}-\sqrt[5]{-1}}$



Answers

1. -2 2. 2 3. 1 4. 0 5. undefined 6. 3 7. -4 8. 0 9. 4 10. 2 11. -1 12. -3 13. undefined 14. undefined 15. 0 16. undefined 17. -1 18. -1 19. undefined 20. 2 21. undefined 22. -3 23. 3 24. 1 25. -1 26. 0 27. 2

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