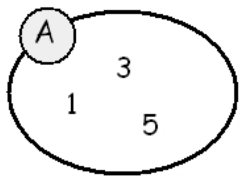
Definition: A **set** is a collection of objects.

Sets are usually denoted by uppercase letters. There are numerous ways a set could be given. For example, let A be the set of odd numbers between 0 and 6. In set theory, we write this set as $A = \{1, 3, 5\}$. When writing a set, the order of listing and repetition does not change a set. For example, if sets B and C are defined as $B = \{5, 1, 3\}$ and $C = \{5, 1, 5, 1, 1, 1, 3, 1\}$, then $A = B = C$.

We can also describe a set using a diagram.



Some famous sets have their own set theory label. For example, the infinite set $\{1, 2, 3, ...\}$ is the set of all natural numbers or counting numbers, is denoted by $\mathbb{N}$.

Definition: A set is a collection of objects. The objects that make up the set are called the **elements** or **members** of the set. If x is an element of a set S, we denote this by $x \in S$.

We also say that x belongs to the set S. In the case of $A = \{1, 3, 5\}$, the following are all true statements.

$$3 \in A \quad \text{read: 3 belongs to } A$$
$$5 \in A \quad \text{read: 5 belongs to } A$$
$$4 \notin A \quad \text{read: 4 does not belong to } A$$

Example 1. Recall that A is still $A = \{1, 3, 5\}$ and that $\mathbb{N} = \{1, 2, 3, ...\}$. Determine whether the given statements are true or false.

a) $1 \in A$ b) $2 \in A$ c) $-1 \in \mathbb{N}$ d) $5 \notin \mathbb{N}$ e) $4 \notin A$

Solution: a) The statement $1 \in A$ reads: 1 *belongs to set* A. This is true as 1 is an element of set A.

b) The statement $2 \in A$ reads: 2 *belongs to set* A. This is not true as 2 is not an element of set A.

c) The statement $-1 \in \mathbb{N}$ reads: -1 *belongs to the set of all natural numbers.* (In short, -1 is a natural number). This statement is false.

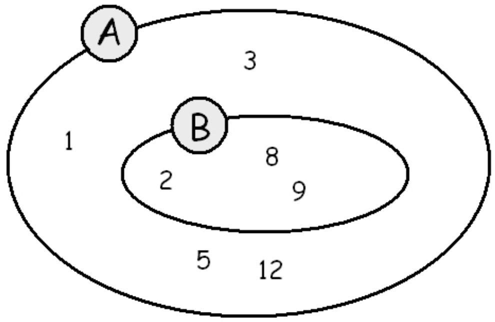
d) The statement $5 \notin \mathbb{N}$ reads: 5 *does not belong to the set of all natural numbers.* (In short, 5 is not a natural number). This statement is false.

e) The statement $4 \notin A$ reads: 4 *does not belong to set* A. This statement is true.

Definition: There is a unique set that contains no elements. It is called the **empty set** and is denoted by $\varnothing$ or by $\{\quad\}$.

Definition: Set B is a **subset** of set A, denoted by $B \subseteq A$, if all elements of B also belong to A.

Example 2. Suppose that $A = \{1, 2, 3, 5, 8, 9, 12\}$ and $B = \{2, 8, 9\}$. Then B is a subset of A.



Example 3. Suppose that $X = \{a, b, d, f\}$ and $Y = \{a, b, c, d, e, f, g\}$. Then $X \subseteq Y$.

Example 4. Suppose that $E = \{2, 4, 6, 8, 10, ...\}$ and recall that $\mathbb{N} = \{1, 2, 3, ...\}$. Then $E \subseteq \mathbb{N}$.

Example 5. Suppose that L is the set of all letters in the English alphabet, and V is the set of vowels in the English alphabet. Then $V \subseteq L$.

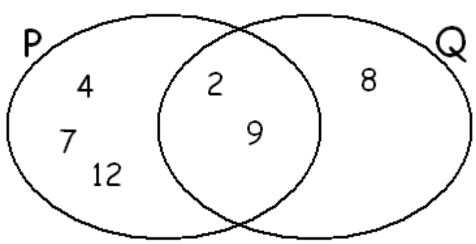
Special sets have their own symbols. The set of all natural numbers is $\mathbb{N} = \{1, 2, 3, ...\}$ and the set of all integers is $\mathbb{Z} = \{0, 1, -1, 2, -2, 3, -3, ...\}$. Then $\mathbb{N}$ is a subset of $\mathbb{Z}$. The same statement can be expressed using notation as $\mathbb{N} \subseteq \mathbb{Z}$.

Example 6. Let M be the set of all mammals and D the set of all dogs. Then D is a subset of M, or, in short, $D \subseteq M$.

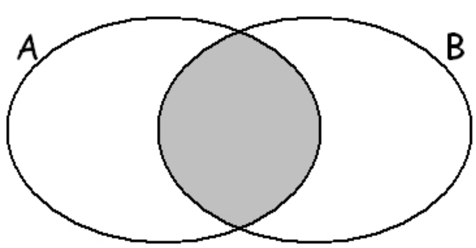
Definition: If A and B are sets, then $A \cap B$ (the intersection of A and B) is the set such that for all x, $x \in A \cap B$ if and only if $x \in A$ and $x \in B$.

The intersection of two sets is the set of all elements that belong to both sets.

Example 7. Suppose that $P = \{2, 4, 7, 9, 12\}$ and $Q = \{2, 8, 9\}$. Then $P \cap Q = \{2, 9\}$.



A picture such as this is called a Venn diagram. Venn diagrams provide very useful visual tools to solve set theory problems. We can depict the intersection using Venn Diagrams.



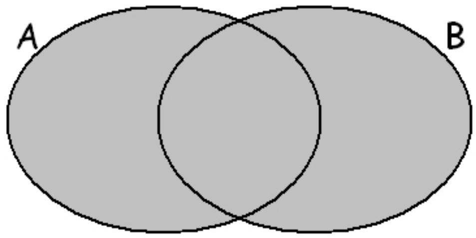
The shaded region is $A \cap B$

Example 8. Suppose that $T = \{3, 4, 7, 10\}$ and $Q = \{1, 6, 8\}$. Then $T \cap Q = \varnothing$.

Definition: If A and B are sets, then $A \cup B$ (the union of A and B) is the set such that for all x, $x \in A \cup B$ if and only if $x \in A$ or $x \in B$.

The union of two sets is the set of all elements from one set, put together with the set of all elements of the other.

Example 9. Suppose that $P = \{2, 4, 7, 9, 12\}$ and $Q = \{2, 8, 9\}$. Then $P \cup Q = \{2, 4, 7, 8, 9, 12\}$.



The shaded region is $A \cup B$

Practice Problems

1. Suppose that S is a set defined as $S = \{-2, 4, 5, 16\}$ and recall that $\mathbb{N} = \{1, 2, 3, ...\}$. Determine whether the given statements are true or false.

   a) $-2 \in S$ b) $-2 \in \mathbb{N}$ c) $-3 \notin \mathbb{N}$ d) $5 \notin S$ e) $1 \in \mathbb{N}$

2. Let $A = \{1, 2, 5, 8, 9\}$ and $B = \{2, 4, 6, 8\}$.

   a) Draw a Venn diagram depicting these sets.

   b) Find each of the following.

      i) $A \cap B$ ii) $A \cup B$ iii) $B \cup (A \cap B)$

   c) Label each of the following statements as true or false.

      i) $A \subseteq A \cap B$ ii) $B \subseteq A \cup B$ iii) $A \cap B \subseteq A \cup B$

3. Let P denote the set of all students taking physics at Truman College. Let M denote the set of all students taking mathematics at Truman College.

   a) describe the set $P \cap M$ b) describe the set $P \cup M$

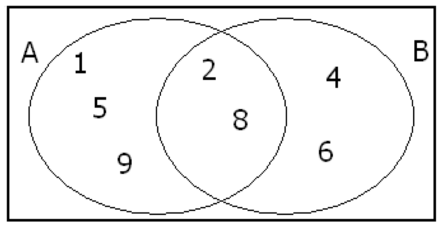
4. Label each of the following as true or false.

   a) $\mathbb{N} \cap \mathbb{Z} = \mathbb{N}$ b) $\mathbb{N} \cap \mathbb{Z} = \mathbb{Z}$ c) $\mathbb{N} \cup \mathbb{Z} = \mathbb{N}$ d) $\mathbb{N} \cup \mathbb{Z} = \mathbb{Z}$

Answers

1. a) true b) false c) true d) false e) true

2. a)



b) i) $\{2, 8\}$ ii) $\{1, 2, 4, 5, 6, 8, 9\}$ iii) $\{2, 4, 6, 8\}$

c) i) false ii) true iii) true

3. a) $P \cap M$ - the set of all students taking mathematics and physics at Truman College.
b) $P \cup M$ - the set of all students taking mathematics or physics or both at Truman College.

4. a) true b) false c) false d) true

