

Sample Problems

1. Find the exact value of $\sin \alpha$ if we know that α is in the fourth quadrant and $\cos \alpha = \frac{1}{3}$.
2. Suppose that x is in the third quadrant and $\sin x = -\frac{2}{5}$. Find the exact value of each of the following.
 - a) $\cos x$
 - b) $\sec x$
 - c) $\tan x$
3. Find the exact value of $\cos \alpha$ where α is an acute angle with $\tan \alpha = \frac{1}{2}$.
4. If $\cot A = 2$, then what is the exact value of $\sin A$?
5. Simplify $\frac{\tan t + \sin(-t) \cos(-t)}{\tan t}$.

Practice Problems

1. Find the exact value of $\cos \alpha$ if we know that α is NOT in the fourth quadrant and $\sin \alpha = -\frac{3}{4}$.
2. Find the exact value of $\tan B$ if we know that $90^\circ < B < 180^\circ$ and $\cos B = -\frac{1}{5}$.
3. Suppose that x is in the second quadrant and $\cot x = -3$. Find the exact value of $\sin x$.
4. Suppose that x is in NOT the second quadrant and $\cot x = -3$. Find the exact value of $\sin x$.

Sample Problems - Answers

$$1.) -\frac{\sqrt{8}}{3} \quad 2.) \text{ a) } -\frac{\sqrt{21}}{5} \quad \text{b) } -\frac{5}{\sqrt{21}} \quad \text{c) } \frac{2}{\sqrt{21}} \quad 3.) \frac{2}{\sqrt{5}} \quad 4.) \pm\frac{1}{\sqrt{5}} \quad 5.) \sin^2 t$$

Practice Problems - Answers

$$1.) -\frac{\sqrt{7}}{4} \quad 2.) -2\sqrt{6} \quad 3.) \frac{1}{\sqrt{10}} \quad 4.) -\frac{1}{\sqrt{10}}$$

Sample Problems - Solutions

1. Find the exact value of $\sin \alpha$ if we know that α is in the fourth quadrant and $\cos \alpha = \frac{1}{3}$.

Solution: We can solve for $\sin \alpha$ in the equation $\sin^2 \alpha + \cos^2 \alpha = 1$.

$$\begin{aligned} \sin^2 \alpha + \cos^2 \alpha &= 1 \\ \sin^2 \alpha &= 1 - \cos^2 \alpha \\ \sin \alpha &= \pm\sqrt{1 - \cos^2 \alpha} = \pm\sqrt{1 - \left(\frac{1}{3}\right)^2} = \pm\sqrt{1 - \frac{1}{9}} = \pm\sqrt{\frac{8}{9}} = \pm\frac{\sqrt{8}}{3} \end{aligned}$$

Since α is in the fourth quadrant, $\sin \alpha$ is negative. Thus $\sin \alpha = -\frac{\sqrt{8}}{3}$.

2. Suppose that x is in the third quadrant and $\sin x = -\frac{2}{5}$. Find the exact value of each of the following.

a) $\cos x$

Solution: We can solve for $\cos x$ in the equation $\sin^2 x + \cos^2 x = 1$.

$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ \cos^2 x &= 1 - \sin^2 x \\ \cos x &= \pm\sqrt{1 - \sin^2 x} = \pm\sqrt{1 - \left(-\frac{2}{5}\right)^2} = \pm\sqrt{1 - \frac{4}{25}} = \pm\sqrt{\frac{21}{25}} = \pm\frac{\sqrt{21}}{5} \end{aligned}$$

Since x is in the third quadrant, $\cos x$ is negative. Thus $\cos x = -\frac{\sqrt{21}}{5}$.

b) $\sec x$

$$\text{Solution: } \sec x = \frac{1}{\cos x} = \frac{1}{\frac{-\sqrt{21}}{5}} = -\frac{5}{\sqrt{21}}$$

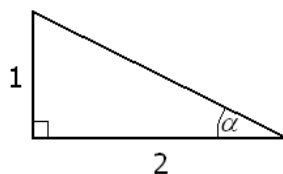
c) $\tan x$

$$\text{Solution: } \tan x = \frac{\sin x}{\cos x} = \frac{-\frac{2}{5}}{\frac{-\sqrt{21}}{5}} = -\frac{2}{5} \left(-\frac{5}{\sqrt{21}} \right) = \frac{2}{\sqrt{21}}$$

3. Find the exact value of $\cos \alpha$ where α is an acute angle with $\tan \alpha = \frac{1}{2}$.

Solution: We will see two different methods of solving for this problem.

Method 1. Since α is an acute angle, all trigonometric functions are positive. Now that we established the signs, let us draw a right triangle where $\tan \alpha = \frac{1}{2}$.



We can easily compute the hypotenuse of this triangle, using the Pythagorean theorem. The hypotenuse is $\sqrt{5}$ long. Then we can read the value of $\cos \alpha$ to be $\frac{2}{\sqrt{5}}$.

Method 2. Let us start with the Pythagorean identity.

$$\begin{aligned} \sin^2 \alpha + \cos^2 \alpha &= 1 && \text{divide both sides by } \cos^2 \alpha \\ \frac{\sin^2 \alpha}{\cos^2 \alpha} + \frac{\cos^2 \alpha}{\cos^2 \alpha} &= \frac{1}{\cos^2 \alpha} \\ \tan^2 \alpha + 1 &= \frac{1}{\cos^2 \alpha} && \text{take the reciprocal of both sides} \\ \frac{1}{\tan^2 \alpha + 1} &= \cos^2 \alpha && \text{square-root property} \\ \pm \sqrt{\frac{1}{\tan^2 \alpha + 1}} &= \cos \alpha \end{aligned}$$

Since α is an acute angle, $\cos \alpha$ is the positive solution:

$$\cos \alpha = \sqrt{\frac{1}{\tan^2 \alpha + 1}} = \sqrt{\frac{1}{\left(\frac{1}{2}\right)^2 + 1}} = \sqrt{\frac{1}{\frac{5}{4}}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

4. If $\cot A = 2$, then what is the exact value of $\sin A$? $\pm \frac{1}{\sqrt{5}}$

Solution:

$$\begin{aligned} \sin^2 A + \cos^2 A &= 1 && \text{divide by } \sin^2 A \\ \frac{\sin^2 A}{\sin^2 A} + \frac{\cos^2 A}{\sin^2 A} &= \frac{1}{\sin^2 A} \\ 1 + \cot^2 A &= \frac{1}{\sin^2 A} && \cot A = 2 \\ 1 + 2^2 &= \frac{1}{\sin^2 A} \\ 5 &= \frac{1}{\sin^2 A} \\ \sin^2 A &= \frac{1}{5} \\ \sin A &= \pm \frac{1}{\sqrt{5}} \end{aligned}$$

5. Simplify $\frac{\tan t + \sin(-t) \cos(-t)}{\tan t}$

Solution: Since $\sin(-t) = -\sin t$ and $\cos(-t) = \cos t$, we can simplify the expression as

$$\begin{aligned} \frac{\tan t + \sin(-t) \cos(-t)}{\tan t} &= \frac{\tan t - \sin t \cos t}{\tan t} = \frac{\frac{\sin t}{\cos t} - \sin t \cos t}{\frac{\sin t}{\cos t}} = \frac{\cos t}{\cos t} \cdot \frac{\frac{\sin t}{\cos t} - \sin t \cos t}{\frac{\sin t}{\cos t}} \\ &= \frac{\cos t \left(\frac{\sin t}{\cos t} - \sin t \cos t \right)}{\cos t \left(\frac{\sin t}{\cos t} \right)} = \frac{\sin t - \sin t \cos^2 t}{\sin t} = \frac{\sin t (1 - \cos^2 t)}{\sin t} \\ &= \frac{\sin t \sin^2 t}{\sin t} = \frac{\sin^3 t}{\sin t} = \sin^2 t \end{aligned}$$

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