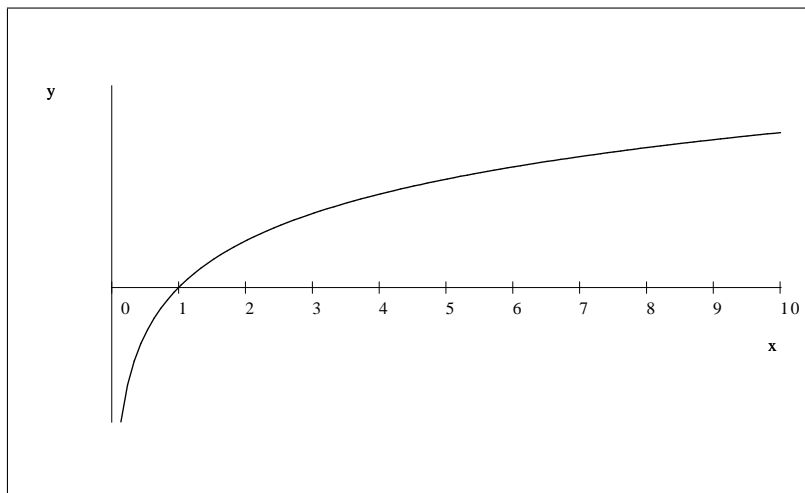
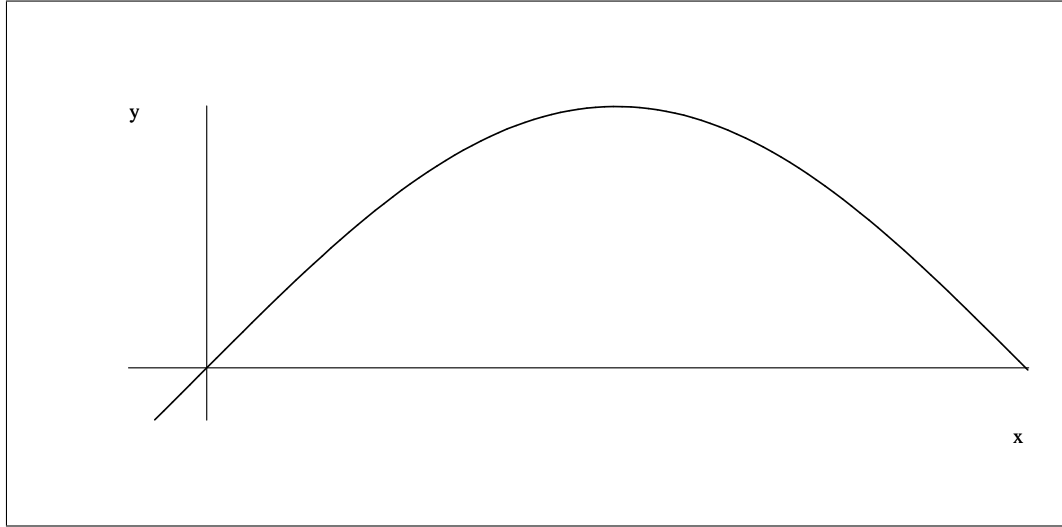


1. Consider the function $f(x) = \ln x$ on the interval $[1, 6]$.



- a) Approximate $\int_1^6 \ln x dx$ by a left Riemann sum using a uniform partition with $n = 5$. Is this an underestimation or overestimation of the area under the graph?
- b) Approximate $\int_1^6 \ln x dx$ by a right Riemann sum using a uniform partition with $n = 5$. Is this an underestimation or overestimation of the area under the graph?
- c) Approximate $\int_1^6 \ln x dx$ by a left Riemann sum using a uniform partition with $n = 10$. Is this an underestimation or overestimation of the area under the graph?
- d) Approximate $\int_1^6 \ln x dx$ by a right Riemann sum using a uniform partition with $n = 10$. Is this an underestimation or overestimation of the area under the graph?

2. Consider the function $f(x) = \sin x$ on the interval $[0, \pi]$.



- a) Compute the left Riemann sum for $\int_0^{\pi} \sin x dx$ on this interval with $n = 4$ subintervals. Make sure to use radians for angles.
- b) Compute the left Riemann sum for $\int_0^{\pi} \sin x dx$ on this interval with $n = 4$ subintervals. Make sure to use radians for angles.
- c) Compute the left Riemann sum for $\int_0^{\pi} \sin x dx$ on this interval with $n = 6$ subintervals. Make sure to use radians for angles.

Answers

1. a) $\ln 2 + \ln 3 + \ln 4 + \ln 5 = \ln 120 \approx 4.787492$ underestimate
b) $\ln 2 + \ln 3 + \ln 4 + \ln 5 + \ln 6 = \ln 720 \approx 6.57925$ overestimate
c) $\frac{1}{2} \left(\ln \frac{2}{2} + \ln \frac{3}{2} + \ln \frac{4}{2} + \ln \frac{5}{2} + \ln \frac{6}{2} + \ln \frac{7}{2} + \ln \frac{8}{2} + \ln \frac{9}{2} + \ln \frac{10}{2} + \ln \frac{11}{2} \right) = \frac{1}{2} \ln \left(\frac{11!}{2^{10}} \right) \approx 5.285418$
underestimate
d) $\frac{1}{2} \left(\ln \frac{3}{2} + \ln \frac{4}{2} + \ln \frac{5}{2} + \ln \frac{6}{2} + \ln \frac{7}{2} + \ln \frac{8}{2} + \ln \frac{9}{2} + \ln \frac{10}{2} + \ln \frac{11}{2} + \ln \frac{12}{2} \right) = \frac{1}{2} \ln \left(\frac{12!}{2^{11}} \right) = 6.1812978$
overestimate
2. a) $\frac{1}{4}\pi + \frac{1}{4}\pi\sqrt{2} \approx 1.89611889793704$ b) $\frac{1}{4}\pi + \frac{1}{4}\pi\sqrt{2} \approx 1.89611889793704$
c) $\frac{1}{3}\pi + \frac{1}{6}\pi\sqrt{3} \approx 1.95409723331371$