Math 98 **Sample Midterm Exam** Solutions

1. Solve: $3y - 9 = -2y + 4$

   (a) $y = -5$

   (b) $y = \dfrac{5}{13}$

   (c) $y = \dfrac{13}{5}$

   (d) $y = 13$

   Solution:

$$\begin{aligned} 3y - 9 &= -2y + 4 && \text{add } 2y \text{ to both sides} \\ 5y - 9 &= 4 && \text{add } 9 \text{ to both sides} \\ 5y &= 13 && \text{divide both sides by } 5 \\ y &= \frac{13}{5} \end{aligned}$$

   We check. If $x = \dfrac{13}{5}$, then

$$\begin{aligned} \text{LHS} &= 3\left(\frac{13}{5}\right) - 9 = \frac{3}{1} \cdot \frac{13}{5} - 9 = \frac{39}{5} - \frac{9}{1} = \frac{39}{5} - \frac{45}{5} = \frac{-6}{5} = -\frac{6}{5} \\ \text{RHS} &= -2\left(\frac{13}{5}\right) + 4 = \frac{-2}{1} \cdot \frac{13}{5} + \frac{4}{1} = \frac{-26}{5} + \frac{20}{5} = \frac{-6}{5} = -\frac{6}{5} \end{aligned}$$

   Thus $x = \dfrac{13}{5}$ is the correct solution, which is choice C.

2. Evaluate: $|4 - 7|$

   (a) $-11$

   (b) $-3$

   (c) 3

   (d) 11

   Solution: $|4 - 7| = |-3| = 3$, which is choice C.

3. If $a = -1$ and $b = -2$, find the value of $2a^3b^2$.

   (a) $-8$

   (b) $-3$

   (c) 2

   (d) 8

   Solution: We first write the expression as it is, only we replace the letters by parentheses.

$$2(\ )^3(\ )^2$$

   Then we copy the values into the expression and work out the problem by applying order of operations.

$$2(-1)^3(-2)^2 = 2(-1)4 = -2 \cdot 4 = -8$$

   which is choice A.

4. Simplify the expression $3(x-2)-2(5x-2)$

   (a) $-7x-2$
   (b) $-7x-10$
   (c) $-7x-4$
   (d) $-3x-2$

   Solution: We first apply the distibutive law and then combine like terms

$$\begin{aligned} 3(x-2)-2(5x-2) &= \text{distribute} \\ 3x-6-10x+4 &= \text{combine like terms} \\ &= -7x-2 \end{aligned}$$

   which is choice A

5. Solve: $4-x=3(x-7)$

   (a) $x=-\dfrac{17}{2}$
   (b) $x=\dfrac{25}{4}$
   (c) $x=\dfrac{17}{4}$
   (d) $x=-\dfrac{25}{2}$

   Solution: We first apply the law of distributivity to simplify the right-hand side.

$$\begin{aligned} 4-x &= 3(x-7) && \text{distribute } 3 \\ 4-x &= 3x-21 && \text{add } x \text{ to both sides} \\ 4 &= 4x-21 && \text{add } 21 \text{ to both sides} \\ 25 &= 4x && \text{divide both sides by } 4 \\ \frac{25}{4} &= x \end{aligned}$$

   We check. If $x=\dfrac{25}{4}$, then

$$\begin{aligned} \text{LHS} &= 4-x=4-\frac{25}{4}=\frac{4}{1}-\frac{25}{4}=\frac{16}{4}-\frac{25}{4}=\frac{16-25}{4}=\frac{-9}{4}=-\frac{9}{4} \\ \text{RHS} &= 3(x-7)=3\left(\frac{25}{4}-7\right)=3\left(\frac{25}{4}-\frac{7}{1}\right)=3\left(\frac{25}{4}-\frac{28}{4}\right)=3\left(\frac{25-28}{4}\right) \\ &= 3\left(\frac{-3}{4}\right)=\frac{3}{1}\cdot\frac{-3}{4}=\frac{-9}{4}=-\frac{9}{4} \end{aligned}$$

   Thus our solution, $x=\dfrac{25}{4}$ is correct. The correct answer is choice B

6. Evaluate: $\dfrac{(-4)(-3)(5)}{-1+5}$

   (a) $-7$
   (b) 11
   (c) $-12$
   (d) 15

   Solution: We apply the order of operations agreement.

$$\frac{(-4)(-3)(5)}{-1+5} = \frac{(12)(5)}{-1+5} = \frac{60}{-1+5} = \frac{60}{4} = 15$$

   which is choice D.

7. Solve: $\dfrac{a-10}{5} = -3$

   (a) $a = -5$
   (b) $a = -1$
   (c) $a = 1$
   (d) $a = 5$

   Solution:

$$\begin{aligned} \frac{a-10}{5} &= -3 \qquad \text{multiply both sides by } 5 \\ a-10 &= -15 \quad \text{add } 10 \text{ to both sides} \\ a &= -5 \end{aligned}$$

   We check: if $a = -5$, then

$$\begin{aligned} \text{LHS} &= \frac{-5-10}{5} = \frac{-15}{5} = -3 \\ \text{RHS} &= -3 \end{aligned}$$

   Thus our solution is correct. The answer is choice A.

8. What is the $y$-intercept of the line with equation $3x + 2y = 30$?

   (a) $(10, 15)$
   (b) $(15, 10)$
   (c) $(0, 15)$
   (d) $(10, 0)$

   Solution: The $y-$intercept of a graph is the point where it intercects the $y-$axis. We can find this point by substituting $x = 0$ into the equation of the line and solve for $y$.

$$\begin{aligned} \text{If } x &= 0, \ y = ? \\ 3(0) + 2y &= 30 \\ 0 + 2y &= 30 \\ 2y &= 30 \qquad \text{divide both sides by } 2 \\ y &= 15 \quad \Longrightarrow \quad \text{we found the point } (0, 15) \end{aligned}$$

   Thus the $y-$intercept is $(0, 15)$, which is choice C.

9. A certain triangle's longest side is one centimeter less than six times the shortest side. The other side is five times the shortest side. The perimeter is thirty-five centimeters. Find the length of the longest side.

   (a) 3 centimeters
   (b) 11 centimeters
   (c) 17 centimeters
   (d) 35 centimeters

   Solution: Let $x$ denote the shortest side. Then the longest side is $6x-1$, and the other side is $5x$. We obtain the equation by expressing the perimeter of the triangle. Then we solve for $x$.

$$\begin{array}{rcll} \underbrace{x}_{\text{shortest side}} + \underbrace{6x-1}_{\text{longest side}} + \underbrace{5x}_{\text{other side}} & = & 35 & \text{combine like terms} \\ 12x-1 & = & 35 & \text{add } 1 \text{ to both sides} \\ 12x & = & 36 & \text{divide both sides by } 12 \\ x & = & 3 & \end{array}$$

   Now we know that $x=3$. Since the longest side was denoted by $6x-1$, it must be $6(3)-1 = 18-1 = 17$ cm long. Thus the answer is choice C.

10. Solve: $2x+3<4x+9$

    (a) $x>1$
    (b) $x<-3$
    (c) $x>-3$
    (d) $x>-1$

    Solution:

$$\begin{array}{rcll} 2x+3 & < & 4x+9 & \text{subtract } 2x \text{ from both sides} \\ 3 & < & 2x+9 & \text{subtract } 9 \text{ from both sides} \\ -6 & < & 2x & \text{divide both sides by } 2 \\ -3 & < & x & \end{array}$$

    The solution is choice C.

11. Simplify: $\frac{3}{4}\cdot 6 - 5\cdot\frac{5}{2}$

    (a) $-8$
    (b) $\frac{15}{8}$
    (c) $4$
    (d) $\frac{7}{2}$

Solution: We apply order of operations.

$$\begin{aligned} \frac{3}{4}\cdot 6 - 5\cdot\frac{5}{2} &= \\ \frac{9}{2} - 5\cdot\frac{5}{2} &= \\ \frac{9}{2} - \frac{25}{2} &= \\ \frac{9-25}{2} &= \\ \frac{-16}{2} &= -8 \end{aligned}$$

$$\frac{3}{4}\cdot 6 = \frac{3}{4}\cdot\frac{6}{1} = \frac{18}{4} = \frac{9}{2}$$

$$5\cdot\frac{5}{2} = \frac{5}{1}\cdot\frac{5}{2} = \frac{25}{2}$$

which is choice A

12. Solve: $\frac{3t}{4} - 10 = -4$

(a) $t = 12$
(b) $t = 8$
(c) $t = -2$
(d) $t = 2$

Solution:

$$\begin{aligned} \frac{3t}{4} - 10 &= -4 && \text{add } 10 \text{ to both sides} \\ \frac{3t}{4} &= 6 && \text{multiply both sides by } 4 \\ 3t &= 24 && \text{divide both sides by } 3 \\ t &= 8 \end{aligned}$$

We check: if $t = 8$, then

$$\begin{aligned} \text{RHS} &= \frac{3t}{4} - 10 = \frac{3(8)}{4} - 10 = \frac{24}{4} - 10 = 6 - 10 = -4 \\ \text{LHS} &= -4 \end{aligned}$$

which is choice B.

13. Solve: $3w - 5 < 5(w - 2)$

(a) $w > \frac{5}{2}$
(b) $w < -\frac{3}{2}$
(c) $w < \frac{3}{2}$
(d) $w > \frac{15}{8}$

Solution: we first apply the law of distributivity to simplify the right-hand side.

$$\begin{array}{rcll} 3w-5 & < & 5\,(w-2) & \\ 3w-5 & < & 5w-10 & \text{subtract } 3w \text{ from both sides} \\ -5 & < & 2w-10 & \text{add } 10 \text{ to both sides} \\ 5 & < & 2w & \text{divide both sides by 2} \\ \frac{5}{2} & < & w & \end{array}$$

which is choice A.

14. Give the $x$-intercept of the line given by $5x - y = -10$.

(a) $(5, -1)$
(b) $(2, 0)$
(c) $(-2, 10)$
(d) $(-2, 0)$

Solution: We substitute $y = 0$ into the equation of the line and solve for $x$.

$$\begin{array}{rcll} \text{If } y & = & 0, \ x = ? & \\ 5x - 0 & = & -10 & \\ 5x & = & -10 & \text{divide both sides by 5} \\ x & = & -2 & \Longrightarrow \quad \text{we found the point } \ (-2, 0) \end{array}$$

Thus the $y-$intercept is $(-2, 0)$, which is choice D.

15. A rectangle has a width which is seven inches less than its length. The perimeter is 46 inches. Find the *area*.

(a) 690 square inches
(b) 529 square inches
(c) 120 square inches
(d) 30 square inches

Solution: Let us call the shorter side (the width) by $x$. Then the longer side, the length must be $x + 7$. We obtain the equation by expressing the perimeter.

$$\begin{array}{rcll} 2x + 2\,(x+7) & = & 46 & \text{distribute} \\ 2x + 2x + 14 & = & 46 & \text{combine like terms} \\ 4x + 14 & = & 46 & \text{subtract 14 from both sides} \\ 4x & = & 32 & \text{divide both sides by 4} \\ x & = & 8 & \end{array}$$

We denoted the width by $x$, thus the width is 8 in long. The length is then $x+7 = 8+7 = 15$ in long. The area of a rectangle can be computed by multiplying the two different sides:

$$A = 8 \text{ in} \cdot 15 \text{ in} = 120\,\text{in}^2$$

which is choice C.

## Other non-multiple choice questions for review:

16. Evaluate the following.

i) $\frac{5}{6} - \frac{5}{4}$

Solution: The common denominator is 12

$$\frac{5}{6} - \frac{5}{4} = \frac{10}{12} - \frac{15}{12} = \frac{10-15}{12} = \frac{-5}{12} = -\frac{5}{12}$$

ii) $\frac{2}{3} \cdot \frac{5}{8}$

Solution: We just multiply the numerator by the numerator, the denominator by the denominator, and then simplify.

$$\frac{2}{3} \cdot \frac{5}{8} = \frac{2 \cdot 5}{3 \cdot 8} = \frac{10}{24} = \frac{5}{12}$$

iii) $\frac{4^2}{5} - 3^2$

Solution:

$$\frac{4^2}{5} - 3^2 = \frac{16}{5} - 9 = \frac{16}{5} - \frac{9}{1} = \frac{16}{5} - \frac{45}{5} = \frac{16-45}{5} = \frac{-29}{5} = -\frac{29}{5}$$

iv) $\frac{5-3}{(-2)(-4)(-6)}$

Solution: We apply order of operations.

$$\frac{5-3}{(-2)(-4)(-6)} = \frac{2}{(-2)(-4)(-6)} = \frac{2}{8(-6)} = \frac{2}{-48} = -\frac{1}{24}$$

v) $9 - |-3|$

Solution: $9 - |-3| = 9 - 3 = 6$

vi) $x - y^2 z$, where $x = -6$, $y = -3$, and $z = 2$

Solution: we first copy the expression, writing little parentheses instead if the letters. Then we copy the values within the parentheses and work out the order of operations problem.

$$\begin{aligned} x - y^2 z &= (\ ) - (\ )^2 (\ ) \\ &= (-6) - (-3)^2 (2) && \text{exponent first} \\ &= -6 - 9 \cdot 2 && \text{multiplication} \\ &= -6 - 18 && \text{subtraction} \\ &= -24 \end{aligned}$$

17. Solve:

i) $2x - 5 = 17$

Solution:

$$\begin{aligned} 2x - 5 &= 17 \quad \text{add 5 to both sides} \\ 2x &= 22 \quad \text{divide by 2} \\ x &= 11 \end{aligned}$$

We check: if $x = 11$, then

$$\begin{aligned} \text{RHS} &= 2(11) - 5 = 22 - 5 = 17 \\ \text{LHS} &= 17 \end{aligned}$$

Thus our solution, $x = 11$ is correct.

ii) $7(j-5) + 8 = 2(j+5) + 5j$

Solution:

$$\begin{aligned} 7(j-5) + 8 &= 2(j+5) + 5j \quad \text{distribute on both sides} \\ 7j - 35 + 8 &= 2j + 10 + 5j \quad \text{combine like terms} \\ 7j - 27 &= 7j + 10 \quad \text{subtract } 7j \\ -27 &= 10 \end{aligned}$$

There is no value of $j$ that could make the statement $-27 = 10$ true. Thus there is no solution of this equation.

iii) $\dfrac{t-5}{12} = 4$

Solution:

$$\begin{aligned} \frac{t-5}{12} &= 4 \quad \text{multiply both sides by 12} \\ t - 5 &= 48 \quad \text{add 5 to both sides} \\ t &= 53 \end{aligned}$$

We check: if $t = 33$, then

$$\text{RHS} = \frac{53-5}{12} = \frac{48}{12} = 4 = \text{LHS}$$

Thus our solution, $t = 53$ is correct.

18. Graph the following lines:

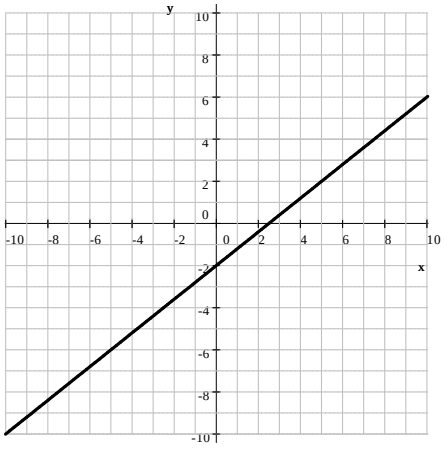
(a) $4x - 5y = 10$

Solution: we find points by picking a value for $x$, and then substitute that value into the equation of the line and solve for $y$. Finally, when we have a few points, we graph them and connect the points.

$$\begin{aligned} \text{If } x &= 0, \ y = ? \\ 4(0) - 5y &= 10 \\ 0 - 5y &= 10 \\ -5y &= 10 \quad \text{divide both sides by } -5 \\ y &= -2 \quad \Longrightarrow \quad \text{we found the point } (0, -2) \end{aligned}$$

$$\begin{aligned} \text{If } x &= 5, \ y = ? \\ 4(5) - 5y &= 10 \\ 20 - 5y &= 10 \quad \text{subtract } 20 \text{ from both sides} \\ -5y &= -10 \quad \text{divide both sides by } -5 \\ y &= 2 \implies \text{we found the point } (5, 2) \end{aligned}$$

$$\begin{aligned} \text{If } x &= -5, \ y = ? \\ 4(-5) - 5y &= 10 \\ -20 - 5y &= 10 \quad \text{add } 20 \text{ to both sides} \\ -5y &= 30 \quad \text{divide both sides by } -5 \\ y &= -6 \implies \text{we found the point } (-5, -6) \end{aligned}$$



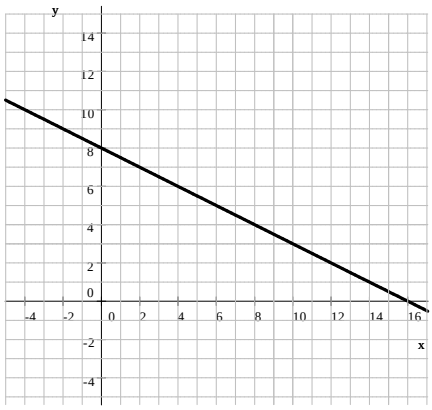

(b) $y = 8 - \dfrac{x}{2}$

Solution: we find points by picking a value for $x$, and then substitute that value into the equation of the line and solve for $y$. Finally, when we have a few points, we graph them and connect the points.

$$\begin{aligned} \text{If } x &= 0, \ y = ? \\ y &= 8 - \frac{0}{2} \\ y &= 8 - 0 \\ y &= 8 \implies \text{we found the point } (0, 8) \end{aligned}$$

$$\begin{aligned} \text{If } x &= 4, \ y = ? \\ y &= 8 - \frac{4}{2} \\ y &= 8 - 2 \\ y &= 6 \implies \text{we found the point } (4, 6) \end{aligned}$$

$$\begin{aligned} \text{If } x &= 6, \ y = ? \\ y &= 8 - \frac{6}{2} \\ y &= 8 - 3 \\ y &= 5 \implies \text{we found the point } (6, 5) \end{aligned}$$



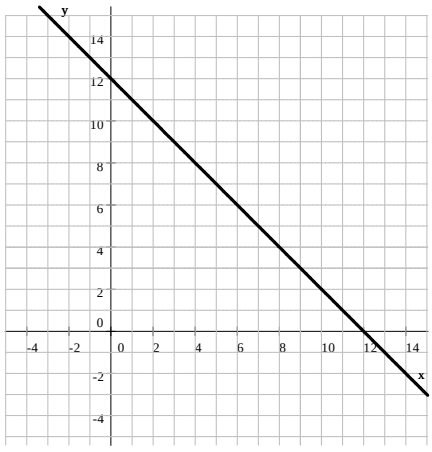

(c) $x + y = 12$

Solution: we find points by picking a value for $x$, and then substitute that value into the equation of the line and solve for $y$. Finally, when we have a few points, we graph them and connect the points.

$$\begin{aligned} \text{If } \ x &= 0, \ \ y = ? \\ 0 + y &= 12 \\ y &= 12 \quad \Longrightarrow \quad \text{we found the point} \quad (0, 12) \end{aligned}$$

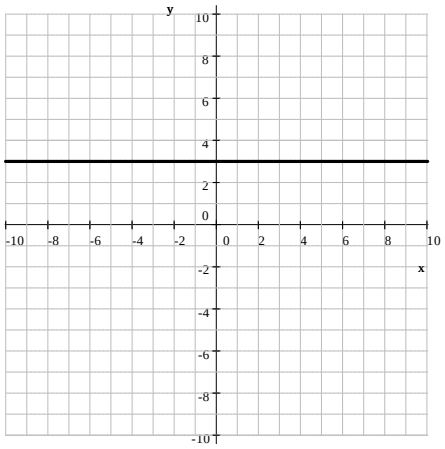
$$\begin{aligned} \text{If } \ x &= 4, \ \ y = ? \\ 4 + y &= 12 \qquad\quad \text{subtract } \ 4 \ \text{ from both sides} \\ y &= 8 \quad \Longrightarrow \quad \text{we found the point} \quad (4, 8) \end{aligned}$$

$$\begin{aligned} \text{If } \ x &= -3, \ \ y = ? \\ -3 + y &= 12 \qquad\quad \text{add } \ 3 \ \text{ to both sides} \\ y &= 15 \quad \Longrightarrow \quad \text{we found the point} \quad (-3, 15) \end{aligned}$$



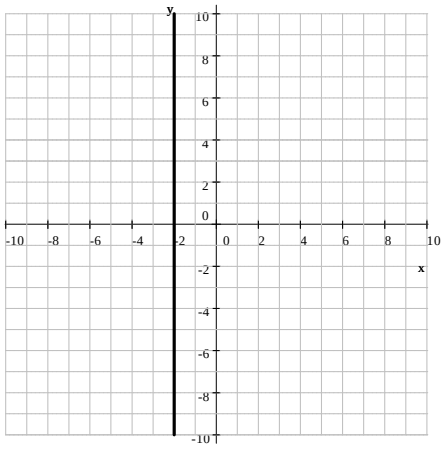

(d) $y = 3$

Solution. No matter what $x$ is, the $y-$value is always 3. We plot points like $(-2, 3)$, $(-5, 3)$, $(0, 3)$ and $(4, 3)$. The line is horizontal.




(e) $x = -2$

Solution. No matter what $y$ is, the $x-$value is always $-2$. We plot points like $(-2,-3)$, $(-2,3)$, $(-2,0)$ and $(-2,5)$. The line is vertical.




19. Mary bought four less than three times the number of books that Jose did. Together they bought sixteen books. How many did Jose buy?

Solution: Let us denote the number of books bought by Jose by $x$. Then Mary baught $3x-4$ many books. We obtain the equation by expressing the total number of books. Then we solve for $x$.

$$\begin{aligned} x+3x-4 &= 16 && \text{combine like terms} \\ 4x-4 &= 16 && \text{add 4 to both sides} \\ 4x &= 20 && \text{divide both sides by 4} \\ x &= 5 \end{aligned}$$

Thus Jose bought 5 books.