**To receive full credit, show all steps.**

1. Simplify each of the following.

(a) $(5a-1)^2 =$
Solution: to square something means to write it down twice and multiply. Then it is a FOIL problem. F stand for first with first, O for outer terms, I for inner terms, and L for last terms.

$$\begin{aligned}(5a-1)^2 &= (5a-1)(5a-1)\\ &= \underbrace{25a^2}_{\text{F}}\ \underbrace{-5a}_{\text{O}}\ \underbrace{-5a}_{\text{I}}\ \underbrace{+25}_{\text{L}} \quad \text{combine like terms}\\ &= 25a^2-10a+25\end{aligned}$$

(b) $\left(3x^5+4y\right)\left(3x^5-4y\right) =$

$$\begin{aligned}\left(3x^5+4y\right)\left(3x^5-4y\right) &= \underbrace{9x^{10}}_{\text{F}}\ \underbrace{-12x^5}_{\text{O}}\ \underbrace{+12x^5}_{\text{I}}\ \underbrace{-16y^2}_{\text{L}} \quad \text{combine like terms}\\ &= 9x^{10}-16y^2\end{aligned}$$

The expressions $3x^5+4y$ and $3x^5-4y$ are called conjugates. Becuase of the same terms and alternating signs, O and I cacel out when "FOIL"-ing, giving us the difference of two squares.

(c) $\dfrac{3a-8}{8-3a} =$
Solution: We need to notice that the numerator and denominator are opposites of each other. Indeed,

$$-1(8-3a) = -8+3a = 3a-8$$

Thus

$$\frac{3a-8}{8-3a} = \frac{-1(8-3a)}{8-3a} = -1$$

(d) $\dfrac{2x+1}{4x^2-1} =$
Solution: We factor the denominator via the difference of squares theorem, and then cancel.

$$\frac{2x+1}{4x^2-1} = \frac{2x+1}{(2x+1)(2x-1)} = \frac{1}{2x-1}$$

(e) $\dfrac{ab-a-b+1}{b^2-1} =$
Solution: We will facotr both numerator and denominator and then cancel. The numerator can be factored by grouping

$$\begin{aligned}\underbrace{ab-a}\ \underbrace{-b+1} &= a(b-1)-1(b-1)\\ &= (a-1)(b-1)\end{aligned}$$

The denominaotr factors by the difference of squares theorem.

$$b^2-1 = (b+1)(b-1)$$

Thus the fraction can be simplified as

$$\frac{ab-a-b+1}{b^2-1} = \frac{(a-1)(b-1)}{(b+1)(b-1)} = \frac{a-1}{b+1}$$

(f) $\dfrac{5x-30}{x^2-36} \cdot \dfrac{3x+18}{5} =$

Solution: we will factor whatever we can and then cancel.

$$\frac{5x-30}{x^2-36} \cdot \frac{3x+18}{5} = \frac{5(x-6)}{(x+6)(x-6)} \cdot \frac{3(x+6)}{5} = 3$$

(g) $\dfrac{3x}{x-2} - \dfrac{x+4}{x-2} =$

Solution: This is a subtraction of fractions. The denominators are the same, the only difficulty is that we are subtracting expressions instead of numbers. The second pair of parentheses is essential.

$$\frac{3x}{x-2} - \frac{x+4}{x-2} = \frac{(3x)-(x+4)}{x-2} = \frac{3x-x-4}{x-2} = \frac{2x-4}{x-2} = \frac{2(x-2)}{x-2} = 2$$

(h) $2^0 + (-2)^0 =$

Solution: $2^0$ and $(-2)^0$ are both equal to zero. Thus $2^0 + (-2)^0 = 2$.

(i) $2^3 \cdot \left(2^{-2}\right)^{-2} =$

Solution:

$$\begin{aligned} 2^3 \cdot \left(2^{-2}\right)^{-2} &= && \text{use rule } (a^n)^m = a^{nm} \\ 2^3 \cdot 2^4 &= && \text{use rule } a^n \cdot a^m = a^{n+m} \\ &= 2^7 = 128 \end{aligned}$$

(j) $\left(2^3 \cdot 2^{-2}\right)^{-2} =$

Solution:

$$\begin{aligned} \left(2^3 \cdot 2^{-2}\right)^{-2} &= && \text{use rule } a^n \cdot a^m = a^{n+m} \\ \left(2^1\right)^{-2} &= && \text{use rule } (a^n)^m = a^{nm} \\ 2^{-2} &= && \text{use rule } a^{-n} = \frac{1}{a^n} \\ \frac{1}{2^2} &= \frac{1}{4} \end{aligned}$$

(k) $2a^3\left(-2ab^{-2}\right)^{-2} ab^0 =$

Solution:

$$\begin{aligned} 2a^3\left(-2ab^{-2}\right)^{-2} ab^0 &= && \text{use rule } (ab)^n = a^n b^n \\ 2a^3(-2)^{-2} a^{-2}\left(b^{-2}\right)^{-2} ab^0 &= && \text{use rule } (a^n)^m = a^{nm} \\ 2a^3(-2)^{-2} a^{-2} b^4 ab^0 &= && \text{use: } b^0 = 1 \text{ and multiplication is commutative} \\ 2(-2)^{-2} a^3 a^{-2} ab^4 &= && \text{use rule } a^n \cdot a^m = a^{n+m} \\ 2(-2)^{-2} a^2 b^4 &= && \text{use rule } a^{-n} = \frac{1}{a^n} \\ 2\left(\frac{1}{2^2}\right) a^2 b^4 &= \frac{2}{1} \cdot \frac{1}{4} \cdot \frac{a^2 b^4}{1} = \frac{a^2 b^4}{2} \end{aligned}$$

(l) $\dfrac{(-2x)^2 y^{-3}}{2x^{-3}y^2} =$

Solution:

$$\frac{(-2x)^2 y^{-3}}{2x^{-3}y^2} = \qquad \text{use rule } (ab)^n = a^n b^n$$

$$\frac{(-2)^2 x^2 y^{-3}}{2x^{-3}y^2} = \frac{4x^2y^{-3}}{2x^{-3}y^2} = \frac{2x^2y^{-3}}{x^{-3}y^2} = \qquad \text{use rule } a^{-n} = \frac{1}{a^n}$$

$$\frac{2x^2 \cdot \frac{1}{y^3}}{\frac{1}{x^3} \cdot y^2} = \qquad \text{make everything a fraction}$$

$$\frac{\frac{2x^2}{1} \cdot \frac{1}{y^3}}{\frac{1}{x^3} \cdot \frac{y^2}{1}} = \frac{\frac{2x^2}{y^3}}{\frac{y^2}{x^3}} \qquad \text{to divide is to multiply by the reciprocal}$$

$$\frac{2x^2}{y^3} \cdot \frac{x^3}{y^2} = \qquad \text{use rule } a^n \cdot a^m = a^{n+m}$$

$$= \frac{2x^5}{y^5}$$

(m) $(x-y)\left(x^5+x^4y+x^3y^2+x^2y^3+xy^4+y^5\right) =$

Solution: We apply the law of distributivity

$$(x-y)\left(x^5+x^4y+x^3y^2+x^2y^3+xy^4+y^5\right) =$$

$$\begin{aligned}
&= x\cdot x^5 + x\cdot x^4y + x\cdot x^3y^2 + x\cdot x^2y^3 + x\cdot xy^4 + x\cdot y^5 \\
&\qquad - y\cdot x^5 - y\cdot x^4y - y\cdot x^3y^2 - y\cdot x^2y^3 - y\cdot xy^4 - y\cdot y^5 \\
&= x^6 + x^5y + x^4y^2 + x^3y^3 + x^2y^4 + xy^5 \\
&\qquad - x^5y - x^4y^2 - x^3y^3 - x^2y^4 - xy^5 - y^6 \\
&= x^6 - y^6
\end{aligned}$$

2. Factor completely each of the following:

(a) $4a^2mn - 15abm^2 - 6abmn + 10a^2m^2 =$

Solution:

$$\begin{aligned}
4a^2mn - 15abm^2 - 6abmn + 10a^2m^2 &= \qquad \text{the GCF is } am \\
am(4an - 15bm - 6bn + 10am) &= \qquad \text{rearrange} \\
am\left(\underbrace{4an - 6bn}\ \underbrace{+10am - 15bm}\right) &= \\
am\left(2n(2a-3b) + 5m(2a-3b)\right) &= am(2n+5m)(2a-3b)
\end{aligned}$$

(b) $a^2x^3 - b^2x - a^2x + b^2x^3 =$
Solution:

$$\begin{aligned}
a^2x^3 - b^2x - a^2x + b^2x^3 &= \qquad \text{the GCF is } x \\
x\left(a^2x^2 - b^2 - a^2 + b^2x^2\right) &= \qquad \text{rearrange} \\
x\left(\underbrace{a^2x^2 - a^2}\quad\underbrace{+b^2x^2 - b^2}\right) &= \\
x\left(a^2\left(x^2-1\right) + b^2\left(x^2-1\right)\right) &= x\left(a^2+b^2\right)\left(x^2-1\right)
\end{aligned}$$

We are not done yet since $\left(x^2-1\right) = \left(x^2-1^2\right)$ further factors via the difference of squares theorem. Thus the answer is

$$\begin{aligned}
x\left(a^2+b^2\right)\left(x^2-1\right) &= x\left(a^2+b^2\right)\left(x^2-1^2\right) \\
&= x\left(a^2+b^2\right)(x+1)(x-1)
\end{aligned}$$

(c) $162a + 162b - 2ax^4 - 2bx^4 =$
Solution:

$$\begin{aligned}
162a + 162b - 2ax^4 - 2bx^4 &= \qquad \text{the GCF is } 2 \\
2\left(\underbrace{81a + 81b}\quad\underbrace{-ax^4 - bx^4}\right) &= \\
2\left(81(a+b) - x^4(a+b)\right) &= 2\left(81 - x^4\right)(a+b)
\end{aligned}$$

We are not done yet, since $81 - x^4 = 9^2 - \left(x^2\right)^2$ further factors via the difference of squares theorem.

$$\begin{aligned}
2\left(81-x^4\right)(a+b) &= 2\left(9^2 - \left(x^2\right)^2\right)(a+b) \\
&= 2\left(9+x^2\right)\left(9-x^2\right)(a+b)
\end{aligned}$$

One factor still further factors: $9 - x^2 = 3^2 - x^2 = (3+x)(3-x)$. Thus the final answer is

$$\begin{aligned}
&= 2\left(9+x^2\right)\left(9-x^2\right)(a+b) \\
&= 2\left(9+x^2\right)\left(3^2-x^2\right)(a+b) \\
&= 2\left(9+x^2\right)(3+x)(3-x)(a+b)
\end{aligned}$$

3. Factor by grouping.

(a) $x^2 - 6x + 8 =$
Solution: we will factor by grouping. First we conduct the "$pq-$game".

$$\begin{aligned}
pq &= 8 \qquad \text{1st coefficient times 3rd coefficient} \\
p+q &= -6 \qquad \text{2nd coefficient}
\end{aligned}$$

We start by expressing 8 as a product of two numbers. there are only two pairs, 1 with 8 and 2 with 4. Since the product $pq$ is positive, $p$ and $q$ have to have the same sign. Since the sum $p+q$ is negative, they both have to be negative. We only need to consider $-1$ with $-8$ and

$-2$ with $-4$. Clearly $-2$ with $-4$ work as $p$ and $q$. We use these these numbers to express the second term as the sum of two terms, and then factor by grouping.

$$\begin{aligned} x^2-6x+8 &= \underbrace{x^2-2x} \quad \underbrace{-4x+8} \\ &= x(x-2)-4(x-2) = (x-2)(x-4) \end{aligned}$$

We check by multiplication:

$$(x-2)(x-4) = x^2-4x-2x+8 = x^2-6x+8$$

Thus our result is correct.

(b) $3a^2-5a-2=$
Solution: we will factor by grouping. First we conduct the "$pq-$game".

$$\begin{aligned} pq &= -6 \qquad \text{1st coefficient times 3rd coefficient} \\ p+q &= -5 \qquad \text{2nd coefficient} \end{aligned}$$

We start by expressing 6 as a product of two numbers. there are only two pairs, 1 with 6 and 2 with 3. Since the product $pq$ is negative, one number must be positive, the other one must be positive.. Since the sum $p+q$ is negative, the negative sign has to be in front of the larger number. We only need to consider 1 with $-6$ and 2 with $-3$. Clearly 1 with $-6$ work as $p$ and $q$. We use these these numbers to express the second term as the sum of two terms, and then factor by grouping.

$$\begin{aligned} 3a^2-5a-2 &= \underbrace{3a^2+a} \quad \underbrace{-6a-2} \\ &= a(3a+1)-2(3a+1) = (a-2)(3a+1) \end{aligned}$$

We check by multiplication:

$$(a-2)(3a+1) = 3a^2+a-6a-2 = 3a^2-5a-2$$

Thus our result is correct.

(c) $4b^2-b-5=$ $(4b-5)(b+1)$
Solution: we will factor by grouping. First we conduct the "$pq-$game".

$$\begin{aligned} pq &= -20 \qquad \text{1st coefficient times 3rd coefficient} \\ p+q &= -1 \qquad \text{2nd coefficient} \end{aligned}$$

We start by expressing 20 as a product of two numbers. the possible pairs are, 1 with 20, 2 with 10, and 4 with 5. Since the product $pq$ is negative, one number must be positive, the other one must be positive.. Because the sum $p+q$ is negative, the negative sign has to be in front of the larger number. We only need to consider 1 with $-20$, 2 with $-10$, and 4 with $-5$. Clearly 4 with $-5$ work as $p$ and $q$. We use these these numbers to express the second term as the sum of two terms, and then factor by grouping.

$$\begin{aligned} 4b^2-b-5 &= \underbrace{4b^2+4b} \quad \underbrace{-5b-5} \\ &= 4b(b+1)-5(b+1) = (4b-5)(b+1) \end{aligned}$$

We check by multiplication:

$$(4b-5)(b+1) = 4b^2+4b-5b-5 = 4b^2-b-5$$

Thus our result is correct.

4. Solve each of the following equations. Make sure to check your solutions.

(a) $\dfrac{2x+1}{5}-\dfrac{5-x}{2}=x-1$

Solution:

$$\begin{aligned}
\frac{2x+1}{5}-\frac{5-x}{2} &= x-1 && \text{re-write everything as a fraction}\\
\frac{2x+1}{5}-\frac{5-x}{2} &= \frac{x-1}{1} && \text{common denominator is } 10\\
\frac{2(2x+1)}{10}-\frac{5(5-x)}{10} &= \frac{10(x-1)}{10} && \text{multiply both sides by 10}\\
2(2x+1)-5(5-x) &= 10(x-1) && \text{distribute}\\
4x+2-25+5x &= 10x-10 && \text{combine like terms}\\
9x-23 &= 10x-10 && \text{subtract } 9x\\
-23 &= x-10 && \text{add } 10\\
-13 &= x
\end{aligned}$$

We check: if $x=13$, then

$$\begin{aligned}
\text{LHS} &= \frac{2(-13)+1}{5}-\frac{5-(-13)}{2}=\frac{-26+1}{5}-\frac{5+13}{2}=\frac{-25}{5}-\frac{18}{2}=-5-9=-14\\
\text{RHS} &= -13-1=-14 \quad \checkmark
\end{aligned}$$

Thus our solution, $-13$ is correct.

(b) $-3(2x-1)-(3-7x)=2(x+1)-(x-1)$

Solution: In case of subtracting algebraic expressions, we either subtract the opposite, or distribute $-1$. We will use the second method here.

$$\begin{aligned}
-3(2x-1)-(3-7x) &= 2(x+1)-(x-1)\\
-3(2x-1)-1(3-7x) &= 2(x+1)-1(x-1) && \text{distribute}\\
-6x+3-3+7x &= 2x+2-x+1 && \text{combine like terms}\\
x &= x+3 && \text{subtract } x\\
0 &= 3
\end{aligned}$$

Since $x$ disappeared from the equation, we are left with an unconditional statement, this time unconditionally false. There is no value of $x$ that can make $0=3$ be true, and thus this equation has no solution. An equation of this type is called a contradiction.

(c) $8x^3=50x^2$

Solution: since this equation is of a higher degree than 1, our only method is to reduce one side to zero, factor, and then apply the special zero property.

$$\begin{aligned}
8x^3 &= 50x^2 && \text{subtract } 50x^2\\
8x^3-50x^2 &= 0 && \text{the GCF is } 2x^2\\
2x^2(4x-25) &= 0
\end{aligned}$$

We now apply the special zero property. If this product is zero, then either $2x^2 = 0$ or $4x - 25 = 0$. We solve these equations for $x$.

$$\begin{aligned} 2x^2 &= 0 & \text{or} \quad & 4x - 25 = 0 \\ 2 \cdot x \cdot x &= 0 & \text{or} \quad & 4x = 25 \\ x &= 0 & \text{or} \quad & x = \frac{25}{4} \end{aligned}$$

We check both solutions. If $x = 0$, then

$$\begin{aligned} \text{LHS} &= 8 \cdot 0^3 = 8 \cdot 0 = 0 \\ \text{RHS} &= 50 \cdot 0^2 = 50 \cdot 0 = 0 \quad \checkmark \end{aligned}$$

If $x = \frac{25}{4}$, then

$$\begin{aligned} \text{LHS} &= 8\left(\frac{25}{4}\right)^3 = \frac{8}{1} \cdot \frac{15\,625}{64} = \frac{15\,625}{8} \\ \text{RHS} &= 50\left(\frac{25}{4}\right)^2 = \frac{50}{1} \cdot \frac{625}{16} = \frac{15\,625}{8} \quad \checkmark \end{aligned}$$

Thus both solutions, $0$ and $\frac{25}{4}$ are correct.

(d) $8p^3 = 50p$

Solution: since this equation is of a higher degree than 1, our only method is to reduce one side to zero, factor, and then apply the special zero property.

$$\begin{aligned} 8p^3 &= 50p & & \text{subtract } 50p \\ 8p^3 - 50p &= 0 & & \text{the GCF is } 2p \\ 2p\left(4p^2 - 25\right) &= 0 & & \\ 2p\left((2p)^2 - 5^2\right) &= 0 & & \text{factor via difference of squares theorem} \\ 2p\,(2p+5)\,(2p-5) &= 0 & & \end{aligned}$$

We now apply the special zero property. If this product is zero, then either $2p = 0$ or $2p+5 = 0$ or $2p - 5 = 0$. We solve these equations for $p$.

$$\begin{aligned} 2p + 5 &= 0 & \text{or} \quad & 2p - 5 = 0 & \text{or} \quad & 2p = 0 \\ 2p &= -5 & \text{or} \quad & 2p = 5 & \text{or} \quad & p = 0 \\ p &= -\frac{5}{2} & \text{or} \quad & p = \frac{5}{2} & & \end{aligned}$$

We check all three solutions. If $p = -\frac{5}{2}$, then

$$\begin{aligned} \text{LHS} &= 8\left(-\frac{5}{2}\right)^3 = \frac{8}{1} \cdot \frac{-125}{8} = -125 \\ \text{RHS} &= 50\left(-\frac{5}{2}\right) = \frac{50}{1} \cdot \frac{-5}{2} = \frac{-250}{2} = -125 \quad \checkmark \end{aligned}$$

If $p = \frac{5}{2}$, then

$$\begin{aligned} \text{LHS} &= 8\left(\frac{5}{2}\right)^3 = \frac{8}{1}\cdot\frac{125}{8} = 125 \\ \text{RHS} &= 50\left(\frac{5}{2}\right) = \frac{50}{1}\cdot\frac{5}{2} = \frac{250}{2} = 125 \quad \checkmark \end{aligned}$$

and if $p = 0$, then

$$\begin{aligned} \text{LHS} &= 8\cdot 0^3 = 8\cdot 0 = 0 \\ \text{RHS} &= 50\cdot 0 = 0 \quad \checkmark \end{aligned}$$

Thus all three solutions, $-\frac{5}{2}$, $0$, and $\frac{5}{2}$ are correct.

(e) $2-(3-x)(2x+5) = (x-1)(2x-1)$

Solution: We have to use the FOIL method on both sides to perform the multiplications. It is very important, however, to keep the expressions in a parentheses since we are dealing with subtraction between *algebraic expressions*.

$$\begin{aligned} 2-(3-x)(2x+5) &= (x-1)(2x-1) && \text{FOIL} \\ 2-\left(6x+15-2x^2-5x\right) &= 2x^2-x-2x+1 && \text{combine like terms} \\ 2-\left(-2x^2+x+15\right) &= 2x^2-3x+1 && \text{perform subtraction} \\ 2+2x^2-x-15 &= 2x^2-3x+1 && \text{combine like terms} \\ 2x^2-x-13 &= 2x^2-3x+1 && \text{subtract } 2x^2 \quad \text{(the equation is linear!)} \\ -x-13 &= -3x+1 && \text{add } 3x \\ 2x-13 &= 1 && \text{add } 13 \\ 2x &= 14 && \text{divide by } 2 \\ x &= 7 \end{aligned}$$

We check: if $x = 7$, then

$$\begin{aligned} \text{LHS} &= 2-(3-7)(2(7)+5) = 2-(-4)(14+5) = 2-(-4)19 = 2-(-76) = 78 \\ \text{RHS} &= (7-1)(2(7)-1) = 6(14-1) = 6\cdot 13 = 78 \quad \checkmark \end{aligned}$$

Thus our solution, $7$ is correct.

(f) $8a+2a^2 = 42$

Solution: since this equation is of a higher degree than 1, our only method is to reduce one side to zero, factor, and then apply the special zero property.

$$\begin{aligned} 8a+2a^2 &= 42 && \text{subtract 42, rearrange} \\ 2a^2+8a-42 &= 0 && \text{the GCF is } 2 \\ 2\left(a^2+4a-21\right) &= 0 \end{aligned}$$

We will factor $a^2+4a-21$ by grouping. First we conduct the "$pq$−game".

$$\begin{aligned} pq &= -21 && \text{1st coefficient times 3rd coefficient} \\ p+q &= 4 && \text{2nd coefficient} \end{aligned}$$

We start by expressing 21 as a product of two numbers. the only possible pairs are, 1 with 21 and 3 with 7. Since the product $pq$ is negative, one number must be positive, the other one must be positive.. Because the sum $p+q$ is positive, the negative sign has to be in front of the smaller number. We only need to consider $-1$ with 20, and $-3$ with 7. Clearly $-3$ with 7 work as $p$ and $q$. We use these these numbers to express the second term as the sum of two terms, and then factor by grouping.

$$\begin{aligned}
2\left(a^2+4a-21\right) &= 0\\
2\left(\underbrace{a^2+7a}\ \ \underbrace{-3a-21}\right) &= 0\\
2\left(a\left(a+7\right)-3\left(a+7\right)\right) &= 0\\
2\left(a-3\right)\left(a+7\right) &= 0
\end{aligned}$$

Thus our equation is

$$2\left(a-3\right)\left(a+7\right)=0$$

We now apply the special zero property. If this product is zero, then either $2=0$ or $a-3=0$ or $a+7=0$. We solve these equations for $a$.

$$\begin{array}{rclcccc}
a-3 &=& 0 & \text{or} & a+7=0 & \text{or} & 2=0\\
a &=& 3 & \text{or} & a=-7 & \text{or} & \text{no solution here}
\end{array}$$

We check both solutions. If $a=3$, then

$$\text{LHS}=8\left(3\right)+2\left(3\right)^2=8\cdot 3+2\cdot 9=24+18=42=\text{RHS}\quad\checkmark$$

If $a=-7$, then

$$\text{LHS}=8\left(-7\right)+2\left(-7\right)^2=8\cdot\left(-7\right)+2\cdot 49=-56+98=42=\text{RHS}\quad\checkmark$$

Thus both solutions, $-7$ and $3$ are correct.

5. Solve each of the following inequalities. Graph the solution set.

(a) $\dfrac{3-4x}{3}-\dfrac{2x-3}{7}\leq -x+7$

Solution:

$$\begin{array}{rcll}
\dfrac{3-4x}{3}-\dfrac{2x-3}{7} &\leq& -x+7 & \text{re-write everything as a fraction}\\
\dfrac{3-4x}{3}-\dfrac{2x-3}{7} &\leq& \dfrac{-x+7}{1} & \text{common denominator is } 21\\
\dfrac{7\left(3-4x\right)}{21}-\dfrac{3\left(2x-3\right)}{21} &\leq& \dfrac{21\left(-x+7\right)}{21} & \text{multiply both sides by } 21\\
7\left(3-4x\right)-3\left(2x-3\right) &\leq& 21\left(-x+7\right) & \text{distribute}\\
21-28x-6x+9 &\leq& -21x+147 & \text{combine like terms}\\
-34x+30 &\leq& -21x+147 & \text{add } 34x\\
30 &\leq& 13x+147 & \text{subtract } 147\\
-117 &\leq& 13x & \text{divide by } 13\\
-9 &\leq& x &
\end{array}$$

(b) $\dfrac{3-2a}{7} > -1$
Solution:

$$\begin{aligned} \frac{3-2a}{7} &> -1 && \text{multiply both sides by 7} \\ 3-2a &> -7 && \text{add } 2a \\ 3 &> 2a-7 && \text{add 7} \\ 10 &> 2a && \text{divide by 2} \\ 5 &> a \end{aligned}$$

(c) $3(2x-3)-(5x+4) > -14$
Solution:

$$\begin{aligned} 3(2x-3)-(5x+4) &> -14 && \text{distribute} \\ 6x-9-5x-4 &> -14 && \text{combine like terms} \\ x-13 &> -14 && \text{add 13} \\ x &> -1 \end{aligned}$$

6. Solve each of the following formulas.

(a) $A = 2a - 3b$ for $a$.
Solution:

$$\begin{aligned} A &= 2a-3b && \text{add } 3b \\ A+3b &= 2a && \text{divide by 2} \\ \frac{A+3b}{2} &= a \end{aligned}$$

Thus $a = \dfrac{A+3b}{2}$ or $\dfrac{A}{2}+\dfrac{3b}{2}$ or $\dfrac{1}{2}A+\dfrac{3}{2}b$

(b) $A = 2a - 3b$ for $b$.
Solution:

$$\begin{aligned} A &= 2a-3b && \text{add } 3b \\ A+3b &= 2a && \text{subtract } A \\ 3b &= 2a-A && \text{divide by 3} \\ b &= \frac{2a-A}{3} \quad \text{or} \quad \frac{2a}{3}-\frac{A}{3} \quad \text{or} \quad \frac{2}{3}a-\frac{1}{3}A \end{aligned}$$

(c) $F = \dfrac{mMG}{d^2}$ for $m$.
Solution:

$$\begin{aligned} F &= \frac{mMG}{d^2} && \text{multiply by } d^2 \\ Fd^2 &= mMG && \text{divide by } MG \\ \frac{Fd^2}{MG} &= m \end{aligned}$$

(d) $3x - 5y = 60$ for $y$.
Solution:

$$\begin{aligned} 3x - 5y &= 60 && \text{add } 5y \\ 3x &= 5y + 60 && \text{subtract } 60 \\ 3x - 60 &= 5y && \text{divide by } 5 \\ \frac{3x-60}{5} &= y \end{aligned}$$

The answer is $y = \dfrac{3x-60}{5}$ or $\dfrac{3x}{5} - 12$ or $\dfrac{3}{5}x - 12$

7. Graph the straight lines $5x - 3y = 11$ and $y = -x - 9$ in the same coordinate system.

(a) Use your graph to find the coordinates of the point where the lines intersect.
Solution: We first graph the line $5x - 3y = 11$.

$$\begin{aligned} \text{If } x &= 1,\ y = ? && \text{substitute } x = 1 \text{ into the equation of the line} \\ 5(1) - 3y &= 11 && \text{solve for } y \\ 5 - 3y &= 11 && \text{subtract } 5 \\ -3y &= 6 && \text{divide by } -3 \\ y &= -2 && \Longrightarrow \text{ we found } (1,-2) \end{aligned}$$

$$\begin{aligned} \text{If } x &= 4,\ y = ? && \text{substitute } x = 4 \text{ into the equation of the line} \\ 5(4) - 3y &= 11 && \text{solve for } y \\ 20 - 3y &= 11 && \text{subtract } 20 \\ -3y &= -9 && \text{divide by } -3 \\ y &= 3 && \Longrightarrow \text{ we found } (4,3) \end{aligned}$$

$$\begin{aligned} \text{If } x &= 7,\ y = ? && \text{substitute } x = 7 \text{ into the equation of the line} \\ 5(7) - 3y &= 11 && \text{solve for } y \\ 35 - 3y &= 11 && \text{subtract } 35 \\ -3y &= -24 && \text{divide by } -3 \\ y &= 8 && \Longrightarrow \text{ we found } (7,8) \end{aligned}$$

$$\begin{aligned} \text{If } x &= -2,\ y = ? && \text{substitute } x = -2 \text{ into the equation of the line} \\ 5(-2) - 3y &= 11 && \text{solve for } y \\ -10 - 3y &= 11 && \text{add } 10 \\ -3y &= 21 && \text{divide by } -3 \\ y &= -7 && \Longrightarrow \text{ we found } (-2,-7) \end{aligned}$$

We graph the points $(1,-2)$, $(4,3)$, $(7,8)$, and $(-2,-7)$ and connect the points. (Green line.)

We now graph the line $y = -x - 9$.

$$\begin{aligned} \text{If } x &= 0, \ y = ? && \text{substitute } x = 0 \text{ into the equation of the line} \\ y &= -(0) - 9 = 0 - 9 = -9 && \Longrightarrow \text{ we found } (0, -9) \end{aligned}$$

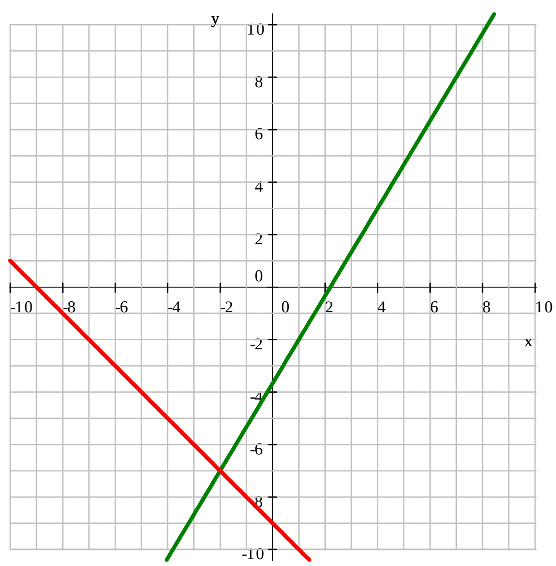
$$\begin{aligned} \text{If } x &= 2, \ y = ? && \text{substitute } x = 2 \text{ into the equation of the line} \\ y &= -(2) - 9 = -11 && \Longrightarrow \text{ we found } (2, -11) \end{aligned}$$

$$\begin{aligned} \text{If } x &= -2, \ y = ? && \text{substitute } x = -2 \text{ into the equation of the line} \\ y &= -(-2) - 9 = 2 - 9 = -7 && \Longrightarrow \text{ we found } (-2, -7) \end{aligned}$$

$$\begin{aligned} \text{If } x &= -4, \ y = ? && \text{substitute } x = -4 \text{ into the equation of the line} \\ y &= -(-4) - 9 = 4 - 9 = -5 && \Longrightarrow \text{ we found } (-4, -5) \end{aligned}$$

$$\begin{aligned} \text{If } x &= -7, \ y = ? && \text{substitute } x = -7 \text{ into the equation of the line} \\ y &= -(-7) - 9 = 7 - 9 = -2 && \Longrightarrow \text{ we found } (-7, -2) \end{aligned}$$

We graph the points $(0, -9)$, $(2, -11)$, $(-2, -7)$, $(-4, -5)$, and $(-7, -2)$ and connect the points. (Red line.)




We read from the graph that the lines intersect at $(-2, -7)$

(b) Use algebraic methods to check your answer for part a).
Solution: Is the point $(-2, -7)$ on the line $5x - 3y = 11$?

$$\text{LHS} = 5(-2) - 3(-7) = -10 - (-21) = -10 + 21 = 11 = \text{RHS} \quad \Longrightarrow \quad \text{yes}$$

Is the point $(-2, -7)$ on the line $y = -x - 9$?

$$\text{RHS} = -(-2) - 9 = 2 - 9 = -7 = \text{LHS} \quad \Longrightarrow \quad \text{yes}$$

8. Word Problems.

(a) A couch went on a 15% sale. The sale price is \$ 697. Find the original price.

Solution: A 15% sale means that we have to pay 85% of the original price. The question is: 85% of what number is 697?

(is) = 697

$F = \frac{85}{100}$

(of) $= x$

We substitute these values into the formula (is) $= F \cdot$ (of) and solve for $x$.

$$\begin{aligned} \text{(is)} &= F \cdot \text{(of)} \\ 697 &= \frac{85}{100} \cdot x && \text{divide bith sides by } \frac{85}{100} \\ 697 \div \frac{85}{100} &= x \\ x &= \frac{697}{1} \cdot \frac{100}{85} = 820 \end{aligned}$$

Thus the original price is \$ 820

(b) The difference between two numbers is 7, their sum is 37. Find these numbers.

Solution: Let us call the smaller number $x$. Then the larger number is $x+7$, since the difference between the two numbers is 7. The equation then expresses the sum of these numbers

$$\begin{aligned} \underbrace{x}_{\text{smaller number}} + \underbrace{x+7}_{\text{larger number}} &= 37 && \text{solve for } x \\ 2x+7 &= 37 && \text{subtract 7} \\ 2x &= 30 && \text{divide by 2} \\ x &= 15 \end{aligned}$$

Thus the smaller number, labeled $x$ is 15. The larger number was labeled $x+7$, so it must be $15+7=22$. Thus the numbers are 15 and 22. We check: the difference between 22 and 15 is $22-(15)=7$, and their sum is indeed $22+15=37$. Thus our solution is correct.

(c) Ann and Betty are roommates. The monthly rent is \$ 950. The amount paid by Ann is \$ 310 less than twice the amount paid by Betty. How much do they each pay for rent?

Solution: Let $x$ denote the amount that is paid by Betty. Then Ann must pay $2x-310$ per month. The equation expresses the monthly rent as the sum of the two payments

$$\begin{aligned} \underbrace{x}_{\text{smaller amount}} + \underbrace{2x-310}_{\text{larger amount}} &= 950 && \text{solve for } x \\ 3x-310 &= 950 && \text{add 310} \\ 2x &= 1260 && \text{divide by 3} \\ x &= 420 \end{aligned}$$

Thus the amount paid by Betty is \$ 420. The amount paid by Betty was labeled as $2x-310$, so Betty must pay $2(420)-310 = 840-310 = 530$. Thus the payments are \$ 420 and \$ 530. We check: \$ 530 = 2 (\$ 420) − \$ 310 and \$ 420 + \$ 530 = \$ 950. Thus our solution is correct.

(d) The population of a town has decreased from 90 000 to 82 800. What percent of a change does this represent?
Solution: The decrease in the population is $90\,000 - 82\,800 = 7200$. The question is then: 7200 is what percent of 90 000?
(is) $= 7200$
$F = x$
(of) $= 90\,000$
We substitute these values into the formula (is) $= F \cdot$ (of) and solve for $x$.

$$\begin{aligned} (\text{is}) &= F \cdot (\text{of}) \\ 7200 &= x \cdot 90\,000 && \text{divide bith sides by } 90\,000 \\ \frac{7200}{90\,000} &= x \\ x &= \frac{8}{100} = 8\% \end{aligned}$$

Thus the change in population is 8%.

(e) One side of a rectangle is 4 ft shorter than three times the other side. Find the sides if the perimeter is 64 ft.
Solution: Let us denote the shorter side by $x$. Then the other side is $3x - 4$. The equation expresses the perimeter of the rectangle.

$$\begin{aligned} 2(x) + 2(3x - 4) &= 64 && \text{multiply out parentheses} \\ 2x + 6x - 8 &= 64 && \text{combine like terms} \\ 8x - 8 &= 64 && \text{add} \\ 8x &= 72 && \text{divide by 8} \\ x &= 9 \end{aligned}$$

If the shorter side was denoted by $x$, we now know it is 9 ft. The longer side was denoted by $3x - 4$, so it must be $3(9 \text{ ft}) - 4 \text{ ft} = 23$ ft. Thus the sides of the rectangle are 9 ft and 23 ft. We check: $P = 2(9 \text{ ft}) + 2(23 \text{ ft}) = 64$ ft and $23 \text{ ft} = 3(9 \text{ ft}) - 4$ ft. Thus our solution is correct.

(f) One side of a rectangle is 4 ft shorter than three times the other side. Find the sides if the area is 84 $\text{ft}^2$.
Solution: Let us denote the shorter side by $x$. Then the other side is $3x - 4$. The equation expresses the area of the rectangle.

$$\begin{aligned} x(3x - 4) &= 84 && \text{multiply out parentheses} \\ 3x^2 - 4x &= 84 && \text{subtract 84} \\ 3x^2 - 4x - 84 &= 0 \end{aligned}$$

Because the equation is quadratic, we need to factor the left-hand side and then apply the zero property. We will factor by grouping. First we conduct the "$pq$−game". The sum of $p$ and $q$ has to be the linear coefficient (the number in front of $x$, with its sign), so it is $-4$. The product of $p$ and $q$ has to be the product of the other coefficients, $3(-84) = -252$.

$$\begin{aligned} pq &= -252 \\ p + q &= -4 \end{aligned}$$

Now we need to find $p$ and $q$. Because the product is negative, we re looking for a positive and a negative number. Becuse the sum is negtive, the larger number must carry the negative sign. We enter $\sqrt{252} = 15.874\,507\,87$ into the calculator and get a decimal:

$$\sqrt{252} = 15.874....$$

So we start looking for factors of 252, starting at 15, and moving down. We soon find 14 and $-18$. These are our values for $p$ and $q$. We use these numbers to express the linear term:

$$-4x = 14x - 18x$$

and factor by grouping.

$$\begin{aligned} 3x^2 - 4x - 84 &= 0 \\ \underbrace{3x^2 + 14x}\underbrace{-18x - 84} &= 0 \\ x\,(3x+14) - 6\,(3x+14) &= 0 \\ (x-6)\,(3x+14) &= 0 \end{aligned}$$

We now apply the zero property. Either $x - 11 = 0$ or $3x + 14 = 0$. We solve both these equations for $x$.

$$\begin{aligned} x - 6 &= 0 \\ x &= 6 \end{aligned}$$

and

$$\begin{aligned} 3x + 14 &= 0 \\ 3x &= -14 \\ x &= -\frac{14}{3} \end{aligned}$$

Since distances can not be negative, the second solution for $x$, $-\dfrac{14}{3}$ is ruled out. Thus $x = 6$. Then the longer side is $3\,(6\ \text{ft}) - 4\ \text{ft} = 14\ \text{ft}$, and so the rectangle's sides are 6 ft and 14 ft long. We check: $6\ \text{ft}\,(14\ \text{ft}) = 84\ \text{ft}^2$ and $14\ \text{ft} = 3\,(6\ \text{ft}) - 4\ \text{ft}$. Thus our solution is correct.