Lecture Notes for Class 1

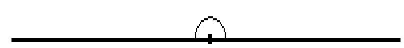
In mathematics, we will be dealing with different types of true statements. Some examples for these are definitions, axioms and theorems. What are these?

A **definition** is a type of statement in which we agree how we will refer to things. It is true in a sense because it just sets an agreement about labeling things.

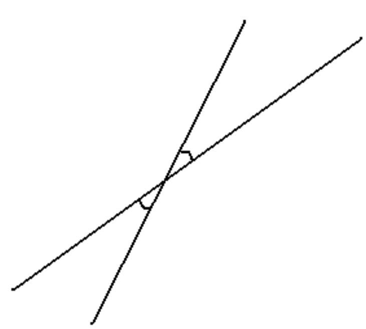
An **axiom** is a statement that we accept as true, without requiring proof of it. It usually agrees with our natural instincts and they "feel true". One example is the statement: "Two points uniquely determine a straight line".

A **theorem** is a statement that we prove to be true. But what does it mean to prove something? It means to derive it from the axioms. Mathematicians set down a set of basic 'truths', the axioms. Everything we prove, we derive them from the axioms. The following proof is a presentation of this. We will use two axioms to prove a theorem. Here is all we need:

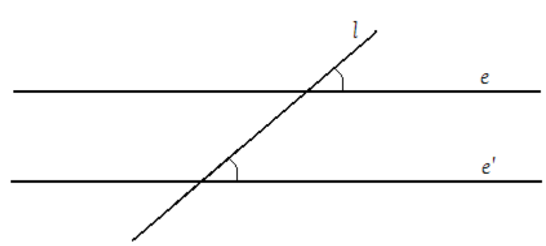
Definition 1 *The angle shown on the picture below is called the straight angle and it measures* $180°$.



Axiom 1 *If two straight lines intersect each other, then the two angles marked on the picture below have equal measures.*



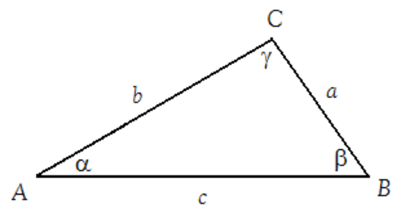
Axiom 2 *If* e *and* e' *are parallel lines, and* l *is a line intersecting these lines, then the two angles marked on the picture below have equal measures. We say that line* l *is a transversal for the lines* e *and* e'.



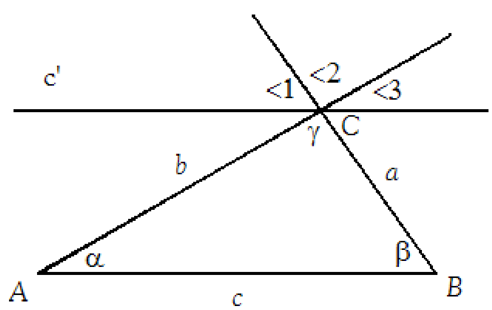
We are now ready to state and prove our first theorem.

Theorem 1 *If* α, β, *and* γ *are interior angles of any triangle, then* $\alpha + \beta + \gamma = 180°$.

Proof. Let ABC be any triangle. Let us denote the angles by α, β, and γ, and the sides by a, b, and c as shown on the picture below.



Let us draw the lines a, b, and c longer, beyond the triangle. As often times in geometry, proofs are based on a smartly drawn, single line. In this case, the 'magical line' that will give us the proof, is a line, we will call it c', that is parallel to c and passes throught the point C. There are three new angles formed, we will label them as $\angle 1$, $\angle 2$, and $\angle 3$, as shown on the picture below.



Because of Axiom 1, $\gamma = \angle 2$. Those two angles are vertical as the lines a and b intersect. Because a is a transversal for the parallel lines c and c', $\angle 1 = \beta$. Because b is a transversal for the parallel lines c and c', $\angle 3 = \alpha$. We can observe that $\angle 1 + \angle 2 + \angle 3 = 180°$, since they are forming a straight angle together. So we have:

$$\begin{aligned} \underbrace{\angle 1}_{\beta} + \underbrace{\angle 2}_{\gamma} + \underbrace{\angle 3}_{\alpha} &= 180° \\ \alpha + \beta + \gamma &= 180° \end{aligned}$$

This concludes our proof. ■