Mixture Problems - SOLUTIONS

1. How many gallons of 3% acid solution must be mixed with 60 gallons of 10% acid solution to obtain an acid solution that is 8%?

   Solution: Let us denote the amount of 3% solution we use. Then

	Amount of Solution (gallons)	Percentage	Amount of Solvant (gallons)
Component 1	x	0.03	$0.03x$
Component 2	60	0.1	$60\,(0.1)$
Mixture	$x+60$	0.08	$0.08\,(x+60) = 0.03x + 60\,(0.1)$

   We obtain the equation by stating that the amount of solvant in the components must add up to the amount of solvant. (In other words, the last entry in the third row can be written in two different ways: the product of $x+60$ and 8%; and the sum of $0.03x$ and $60\,(0.1)$)

$$\begin{aligned}
0.08\,(x+60) &= 0.03x + 60\,(0.1) \\
0.08\,(x+60) &= 0.03x + 6 && \text{multiply by 100 to make numbers 'nice'} \\
8\,(x+60) &= 3x + 600 && \text{distribute} \\
8x + 480 &= 3x + 600 && \text{subtract } 3x \\
5x + 480 &= 600 && \text{subtract 480} \\
5x &= 120 && \text{divide by 5} \\
x &= 24
\end{aligned}$$

   Thus we need to mix 24 gallons of 3% solution with 60 gallons of 10% solution.

   We check: suppose we mix the two solutions specified above. We need to find how much solution and how much solvant we have, hoping that the amount of solvant indeed will be 8% of the amount of mixture.

	Amount of Solution		Amount of Solvant
Component 1	60 gallons of 10% solution	$\rightarrow$	$0.1\,(60) = 6$ gallons
Component 2	24 gallons of 3% solution	$\rightarrow$	$0.03\,(24) = 0.72$ gallons
	$\Downarrow$		$\Downarrow$
Mixture	84 gallons		6.72 gallons

   8% of 84 is $0.08\,(84) = 6.72$. Thus our solution has the right concentration.

2. How many gallons of each of a 4% and an 11% salt solutions should be mixed to obtain 35 gallons of a 7% solution?

   Solution 1 (Application of Equations, 2.5) Let us denote by x the amount of 4% solution. Since we need to mix 35 gallons of a mixture, the amount of 11% solution must be $35 - x$ gallons.

	Amount of Solution (gallons)	Percentage	Amount of Solvant (gallons)
Component 1	x	0.04	$0.04x$
Component 2	$35 - x$	0.11	$0.11\,(35 - x)$
Mixture	35	0.07	

   We obtain the equation by stating that the amount of solvant in the components must add up to the amount of solvant. (In other words, the last entry in the third row can be written in two

different ways: the product of 35 and 7%; and the sum of $0.04x$ and $0.11(35-x)$)

$$\begin{aligned} 0.07(35) &= 0.04x + 0.11(35-x) \\ 2.45 &= 0.04x + 0.11(35-x) && \text{multiply by 100 to make numbers 'nice'} \\ 245 &= 4x + 11(35-x) && \text{distribute} \\ 245 &= 4x + 385 - 11x && \text{combine like terms} \\ 245 &= -7x + 385 && \text{subtract 385} \\ -140 &= -7x && \text{divide by } -7 \\ x &= 20 \end{aligned}$$

If $x = 20$, then the other amount, denoted by $35 - x$ must be $35 - 20 = 15$.

Thus we need to mix 20 gallons of 4% solution with 15 gallons of 11% solution.

We check: suppose we mix the two solutions specified above. We need to find how much solution and how much solvant we have, hoping that the amount of solvant indeed will be 8% of the amount of mixture.

	Amount of Solution			Amount of Solvant
Component 1	20 gallons	of 4% solution	$\Longrightarrow$	$0.04(20) = 0.8$ gallons
Component 2	15 gallons	of 11% solution	$\Longrightarrow$	$0.11(15) = 1.65$ gallons
	$\Downarrow$			$\Downarrow$
	35 gallons			$0.8 + 1.65 = 2.45$ gallons

7% of 35 is $0.07(35) = 2.45$ Thus our solution has the right concentration.

Solution 2 (Application of System of Equations, 3.6) Let us denote by x the amount of 4% solution, and by y the amount of 11% solution. Then

	Amount of Solution (gallons)	Percentage	Amount of Solvant (gallons)
Component 1	x	0.04	$0.04x$
Component 2	y	0.11	$0.11y$
Mixture	$x + y = 35$	0.07	$0.07(35) = 2.45$

We need to obtain two equations:

$$\begin{aligned} x + y &= 35 \\ 0.04x + 0.11y &= 2.45 \end{aligned}$$

We will solve the system by elimination. To make numbers nicer, first we multiply both sides of the second equation by 100.

$$\begin{aligned} x + y &= 35 \\ 4x + 11y &= 245 \end{aligned}$$

To eliminate x, we will multiply both sides of the first equation by -4 and then add the two equations.

$$\begin{aligned} -4x - 4y &= -140 \\ 4x + 11y &= 245 \end{aligned}$$

$$\begin{aligned} 7y &= 105 && \text{divide by 7} \\ y &= 15 \end{aligned}$$

Now that we know the value of y, we find x using the simplest equation:

$$\begin{aligned} x+y &= 35 && \text{we know } y=15 \\ x+15 &= 35 && \text{subtract 15} \\ x &= 20 \end{aligned}$$

Thus we need to mix 20 gallons of 4% solution with 15 gallons of 11% solution.

We check the same way as in solution 1.

3. How much water should we add to 20 gallons of 15% acid solution to dilute it to a concentration of 12%?

   Solution: The trick is to think of pure water as a 0% solution. The rest of the problem goes as usual. Let us denote the amount of 3% solution we use. Then

	Amount of Solution (gallons)	Percentage	Amount of Solvant (gallons)
Component 1	x	0	0
Component 2	20	0.15	$20\,(0.15)=3$
Mixture	$x+20$	0.12	$0.12\,(x+20)=20\,(0.15)$

   $0.12\,(x+20)=3$, Solution is: 5.0

$$\begin{aligned} 0.12\,(x+20) &= 20\,(0.15) \\ 0.12\,(x+20) &= 3 && \text{multiply by 100 to make numbers 'nice'} \\ 12\,(x+20) &= 300 && \text{distribute} \\ 12x+240 &= 300 && \text{subtract 240} \\ 12x &= 60 && \text{divide by 5} \\ x &= 5 \end{aligned}$$

   Thus we need to mix 5 gallons of water with 20 gallons of 15% solution.

   We check:

	Amount of Solution		Amount of Solvant
Component 1	20 gallons of 15% solution	$\rightarrow$	$0.15\,(20)=3$ gallons
Component 2	5 gallons of 0% solution	$\rightarrow$	0 gallons
	$\Downarrow$		$\Downarrow$
Mixture	25 gallons		3 gallons

   12% of 25 is $0.12\,(25)=3$ Thus our solution has the right concentration.