Math 99 # Sample Problems 1 Solutions

1. Perform each of the indicated operations.

   (a) $\sqrt{60 \div 4 \div 5 + 1} =$

   Solution: The square root sign stretching across other operations is a case of an "invisible parentheses." We will work out everything else and then take the square root of the result. We need to be careful; the two divisions have to be executed left to right.

$$\begin{aligned} \sqrt{60 \div 4 \div 5 + 1} &= \quad \text{division first, left to right!} \\ \sqrt{15 \div 5 + 1} &= \quad \text{division} \\ \sqrt{3+1} &= \quad \text{addition} \\ \sqrt{4} &= \quad \text{square root} \\ &= 2 \end{aligned}$$

   (b) $20 - |32 \div (-2)| =$

   Solution: We start with the absolute value sign since it also functions as parentheses. We work out what's inside and then take the absolute value of the result. This absolute value is what we subtract from 20.

$$\begin{aligned} 20 - |32 \div (-2)| &= \quad \text{division within absolute value sign} \\ 20 - |-16| &= \quad \text{absolute value of } -16 \text{ is } 16 \\ 20 - 16 &= \quad \text{subtraction} \\ &= 4 \end{aligned}$$

   (c) $\dfrac{-3^2 + 30 \div (-2)}{-4} =$

   Solution: The division bar stretching over an expression is a case of an "invisible parentheses." We completely work out the top and the bottom, and finally divide.

$$\begin{aligned} \frac{-3^2 + 30 \div (-2)}{-4} &= \quad \text{exponents} \\ \frac{-9 + 30 \div (-2)}{-4} &= \quad \text{division upstrairs} \\ \frac{-9 + (-15)}{-4} &= \quad \text{addition upstairs} \\ \frac{-24}{-4} &= \quad \text{division} \\ &= 6 \end{aligned}$$

   (d) $(9-6)^2 =$

   Solution: We start with the parentheses.

$$\begin{aligned} (9-6)^2 &= \quad \text{subtraction within parentheses} \\ (3)^2 &= \quad \text{drop parentheses} \\ 3^2 &= \quad \text{exponents} \\ &= 9 \end{aligned}$$

(e) $9^2 - 6^2 =$

Solution: This and the previous problem is here to remind you that $a^2 - b^2$ and $(a-b)^2$ are different! In this case there is no parentheses, and so we start with the exponents.

$$\begin{aligned} 9^2 - 6^2 &= \quad \text{exponents} \\ 81 - 36 &= \quad \text{subtraction} \\ &= 45 \end{aligned}$$

(f) $16 - \sqrt{2^3 - 2^2} - 5 =$

Solution: The square root sign stretching across other operations is a case of an "invisible parentheses." We start there.

$$\begin{aligned} 16 - \sqrt{2^3 - 2^2} - 5 &= \quad \text{exponents under square root} \\ 16 - \sqrt{8-4} - 5 &= \quad \text{subtraction under square root} \\ 16 - \sqrt{4} - 5 &= \quad \text{square root} \\ 16 - 2 - 5 &= \quad \text{subtractions, left to right} \\ 14 - 5 &= \\ &= 9 \end{aligned}$$

(g) $\sqrt{25} \cdot \dfrac{-3^5 + (-1)^3}{\left|(-5)^3 - \left|6 + (-5)^3\right|\right|} =$

Solution: There are four parentheses in this problem, although none visible. The division bar stretching over an expression is a case of an "invisible parentheses." We completely work out the top and the bottom, then divide. The nested pairs of absolute value signs also function as parentheses. We start with the innermost absolute value sign.

$$\begin{aligned} \sqrt{25} \cdot \frac{-3^5 + (-1)^3}{\left|(-5)^3 - \left|6 + (-5)^3\right|\right|} &= \quad \text{exponents within innermost absolute value sign} \\ \sqrt{25} \cdot \frac{-3^5 + (-1)^3}{\left|(-5)^3 - |6 + (-125)|\right|} &= \quad \text{addition within innermost absolute value sign} \\ \sqrt{25} \cdot \frac{-3^5 + (-1)^3}{\left|(-5)^3 - |-119|\right|} &= \quad \text{the absolute value of } -119 \text{ is } 119 \\ \sqrt{25} \cdot \frac{-3^5 + (-1)^3}{\left|(-5)^3 - 119\right|} &= \quad \text{exponents within innermost absolute value sign} \\ \sqrt{25} \cdot \frac{-3^5 + (-1)^3}{|-125 - 119|} &= \quad \text{subtraction within innermost absolute value sign} \\ \sqrt{25} \cdot \frac{-3^5 + (-1)^3}{|-244|} &= \quad \text{the absolute value of } -244 \text{ is } 244 \\ \sqrt{25} \cdot \frac{-3^5 + (-1)^3}{244} &= \quad \text{exponents on top} \end{aligned}$$

$$\sqrt{25}\cdot\frac{-243+(-1)}{244} = \text{addition on top}$$
$$\sqrt{25}\cdot\frac{-244}{244} = \text{division}$$
$$\sqrt{25}\cdot(-1) = \text{square root}$$
$$5\cdot(-1) = -5$$

(h) $\dfrac{3+\left(6^2-3^2\right)+\left(3^3-2^3\right)-9}{4^2+(5-3)^2} =$

Solution: The division bar stretching over an expression is a case of an "invisible parentheses" around the top and the bottom expressions. We completely work out the top and the bottom, and then divide. We start within the three parentheses:

$$\frac{3+\left(6^2-3^2\right)+\left(3^3-2^3\right)-9}{4^2+(5-3)^2} = \quad \text{first parentheses on the top}$$
$$\frac{3+(36-9)+\left(3^3-2^3\right)-9}{4^2+(5-3)^2} =$$
$$\frac{3+27+\left(3^3-2^3\right)-9}{4^2+(5-3)^2} = \quad \text{parentheses on the top}$$
$$\frac{3+27+(27-8)-9}{4^2+(5-3)^2} =$$
$$\frac{3+27+19-9}{4^2+(5-3)^2} = \quad \text{parentheses on the bottom}$$
$$\frac{3+27+19-9}{4^2+(2)^2} = \quad \text{drop parentheses}$$
$$\frac{3+27+19-9}{4^2+2^2} = \quad \text{exponents}$$
$$\frac{3+27+19-9}{16+4} = \quad \text{additions, left to right}$$
$$\frac{30+19-9}{20} = \quad \text{addition}$$
$$\frac{49-9}{20} = \quad \text{subtraction}$$
$$\frac{40}{20} = 2$$

(i) $\sqrt{\sqrt{3^6} - 2(3^2 + 3^3) + 2(4(17 - 2^3) - 3^2)} =$

Solution: The square root sign stretching across other operations is a case of an "invisible parentheses." We will work out everything else and then take the square root of the result. We start with the innermost parentheses.

$$\begin{aligned}
\sqrt{\sqrt{3^6} - 2(3^2 + 3^3) + 2(4(17 - 2^3) - 3^2)} &= && \text{exponent in first parentheses from left} \\
\sqrt{\sqrt{3^6} - 2(9 + 3^3) + 2(4(17 - 2^3) - 3^2)} &= && \text{exponent in first parentheses from left} \\
\sqrt{\sqrt{3^6} - 2(9 + 27) + 2(4(17 - 2^3) - 3^2)} &= && \text{addition in first parentheses from left} \\
\sqrt{\sqrt{3^6} - 2 \cdot 36 + 2(4(17 - 2^3) - 3^2)} &= && \text{exponent in innermost parentheses} \\
\sqrt{\sqrt{3^6} - 2 \cdot 36 + 2(4(17 - 8) - 3^2)} &= && \text{subtraction in innermost parentheses} \\
\sqrt{\sqrt{3^6} - 2 \cdot 36 + 2(4 \cdot 9 - 3^2)} &= && \text{exponent in parentheses} \\
\sqrt{\sqrt{3^6} - 2 \cdot 36 + 2(4 \cdot 9 - 9)} &= && \text{multiplication in parentheses} \\
\sqrt{\sqrt{3^6} - 2 \cdot 36 + 2(36 - 9)} &= && \text{subtraction in parentheses} \\
\sqrt{\sqrt{3^6} - 2 \cdot 36 + 2 \cdot 27} &= && \text{exponent within square root sign} \\
\sqrt{\sqrt{729} - 2 \cdot 36 + 2 \cdot 27} &= && \text{square root} \\
\sqrt{27 - 2 \cdot 36 + 2 \cdot 27} &= && \text{multiplications, left to right} \\
\sqrt{27 - 72 + 54} &= && \text{subtraction} \\
\sqrt{-45 + 54} &= && \text{addition} \\
\sqrt{9} &= && \text{square root} \\
&= 3
\end{aligned}$$

(j) $2 \cdot 3^2 - \left(6 - 4 + \left(6 \cdot 4 - 3\left(2^4 - 12\right)\right)\right) =$

Solution: We start with the innermost parentheses. Remember, every step we write down is a new, easier problem we have to solve now.

$$\begin{aligned}
2 \cdot 3^2 - \left(6 - 4 + \left(6 \cdot 4 - 3\left(2^4 - 12\right)\right)\right) &= && \text{exponents in innermost parentheses} \\
2 \cdot 3^2 - (6 - 4 + (6 \cdot 4 - 3(16 - 12))) &= && \text{subtraction in innermost parentheses} \\
2 \cdot 3^2 - (6 - 4 + (6 \cdot 4 - 3(4))) &= && \text{drop parentheses} \\
2 \cdot 3^2 - (6 - 4 + (6 \cdot 4 - 3 \cdot 4)) &= && \text{multiplication in innermost parentheses, left to right} \\
2 \cdot 3^2 - (6 - 4 + (24 - 12)) &= && \text{subtraction in innermost parentheses} \\
2 \cdot 3^2 - (6 - 4 + 12) &= && \text{addition and subtraction in parentheses, left to right} \\
2 \cdot 3^2 - (2 + 12) &= \\
2 \cdot 3^2 - 14 &= && \text{exponent} \\
2 \cdot 9 - 14 &= && \text{multiplication} \\
18 - 14 &= 4
\end{aligned}$$

2. Let $a = -4$, $b = 2$, and $x = -3$. Evaluate each of the following expressions.

(a) $a^2 - b^2 =$

Solution: First we re-write the expression with one change, we write little pairs of parentheses instead of the letters.

$$a^2 - b^2 = (\ \ )^2 - (\ \ )^2$$

We now write the values inside the parentheses. From here on this is an order of operations problem.

$$\begin{aligned} a^2 - b^2 &= (-4)^2 - (2)^2 && \text{drop extra parentheses} \\ &= (-4)^2 - 2^2 && \text{exponents} \\ &= 16 - 4 && \text{subtraction} \\ &= 12 \end{aligned}$$

(b) $(a-b)^2 =$

Solution: First we re-write the expression with one change, we write little pairs of parentheses instead of the letters.

$$(a-b)^2 = ((\ \ ) - (\ \ ))^2$$

We now write the values inside the parentheses. From here on this is an order of operations problem.

$$\begin{aligned} (a-b)^2 &= ((-4) - (2))^2 && \text{drop extra parentheses} \\ &= (-4-2)^2 && \text{subtraction in parentheses} \\ &= (-6)^2 && \text{exponent} \\ &= 36 \end{aligned}$$

This and the previous problem is here to remind you that $(a-b)^2$ and $a^2 - b^2$ are two different expressions.

(c) $a^b - 2bx + x - |2x| =$

Solution: First we re-write the expression with one modification only: we write little pairs of parentheses instead of the letters.

$$a^b - 2bx + x - |2x| = (\ \ )^{(\ \ )} - 2(\ \ )(\ \ ) + (\ \ ) - |2(\ \ )|$$

We now write the values inside the parentheses. From here on this is an order of operations problem.

$a^b - 2bx + x - |2x| =$

$$\begin{aligned} &= (\ \ )^{(\ \ )} - 2(\ \ )(\ \ ) + (\ \ ) - |2(\ \ )| \\ &= (-4)^{(2)} - 2(2)(-3) + (-3) - |2(-3)| && \text{drop extra parentheses} \\ &= (-4)^2 - 2(2)(-3) + (-3) - |2(-3)| && \text{multiplication within absolute value sign} \\ &= (-4)^2 - 2(2)(-3) + (-3) - |-6| && \text{the absolute value of } -6 \text{ is } 6 \\ &= (-4)^2 - 2(2)(-3) + (-3) - 6 && \text{exponent} \end{aligned}$$

$$
\begin{aligned}
&= 16-2\,(2)\,(-3)+(-3)-6 && \text{multiplication, left to right}\\
&= 16-4\,(-3)+(-3)-6 &&\\
&= 16-(-12)+(-3)-6 && \text{additions, subtractions, left to right}\\
&= 16+12+(-3)-6 &&\\
&= 28+(-3)-6 &&\\
&= 25-6 &&\\
&= 19 &&
\end{aligned}
$$

(d) $\dfrac{-x^2+(x+2)^2}{(x-1)} =$

Solution: First we re-write the expression with only one modification: we write little pairs of parentheses instead of the letters.

$$\frac{-x^2+(x+2)^2}{(x-1)} = \frac{-(\ \ )^2+((\ \ )+2)^2}{((\ \ )-1)}$$

We now write the values inside the parentheses. From here on this is an order of operations problem.

$$
\begin{aligned}
\frac{-x^2+(x+2)^2}{(x-1)} &= \frac{-(-3)^2+((-3)+2)^2}{((-3)-1)} && \text{drop parentheses}\\
&= \frac{-(-3)^2+(-3+2)^2}{(-3-1)} && \text{addition in parentheses upstairs}\\
&= \frac{-(-3)^2+(-1)^2}{(-3-1)} && \text{subtraction downstairs in parentheses}\\
&= \frac{-(-3)^2+(-1)^2}{(-4)} && \text{drop parentheses}\\
&= \frac{-(-3)^2+(-1)^2}{-4} && \text{exponents upstairs}\\
&= \frac{-9+1}{-4} && \text{addition}\\
&= \frac{-8}{-4} && \text{division}\\
&= 2 &&
\end{aligned}
$$

(e) $\dfrac{x-1}{x+3} =$

Solution: First we re-write the expression with only one modification: we write little pairs of parentheses instead of the letters.

$$\frac{x-1}{x+3} = \frac{(\ \ )-1}{(\ \ )+3}$$

We write the values inside the parentheses and evaluate the expression.

$$\frac{x-1}{x+3} = \frac{(-3)-1}{(-3)+3} = \frac{-4}{0} = \text{ undefined}$$

3. Simplify each of the following expressions. Show all steps.

(a) $(2a+b)-(a-b)=$

Solution: To subtract is to add the opposite.

$$\begin{aligned}(2a+b)-(a-b) &= (2a+b)+(-a+b) && \text{drop parentheses}\\ &= 2a+b+(-a)+b && \text{combine like terms}\\ &= a+2b\end{aligned}$$

(b) $4(2a-b)-3(2a-4b)=$

Solution: We apply the law of distributivity first:

$$\begin{aligned}&= 4(2a-b)-3(2a-4b)\\ &= 4(2a)-4b-3(2a)-3(-4b) && \text{simplify products}\\ &= 8a-4b-6a+12b && \text{combine like terms}\\ &= 2a+8b\end{aligned}$$

(c) $(2a-2b)+(b-2a)=$

Solution: Since we add two expressions, we can drop the parentheses and simply combine like terms.

$$(2a-2b)+(b-2a)=2a-2b+b-2a=-b$$

(d) $(2a-2b)-(b-a)=$

Solution: To subtract is to add the opposite.

$$\begin{aligned}(2a-2b)-(b-a) &= (2a-2b)+(-b+a) && \text{drop parentheses}\\ &= 2a-2b-b+a=3a-3b\end{aligned}$$

(e) $-(3a-2)-(1-4a)=$

Solution:

$$\begin{aligned}-(3a-2)-(1-4a) &= -1(3a-2)-(1-4a) && \text{multiplication}\\ &= (-3a+2)-(1-4a) && \text{to subtract is to add the opposite}\\ &= (-3a+2)+(-1+4a) && \text{drop parentheses}\\ &= -3a+2+(-1)+4a && \text{combine like terms}\\ &= a+1\end{aligned}$$