Math 99 | Sample Exam 1 | Solutions

To receive full credit, show all work.

1. Simplify each of the following expressions. Show all work.

(a) $\sqrt{(-1)^4 - 6\left(2^2 - (-3)^2\right) - (-1)^3 + 10 \div 5 \cdot 2} =$

Solution: We apply order of operations. Parentheses first.

$$\sqrt{(-1)^4 - 6\left(2^2 - (-3)^2\right) - (-1)^3 + 10 \div 5 \cdot 2} = \sqrt{(-1)^4 - 6(4-9) - (-1)^3 + 10 \div 5 \cdot 2} =$$
$$= \sqrt{(-1)^4 - 6(-5) + - (-1)^3 + 10 \div 5 \cdot 2}$$

Now exponents:

$$\sqrt{(-1)^4 - 6(-5) + - (-1)^3 + 10 \div 5 \cdot 2} = \sqrt{1 - 6(-5) - (-1) + 10 \div 5 \cdot 2}$$

Now multiplications, divisions, left to right.

$$\sqrt{1 - 6(-5) - (-1) + 10 \div 5 \cdot 2} = \sqrt{1 - (-30) - (-1) + 2 \cdot 2} = \sqrt{1 - (-30) - (-1) + 4}$$

Now additions, subtractions, left to right.

$$\sqrt{1 - (-30) - (-1) + 4} = \sqrt{1 + 30 - (-1) + 4} = \sqrt{31 - (-1) + 4} = \sqrt{31 + 1 + 4} = \sqrt{32 + 4} = \sqrt{36}$$

Now we take the square root. (The fact that this operation is last is because the long square root is a case of an "invisible parentheses")

$$\sqrt{36} = 6$$

Thus the solution is 6.

(b) $\dfrac{-3^2 - (-3)^2 - 16 \div (-2) \cdot (-2) + (-2)^2}{|(-4)(-7) - (-2)|} = -1$

Solution: We apply order of operations. The big bar is an "invisible parentheses". It means that we have to completely work out the numerator and the denominator and then apply the division. In the numerator, there is no parentheses, so we start with exponents, left to right. Notice that $-3^2 = -9$ and not 9.

$$-3^2 - (-3)^2 - 16 \div (-2) \cdot (-2) + (-2)^2 = -9 - 9 - 16 \div (-2) \cdot (-2) + 4$$

We now perform all multiplications and divisions, left to right.

$$-9 - 9 - 16 \div (-2) \cdot (-2) + 4 = -9 - 9 - (-8) \cdot (-2) + 4 = -9 - 9 - 16 + 4$$

Now we perform all additions, subtractins, left to right.

$$-9 - 9 - 16 + 4 = -18 - 16 + 4 = -34 + 4 = -30$$

Now the denominator. It has a parentheses, since the absolute value sign also functions as parentheses. We start with the multiplication.

$$|(-4)(-7) - (-2)| = |28 - (-2)| = |28 + 2| = |30| = 30$$

The answer is thus $\dfrac{-30}{30} = -1$

(c) $\dfrac{(-1)^2-\left(-\frac{1}{2}\right)^2}{\left(5\frac{5}{8}\right)}+\dfrac{1}{5}=\dfrac{1}{3}$

Solution: We apply order of operations. We will keep all negative signs in the denominator. We start with exponents, left to right. Every step will be shown.

$$\frac{(-1)^2-\left(\frac{-1}{2}\right)^2}{\left(5\frac{5}{8}\right)}+\frac{1}{5} = \qquad (-1)^2=-1(-1)=1$$

$$\frac{1-\left(\frac{-1}{2}\right)^2}{\left(5\frac{5}{8}\right)}+\frac{1}{5} = \qquad \left(\frac{-1}{2}\right)^2=\frac{-1}{2}\left(\frac{-1}{2}\right)=\frac{1}{4}$$

$$= \frac{1-\frac{1}{4}}{\left(5\frac{5}{8}\right)}+\frac{1}{5}$$

Subtraction in the denominator is next.

$$\frac{1-\frac{1}{4}}{\left(5\frac{5}{8}\right)}+\frac{1}{5} = \qquad 1-\frac{1}{4}=\frac{1}{1}-\frac{1}{4}=\frac{4}{4}-\frac{1}{4}=\frac{3}{4}$$

$$= \frac{\frac{3}{4}}{\left(5\frac{5}{8}\right)}+\frac{1}{5}$$

Converting the mixed number to improper fraction is an addition:

$$\frac{\frac{3}{4}}{\left(5\frac{5}{8}\right)}+\frac{1}{5} = \qquad 5+\frac{5}{8}=\frac{5}{1}+\frac{5}{8}=\frac{40}{8}+\frac{5}{8}=\frac{45}{8}$$

$$= \frac{\frac{3}{4}}{\frac{45}{8}}+\frac{1}{5}$$

Division and then finally the addition:

$$\frac{\frac{3}{4}}{\frac{45}{8}}+\frac{1}{5} = \qquad \frac{\frac{3}{4}}{\frac{45}{8}}=\frac{3}{4}\cdot\frac{8}{45}=\frac{24}{180}=\frac{2}{15}$$

$$= \frac{2}{15}+\frac{1}{5}=\frac{2}{15}+\frac{3}{15}=\frac{5}{15}=\frac{1}{3}$$

And so the answer is $\frac{1}{5}$.

2. Evaluate $\dfrac{3ab+2a^2-2b^2}{a+2b}$ if

(a) $a=2$ and $b=-3$

Solution: We need to plug in $a=2$ and $b=-3$ into the expression given and then evaluate it by applying order of operations.

$$\begin{aligned}\frac{3ab+2a^2-2b^2}{a+2b} &= \frac{3(2)(-3)+2(2)^2-2(-3)^2}{(2)+2(-3)} = \qquad \text{exponents}\\ &= \frac{3(2)(-3)+2\cdot 4-2\cdot 9}{(2)+2(-3)}\end{aligned}$$

Now we perform all multiplications and divisions, left to right

$$\begin{aligned}\frac{3(2)(-3)+2\cdot 4-2\cdot 9}{(2)+2(-3)} &= \frac{6(-3)+2\cdot 4-2\cdot 9}{(2)+2(-3)} = \frac{-18+2\cdot 4-2\cdot 9}{(2)+2(-3)} = \\ &= \frac{-18+8-2\cdot 9}{(2)+2(-3)} = \frac{-18+8-18}{(2)+2(-3)} = \frac{-18+8-18}{(2)+(-6)}\end{aligned}$$

Now we perform all additions and subtractions, left to right. Finally we divide.

$$\frac{-18+8-18}{(2)+(-6)} = \frac{-10-18}{(2)+(-6)} = \frac{-28}{(2)+(-6)} = \frac{-28}{-4} = 7$$

Thus the answer is 7.

(b) $a=-1$ and $b=-2$.

Solution: We need to plug in $a=-1$ and $b=-2$ into the expression given and then evaluate it by applying order of operations. Since there is no parentheses, we start with exponents.

$$\frac{3ab+2a^2-2b^2}{a+2b} = \frac{3(-1)(-2)+2(-1)^2-2(-2)^2}{(-1)+2(-2)} = \frac{3(-1)(-2)+2\cdot 1-2\cdot 4}{(-1)+2(-2)}$$

Now we perform all multiplications and divisions, left to right (we individually show them)

$$\begin{aligned}\frac{3(-1)(-2)+2\cdot 1-2\cdot 4}{(-1)+2(-2)} &= \frac{(-3)(-2)+2\cdot 1-2\cdot 4}{(-1)+2(-2)} = \frac{6+2\cdot 1-2\cdot 4}{(-1)+2(-2)} = \\ &= \frac{6+2-2\cdot 4}{(-1)+2(-2)} = \frac{6+2-8}{(-1)+2(-2)} = \frac{6+2-8}{(-1)+(-4)}\end{aligned}$$

Now we perform all additions and subtractions, left to right. Finally we divide.

$$\frac{6+2-8}{(-1)+(-4)} = \frac{8-8}{-1+(-4)} = \frac{0}{-1+(-4)} = \frac{0}{-5} = 0$$

Thus the answer is 0.

(c) $a=-6$ and $b=3$.

Solution: We need to plug in $a=-6$ and $b=3$ into the expression given and then evaluate it by applying order of operations. Since there is no parentheses, we start with exponents.

$$\frac{3ab+2a^2-2b^2}{a+2b} = \frac{3(-6)(3)+2(-6)^2-2(3)^2}{(-6)+2(3)} = \frac{3(-6)(3)+2\cdot 36-2\cdot 9}{(-6)+2(3)}$$

Now we perform all multiplications and divisions, left to right (we individually show them)

$$\frac{3(-6)(3)+2\cdot 36-2\cdot 9}{(-6)+2(3)} = \frac{(-18)(3)+2\cdot 36-2\cdot 9}{(-6)+2(3)} = \frac{-54+2\cdot 36-2\cdot 9}{(-6)+2(3)} = $$
$$= \frac{-54+72-2\cdot 9}{(-6)+2(3)} = \frac{-54+72-18}{(-6)+2(3)} = \frac{-54+72-18}{(-6)+6}$$

Now we perform all additions and subtractions, left to right. Finally we divide (IF WE CAN).

$$\frac{-54+72-18}{(-6)+6} = \frac{18-18}{(-6)+6} = \frac{0}{0} = \text{undefined}$$

since division by 0 is not allowed. The answer is: undefined

(d) $a=-\frac{1}{2}$ and $b=\frac{3}{4}$

Solution: We need to plug in $a=-\frac{1}{2}$ and $b=\frac{3}{4}$ into the expression given and then evaluate it by applying order of operations.

$$\frac{3ab+2a^2-2b^2}{a+2b} = \frac{3\left(-\frac{1}{2}\right)\left(\frac{3}{4}\right)+2\left(-\frac{1}{2}\right)^2-2\left(\frac{3}{4}\right)^2}{\left(-\frac{1}{2}\right)+2\left(\frac{3}{4}\right)}$$

Since there is no parentheses, we start with exponents. We proceed left to right. Keep the negative signs in the numerator.

$$\frac{3\left(\frac{-1}{2}\right)\left(\frac{3}{4}\right)+2\left(\frac{-1}{2}\right)^2-2\left(\frac{3}{4}\right)^2}{\left(\frac{-1}{2}\right)+2\left(\frac{3}{4}\right)} = \qquad \left(\frac{-1}{2}\right)^2 = \left(\frac{-1}{2}\right)\left(\frac{-1}{2}\right) = \frac{1}{4}$$

$$\frac{3\left(\frac{-1}{2}\right)\left(\frac{3}{4}\right)+2\left(\frac{1}{4}\right)-2\left(\frac{3}{4}\right)^2}{\left(\frac{-1}{2}\right)+2\left(\frac{3}{4}\right)} = \qquad \left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = \frac{9}{16}$$

$$= \frac{3\left(\frac{-1}{2}\right)\left(\frac{3}{4}\right)+2\left(\frac{1}{4}\right)-2\left(\frac{9}{16}\right)}{\left(\frac{-1}{2}\right)+2\left(\frac{3}{4}\right)}$$

Now we perform all multiplications and divisions, left to right (we individually show them)

$$\frac{3\left(\frac{-1}{2}\right)\left(\frac{3}{4}\right)+2\left(\frac{1}{4}\right)-2\left(\frac{9}{16}\right)}{\left(\frac{-1}{2}\right)+2\left(\frac{3}{4}\right)} = \qquad 3\left(-\frac{1}{2}\right) = \frac{3}{1}\cdot\frac{-1}{2} = \frac{-3}{2}$$

$$\frac{\frac{-3}{2}\left(\frac{3}{4}\right)+2\left(\frac{1}{4}\right)-2\left(\frac{9}{16}\right)}{\left(\frac{-1}{2}\right)+2\left(\frac{3}{4}\right)} =$$

$$\frac{\frac{-3}{2}\left(\frac{3}{4}\right)+2\left(\frac{1}{4}\right)-2\left(\frac{9}{16}\right)}{\left(\frac{-1}{2}\right)+2\left(\frac{3}{4}\right)} = \qquad \frac{-3}{2}\left(\frac{3}{4}\right)=\frac{-9}{8}$$

$$\frac{\frac{-9}{8}+2\left(\frac{1}{4}\right)-2\left(\frac{9}{16}\right)}{\left(\frac{-1}{2}\right)+2\left(\frac{3}{4}\right)} = \qquad 2\left(\frac{1}{4}\right)=\frac{2}{1}\cdot\frac{1}{4}=\frac{2}{4}=\frac{1}{2}$$

$$\frac{\frac{-9}{8}+\frac{1}{2}-2\left(\frac{9}{16}\right)}{\left(\frac{-1}{2}\right)+2\left(\frac{3}{4}\right)} = \qquad 2\left(\frac{9}{16}\right)=\frac{2}{1}\cdot\frac{9}{16}=\frac{18}{16}=\frac{9}{8}$$

$$\frac{\frac{-9}{8}+\frac{1}{2}-\frac{9}{8}}{\left(\frac{-1}{2}\right)+2\left(\frac{3}{4}\right)} = \qquad 2\left(\frac{3}{4}\right)=\frac{2}{1}\cdot\frac{3}{4}=\frac{6}{4}=\frac{3}{2}$$

$$= \frac{\frac{-9}{8}+\frac{1}{2}-\frac{9}{8}}{\left(\frac{-1}{2}\right)+\frac{3}{2}}$$

Now we perform all additions and subtractions, left to right.

$$\frac{\frac{-9}{8}+\frac{1}{2}-\frac{9}{8}}{\left(\frac{-1}{2}\right)+\frac{3}{2}} = \qquad \frac{-9}{8}+\frac{1}{2}=\frac{-9}{8}+\frac{4}{8}=\frac{-9+4}{8}=\frac{-5}{8}$$

$$\frac{\frac{-5}{8}-\frac{9}{8}}{\left(\frac{-1}{2}\right)+\frac{3}{2}} = \qquad \frac{-5}{8}-\frac{9}{8}=\frac{-5-9}{8}=\frac{-14}{8}=\frac{-7}{4}$$

$$\frac{\frac{-7}{4}}{\left(\frac{-1}{2}\right)+\frac{3}{2}} = \qquad \left(\frac{-1}{2}\right)+\frac{3}{2}=\frac{-1+3}{2}=\frac{2}{2}=1$$

$$= \frac{\left(\frac{-7}{4}\right)}{1}=-\frac{7}{4}$$

3. Solve each of the following equations. Make sure to check your solutions.

(a) $8(x-3)-3(5-2x)=x$
Solution:

$$\begin{aligned} 8(x-3)-3(5-2x) &= x && \text{multiply out parentheses} \\ 8x-24-15+6x &= x && \text{combine like terms} \\ 14x-39 &= x && \text{subtract } x \\ 13x-39 &= 0 && \text{add 39} \\ 13x &= 39 && \text{divide by 13} \\ x &= 3 \end{aligned}$$

We check our solution by evaluating the left hand side and the right hand side of the original equation with $x=3$.

$$\begin{aligned} \text{RHS} &= 8(3-3)-3(5-2(3)) = 8\cdot 0-3(5-6) = 8\cdot 0-3(-1) = \\ &= 0-(-3) = 3 \\ \text{LHS} &= 3 \end{aligned}$$

Since the left-had side equals to the right-hand side, our solution $x=3$ is correct.

(b) $\dfrac{3x-1}{4}+\dfrac{8-4x}{3}=-3-x$
Solution:

$$\begin{aligned} \frac{3x-1}{4}+\frac{8-4x}{3} &= -3-x && \text{we express the left hand side as a fraction} \\ \frac{3x-1}{4}+\frac{8-4x}{3} &= \frac{-3-x}{1} && \text{express fractions with common denominator} \\ \frac{3(3x-1)}{12}+\frac{4(8-4x)}{12} &= \frac{12(-3-x)}{12} && \text{multiply both sides by 12} \\ 3(3x-1)+4(8-4x) &= 12(-3-x) && \text{mulitply out parentheses (distribute)} \\ 9x-3+32-16x &= -36-12x && \text{combine like terms} \\ -7x+29 &= -12x-36 && \text{add } 12x \text{ to both sides} \\ 5x+29 &= -36 && \text{subtract 29 from both sides} \\ 5x &= -65 && \text{divide by 5} \\ x &= -13 \end{aligned}$$

We check our solution by evaluating the left hand side and the right hand side of the original equation with $x=-13$.

$$\begin{aligned} \text{RHS} &= \frac{3(-13)-1}{4}+\frac{8-4(-13)}{3} = \frac{-39-1}{4}+\frac{8-(-52)}{3} = \frac{-40}{4}+\frac{8+52}{3} = \\ &= \frac{-40}{4}+\frac{60}{3} = -10+20 = 10 \\ \text{LHS} &= -3-(-13) = -3+13 = 10 \end{aligned}$$

Since the left-had side equals to the right-hand side, our solution $x=-13$ is correct.

(c) $\frac{3x-2}{5}+\frac{x+4}{3}=\frac{14(x+1)}{15}$
Solution:

$$\begin{aligned} \frac{3x-2}{5}+\frac{x+4}{3} &= \frac{14(x+1)}{15} && \text{express fractions with common denominator} \\ \frac{3(3x-2)}{15}+\frac{5(x+4)}{15} &= \frac{14(x+1)}{15} && \text{multiply both sides by 15} \\ 3(3x-2)+5(x+4) &= 14(x+1) && \text{mulitply out parentheses (distribute)} \\ 9x-6+5x+20 &= 14x+14 && \text{combine like terms} \\ 14x+14 &= 14x+14 \end{aligned}$$

Since the left hand side and the right hand side are identical, every number will work if substituted. Thus this is an identity, all numbers are solution(s).

(d) $\frac{3}{8}x+\left(1\frac{4}{5}\right)=\frac{3}{10}$
Solution: this is a very simple equation, much like $2x+1=7$, only the numbers are fractions. But the principles and operations regarding equations are the same.

$$\begin{aligned} \frac{3}{8}x+\left(1\frac{4}{5}\right) &= \frac{3}{10} && \text{convert mixed number to improper fraction} \\ \frac{3}{8}x+\frac{9}{5} &= \frac{3}{10} && \text{subtract } \frac{9}{5} \text{ from both sides; } \frac{3}{10}-\frac{9}{5}=\frac{3}{10}-\frac{18}{10}=\frac{3-18}{10}=\frac{-15}{10}=\frac{-3}{2} \\ \frac{3}{8}x &= \frac{-3}{2} && \text{divide both sides by } \frac{3}{8} \\ x &= -4 && \left(\frac{-3}{2}\right)\div\left(\frac{3}{8}\right)=\frac{-3}{2}\cdot\frac{8}{3}=\frac{-24}{6}=-4 \end{aligned}$$

We check:

$$\begin{aligned} \text{RHS} &= \frac{3}{8}(-4)+\left(1\frac{4}{5}\right)=\frac{3}{8}\cdot\frac{-4}{1}+\frac{9}{5}=\frac{-12}{8}+\frac{9}{5}=\frac{-3}{2}+\frac{9}{5}=\frac{-15}{10}+\frac{18}{10}=\frac{3}{10} \\ \text{LHS} &= \frac{3}{10} \end{aligned}$$

Thus our solution, -4 is correct.

(e) $(x-3)(5x+1)=0$
We apply the special 0 property. If a product of two numbers is 0, one of the factors must be 0. Furthermore, once we have guarantee that a factor is 0, the value of the other factors doesn't matter.

$$(x-3)(5x+1)=0 \text{ means that either } x-3=0 \quad \text{or} \quad 5x+1=0$$

We solve these equations separately:

$$\begin{aligned} x-3 &= 0 && \text{add 3 to both sides} \\ x &= 3 \end{aligned}$$

and

$$\begin{aligned} 5x+1 &= 0 && \text{subtract 1 from both sides} \\ 5x &= -1 && \text{divide by 5} \\ x &= -\frac{1}{5} \end{aligned}$$

We check both solutions. If $x = 3$, then

$$\begin{aligned} \text{LHS} &= (3-3)(5(3)+1) = 0 \cdot 16 = 0 \\ \text{RHS} &= 0 \end{aligned}$$

And if $x = -\frac{1}{5}$, then

$$\begin{aligned} \text{LHS} &= \left(-\frac{1}{5} - 3\right)\left(5\left(-\frac{1}{5}\right) + 1\right) = \left(-\frac{1}{5} - \frac{15}{5}\right)(-1+1) = -\frac{16}{5} \cdot 0 = 0 \\ \text{RHS} &= 0 \end{aligned}$$

Thus the solution is: 3 and $-\frac{1}{5}$.

4. Find the average of 55, 98, −20, −90, −15, and −34.

   Solution: To find the average, we need to add all the numbers first. We divide this sum by the number of numbers.

$$Av = \frac{55 + 98 - 20 - 90 - 15 - 34}{6} = \frac{-6}{6} = -1$$